PREFACE.

The readiness with which our "Algebra Made Simple," was received by the teachers in our schools and the success that we achieved in that book prompted us to write this volume too.

We do not claim anything new or original in the book but *the system of grading almost the whole lot of examples and the method of solving them will be found by the teachers to be interesting and unique.* The examples in each Chapter have been very carefully selected and then graded and we have freely used simple algebraical artifices to solve some of them which would have been otherwise difficult to do.

Throughout the book will be found a very large number of examples ; most of them, with the exception of a few at the end of some of exercises and the three miscellaneous exercises, are rather of an easy type. We have made each Chapter of the book complete as far as it goes and hence some of the elementary portion of the book will be found to be rather difficult to students reading in the lower classes.

Many examples, especially those given in the Miscellaneous sets, have been taken from various Indian and foreign University papers, some of which have been appended, intact, at the end of the book only to show the students the nature of the questions set at University examinations.

We can boldly assert that there is no other book which contains all that is required from students preparing for the Matriculation Examination. We have explained all theorems as simply and methodically as possible.

In spite of our desire and our best efforts in that respect, we could not make the volume smaller but the teachers using the book in the class, might here and there omit the detailed descriptions of certain portions to save time.

We spared no pains to make the printing and the general get-up of the book as best as we could but we are painfully conscious that there is yet much room for improvement and

we intend to do the same in the next edition of the book, if the necessity for it ever arises.

It is not impossible that there might have crept in some typographical errors here and there and we shall be greatly obliged if those, who use the book would kindly communicate to us any such mistake they might come across.

In conclusion, we shall feel our labours amply rewarded if those for whom the book has been written derive any the least benefit from it.

LAHORE : K. M. G.
10 February, 1930 L. R. D.

Preface to the twentieth Edition.

In this edition, the book has been thoroughly revised. All the examples have been worked anew to ensure accuracy in the examples and the answers. We hope that no error now remains in this edition.

We render our hearty thanks to the teachers of Mathematics in the Province and outside who pointed out errors and communicated suggestions for the improvement of this book.

LAHORE : L. R. D.
March 15, 1947

Hindi Edition of this unique book will be avilable from May 1949.

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Arithmetic Made Easy

Note. Since this book is meant for secondary schools, the first three chapters have, therefore, been written briefly.

CHAPTER I

NUMERATION AND NOTATION

§ 1. Arithmetic may be regarded as a science of numbers but it has a theoretical and also a practical side. In its theoretical part it deals with numbers and enunciates general principles obtained from an enquiry into their properties. In its practical part, it concerns itself with the application of these principles to specific numbers for practical purposes. While a knowledge of the theory is essential for a clear understanding of the subject, an intelligent grasp of the principles underlying the solution of a problem or involved in the *modus operandi* of it or in deducing a formula to economise time and save labour is also important. Accuracy and rapidity are the principal objects to be borne in mind in actual practice.

§ 2. The digits used in Arithmetic are the following :—

0,	1,	2,	3,	4,	5,	6,	7,	8,	9.
zero,	one,	two,	three,	four,	five,	six,	seven,	eight,	nine.

Out of these, the figures 1 to 9 are called **significant** digits as distinguished from 0 which is called **insignificant** digit. The fundamental digit is 1 which is often called a unit or unity meaning a single object; the other digits 2, 3, 4, etc., therefore, mean 2, 3, 4, etc., units respectively. Various numbers may be formed with these digits by repeating one or more of them any number of times or by using different digits and their repetitions.

These digits have also another value. Thus in 27, the value of the digit 2 is not two but it means 2 tens, *i.e.*, twenty. Hence every digit has a **local value** and an **intrinsic value.** In 33707 the *local value* of the digits 3, 3, 7, 0, 7 are 30000 (*viz.* 3 ten thousands), 3000 (*viz.* 3 thousands), 700 (*viz.* 7

hundreds), 0 tens and 7 units respectively ; while the *intrinsic values* of 3, 7, 0, are 3 units, 7 units and nothing. 0 is a digit having no digit value but it increases the local value of each digit to its left and does not affect the value of any digit succeeding it. The others are significant digits. In any number, the first significant digit is the one which has the highest local value.

§ 3. The above is the Arabic System of Notation. The other system, *viz.* the Roman System of Notation is as follows :—

I.=1, V=5, X=10, L=50, C=100, D=500, M=1,000

I= 1	XI= 11	XXX= 30	CCC= 300
II= 2	XII= 12	XL= 40	CD= 400
III= 3	XIII= 13	L= 50	D= 500
IV= 4	XIV= 14	LX= 60	DC= 600
V= 5	XV= 15	LXX= 70	DCC= 700
VI= 6	.....	LXXX= 80	DCCC= 800
VII= 7	and so on until	XC= 90	CM= 900
VIII= 8	we come to	C=100	LM= 950
IX= 9	XX= 20	CC=200	M=1000
X= 10			ML=1050

☞ It is clear from the above that I or X or L or C when placed to the right of a bigger number indicates addition and when placed to the left denotes subtraction. A bar placed over a number multiplies its value by one thousand. Thus DC=600, and $\overline{DC}$=600,000.

§4. The Indian System is like the Arabic System up to ten thousand [see §7]. But in this system hundred thousand is called a lac and hundred lacs a **crore.**

§5. **Abstract** and **Concrete** numbers. **Two** books or **Three** rupees is said to be a concrete quantity because the number tells us not only *how many* things are taken but also of *what kind.* If there is no mention of the kind, the number is said to be abstract, *e.g.* 2, 3.

The numbers 1, 2, 3,.....are called **natural** numbers. The alternate numbers 1, 3, 5.....are called **odd** numbers and the alternate numbers. 2, 4, 6.....are called **even** numbers. Two

or more numbers taken successively in any series are said to be **consecutive**, as 57, 58, 59 or 103, 104, 105, 106, etc.

§ 6. The method of expressing numbers by means of the symbols or figures is called **Notation** and the art of reading in words a number expressed by figures is called **Numeration**, which is, therefore, the *reverse* of Notation.

§ 7. The follwing scheme, known as the **Numeration Table** will show how to read a number in words:—

18.	17.	16.	15.	14.	13.	12.	11.	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.
Hundreds of thousands of billions.	Tens of thousands of billions.	Thousands of billions.	Hundreds of billions	Tens of billions.	**Billions.**	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	**Millions.**	Hundreds of thousands.	Tens of thousands.	**Thousands.**	Hundreds.	Tens.	**Units.**

Note. A million billion is called **trillion**, a million trillion is called **quadrillion**. Similarly we have **quintillion, sextillion, septillion, octillion, nonillion.**

Solved Examples

Example 1. Express the following numbers in words:—

(*a*) 35000478 (*b*) 760829145 (*c*) 31200780020.

Rule. *Put a comma after the first 3 figures from right, another comma after next three figures, then a comma after every 6 figures respectively. Now the first figure in the first period stands for the unit's place, the second figure for ten's place, the third figure for the hundred's place. The 3 digits*

in the next period will give so many thousands and the next period of six will give so many millions, next billions and so on. Hence

(*a*) 35000478 =35,000, 478
=Thirty five million, four hundred and seventy-eight. **Ans.**

(*b*) 760829145 =760, 829, 145
=Seven hundred and sixty million, eight hundred and twenty-nine thousand, one hundred and forty-five. **Ans.**

(*c*) 31200780020 =31200,780, 020
=Thirty-one thousand and two hundred million, seven hundred and eighty thousand and twenty. **Ans.**

Example 2. Express the following in figures :—

(*a*) Four hundred and six million, seven thousand and six.

(*b*) Four hundred thousand million and twenty-five.

(*c*) Six trillion, two hundred billion, twenty-seven thousand million, five hundred thousand and nine.

These questions are the reverse of the first question. Hence

(*a*) Four hundred and six million=406,000,000
Seven thousand and six = 7,006.
Now write at once as 406007006. **Ans.**

(*b*) Four hundred thousand millions = 400000,000,000
Twenty-five = 25
Now write at once as 400000000025 **Ans.**

(*c*) Six trillion =6,000000,000000,000,000
Two hundred billion = 200,000000,000,000
Twenty-seven thousand million = 27000,000,000
Five hundred thousand = 500,000
Nine 9
Now write at once as 6000200027000500009 **Ans.**

Example 3. Write down the local value of each digit in the following numbers :—

(*a*) 1213456 (*b*) 4446607 (*c*) 37960012456

(*a*) The value of 1 = 1,000,000= one million
" " " 2 = 200,000=two hundred thousand
" " " 1 = 10,000=ten thousand
" " " 3 = 3,000=three thousand
" " " 4 = 400=four hundred
" " " 5 = 50=fifty.
" " " 6 = 6=six

(*b*) The value of first 4 = 4,000,000 = Four million
" " next 4 = 400,000 = four hundred thousand
" " " 4 = 40,000 = forty thousand
" " first 6 = 6,000 = six thousand
" " next 6 = 600 = six hundred
" " 7 = 7 = seven.

(*c*) The value of 3=30000,000,000 = Thirty thousand million
" " " 7= 7000,000,000 = seven thousand million
" " " 9= 900,000,000 = nine hundred million
" " " 6= 60,000,000 = sixty million
" " " 1= 10,000 = ten thousand
" " " 2= 2,000 = two thousand
" " " 4= 400 = four hundred
" " " 5= 50 = fifty
" " " 6= 6 = six

Example 4. Express the following numbers in Roman Notation :—

(*a*) 200018 (*b*) 1884 (*c*) 560844

Remember :—

(*i*) I, V, X, L, C, D and M respectively denote 1, 5, 10, 50, 100, 500 and 1000.

(*ii*) I or X or L or C when placed to the right of a bigger figure indicates addition and when placed to the left indicates subtraction.

(*iii*) A bar or line placed over a number increases its value a *thousand fold.* Hence

(*a*) 200018=200,018
200,000=$\overline{CC}$ [line indicates thousand fold]
10=X
8=VIII

$\therefore$ 200018=$\overline{\text{C}}$CXVIII. **Ans.**

(*ii*) 1884=1,884
1,000=M
800=DCCC [C to the right indicates addition.
80=LXXX [X " " "
4=IV [I to the left indicates subtraction.

$\therefore$ 1884=MDCCCLXXXIV. **Ans.**

(*iii*) 560844=560,844
500,000=$\overline{\text{D}}$ line indicates thousand fold
60,000=$\overline{\text{LX}}$ [" " "
800=DCCC [C to the right indicstes addition
40=XL [X to the left indicates subtraction
4=IV [I " " " " "

$\therefore$ 560844=$\overline{\text{DLX}}$DCCCXLIV. **Ans.**

Example 5. Express in Arabic Notation the following Numbers :—

(*a*) MDCCLXVI (*b*) $\overline{\text{MM}}$MMDCX (*c*) $\overline{\text{DLX}}$CCXXVII.

This question is reverse to question 4. Hence

(*a*) M =1000
DCC =500+100+100= 700
LX =50+10 = 60
VI =5+1 = 6
$\therefore$ MDCCLXVI=1766. **Ans.**

(*b*) $\overline{\text{MM}}$ =(1000+1000) 1000=2000000
MM =1000+1000 = 2000
DC =500+ 100 = 600
X = 10
$\therefore$ $\overline{\text{MM}}$MMDCX=2002610. **Ans.**

(*c*) $\overline{\text{DLX}}$ = (500+50+10)1000=560000
CC =100+100 = 200
XX =10+10 = 20
VII =5+1+1 = 7
$\therefore$ $\overline{\text{DLX}}$CCXXVII =560227. **Ans.**

Note :-After a little practice the students will be able to write the answers of all such questions at once.

EXERCISE 10.

Express each of the following in words :—

1. 3460
2. 15070.
3. 200903.
4. 10005008.
5. 506720891.
6. 407080293.

Express the following in figures :—

7. Three hundred thousand and three hundred.
8. Three million, five thousand and six.
9. Four hundred and six million, seven thousand and five.
10. Four hundred thousand million and twenty one.
11. Six trillion, two hundred billion, twenty-seven thousand million, five hundred thousand and nine.

Write down the local value of each digit in the following numbers :—

12. 47012.
13. 1213456.
14. 4446607.
15. 37960012456.

Express in Roman Notation the following numbers :—

16. 37 ; 67 ; 79 ; 84 ; 99.
17. 107 ; 289 ; 702 ; 868.
18. 1920 ; 2070 ; 4659 ; 7200 ;
19. 200018 ; 10010001.

Express in Arabic Notation the following numbers :—

20. $\overline{\text{DCLX}}$MXXXII.
21. MDCCCLXIII.
22. MDCCCLXXX
23. MDCCLXVI.
24. $\overline{\text{MM}}$MMDCX.
25. $\overline{\text{DLX}}$CCXXVII.
26. Write the largest and the smallest numbers of five digits.
27. Write the largest and the smallest numbers with symbols 2, 5, 9, 0, 8.
28. Write down all the numbers that can be formed by the digits 5, 4, 8 taken altogether.
29. What change takes place in the value of 5 in 23405 if it changes place with 3 ?
30. What is the greatest number of five figureswhich begins with 7 and ends with 3 ?
31. What is the greatest number of six figures which begins with 8 and ends with 2 ?
32. Form the biggest number and the smallest number by the digits of the number 2304951.

CHAPTER II

IE FOUR FUNDAMENTAL OPERATIONS

ADDITION

§1. To add is to find a number equal to two or more numbers taken together. The number thus obtained is called their **sum** or **amount** and the numbers to be added are called **summands.**

§2. **The sign of addition.** The sign + is read plus. It is used to indicate that the numbers between which it is placed are to be added. Thus 8+9 means that 9 is to be added to 8. It is read 'eight plus nine.'

§3. **The sign of equality.** The sign = is called the sign of equality. It is read **equal or equal to.** Thus 8+9=17 indicates that the sum of 8 and 9 is equal to 17. It will be read " eight plus nine is equal to seventeen."

§4. Accuracy and rapidity in the process of addition depends upon learning thoroughly the addition table which we subjoin here.

1 and	2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
1 are 2	1 are 3	1 are 4	1 ,, 5	1 are 6	1 ,, 7	1 are 8	1 are 9	1 are 10
2 ,, 3	2 ,, 4	2 ,, 5	2 ,, 6	2 ,, 7	2 ,, 8	2 ,, 9	2 ,, 10	2 ,, 11
3 ,, 4	3 ,, 5	3 ,, 6	3 ,, 7	3 ,, 8	3 ,, 9	3 ,, 10	3 ,, 11	3 ,, 12
4 ,, 5	4 ,, 6	4 ,, 7	4 ,, 8	4 ,, 9	4 ,, 10	4 ,, 11	4 ,, 12	4 ,, 13
5 ,, 6	5 ,, 7	5 ,, 8	5 ,, 9	5 ,, 10	5 ,, 11	5 ,, 12	5 ,, 13	5 ,, 14
6 ,, 7	6 ,, 8	6 ,, 9	6 ,, 10	6 ,, 11	6 ,, 12	6 ,, 13	6 ,, 14	6 ,, 15
7 ,, 8	7 ,, 9	7 ,, 10	7 ,, 11	7 ,, 12	7 ,, 13	7 ,, 14	7 ,, 15	7 ,, 16
8 ,, 9	8 ,, 10	8 ,, 11	8 ,, 12	8 ,, 13	8 ,, 14	8 ,, 15	8 ,, 16	8 ,, 17
9 ,, 10	9 ,, 11	9 ,, 12	9 ,, 13	9 ,, 14	9 ,, 15	9 ,, 16	9 ,, 17	9 ,, 18

This table can be extended for numbers larger than ten. We give here some questions for practice :—

1. Add 2 to 2, 4 6 and so on up to 100.
2. ,, 3 ,, 3 6 9 ,, ,, 99.
3. ,, 4 ,, 4, 8, 12 ,, ,, 100.
4. ,, 5 ,, 5, 10, 15 ,, ,, 100.

5. Add 6 to 6, 12, 18 and so on up to 96.
6. „ 7 „ 7, 14, 21 „ „ 98.
7. „ 8 „ 8, 16, 24 „ „ 96.
8. „ 9 „ 9, 18, 27 „ „ 99.
9. „ 10 „ 10, 20, 30 „ „ 100.

Such exercises are very useful. The teacher can make the boy practise some more exercises beginning each exercise with any other number.

The following is the rule of addition :—

Rule. *Write down the numbers to be added one under another so that units may come under units, tens under tens, hundreds under hundreds, and so on, then draw a straight line under the last number. Now add up the figures in the first vertical column on the right hand. Put down the last figure of this sum beneath the line and carry the remaining figure or figures to the preceeding column. Proceed in the same manner till the last column is added. Put down the complete sum of the last column. The result thus obtained will be the sum or amount of the summands.*

Example 1. Add together 8935, 8205, 5576, 89375 and 27.

Sol. Arrange the numbers according to the rule given above and proceed thus :—

8935	1st col.	5+5+6+5+7	=28 ; put down 8 & carry	2
8205	2nd „	2+3+0+7+7+2	=21 , „ „ 1 „	2
5576	3rd „	2+9+2+5+3	=21 ; „ „ 1 „	2
89375	4th „	2+8+8+5+9	=32 ; „ „ 2 „	3
27	5th „	8+8	=11 ; „ „ 11	
112118	The sum is 112118. **Ans.**			

Test of Correctness. We can test the correctness of the addition by casting out nines. Thus :—

Cast out nines from the digits of each summand and note down the remainders ; then cast out nines from the sum of the remainders and note down the remainder. This remainder will correspond with the remainder if we cast out nines from the digits of the sum.

Observe the test of the above example.

8935	8+9+3+5	=25=18+7 ; 7 is rem of 1st smd.
8205	8+2+0+5	=15=9 +6 ; 6 ,, ,, ,, 2nd ,,
5576	5+5+7+6	=23=18+5 ; 5 ,, ,, ,, 3rd ,,
89375	8+9+3+7+5	=32=27+5 ; 5 ,, ,, ,, 4th ,,
27	2+7	= 9=9 +0 ; 0 ,, ,, ,, 5th ,,
112118	7+6+5+5+0	=23=18+5 ; 5 is the final rem.
	1+1+2+1+1+8	=14=9 + 5 ; 5 is the rem. of the sum.

Since the final remainder corresponds with the remainder of the sum, the operation is correct.

EXERCISE 2.

Add together :—

1.	21	2.	28	3.	38	4.	58
	25		56		48		59
	35		46		59		75
	55		57		81		96
5.	872	6.	721	7.	981	8.	321
	537		86		302		526
	935		156		856		905
	821		322		706		738
9.	5937	10.	5504	11.	89356	12.	79356
	531		728		3562		215302
	7431		15631		302		18019
	295		439		160025		514
	302		8050		56829		918202

Find the value of:—

13. 8502+502+90321+83562+1613204.
14. 5230+1519+21203+25625+1635209.
15. 1802+27+124+335+95639+1835234.

Find the sum of the following and test the correctness :—

16. 15935, 8102, 9356304, 1650235.
17. 89, 514, 893521, 89356201, 21935.
18. 165, 9356, 85312, 5010101, 89356.

Add together (19 & 20) :—

19. Nine hundred and twenty seven ; fifty thousand and eleven ; five thousand and nine ; eighty-five thousand six hundred and twenty six ; two thousand six hundred and nine.

20. Three hundred and forty-nine thousand and eleven; ten million, five hundred and six ; nine thousand and sixteen; sixty-five thousand and two ; five hundred and sixty thousand and seven.

21. There are 3 trees in my garden ; one of them has 574 apples on it, the second has 200 and the third has 426 on it. How many apples are there in my garden ?

22. I was born in the year 1875 A. D., in which year shall I be 60 years old ?

23. A student has to read 5 books for his examination, there are 400 pages in one book, 526 in the second, 74 in the third, 129 in the fourth and 83 in the fifth. How many pages is he to read altogether ?

24. The Bombay Mail carried yesterday 37 first class passengers, 80 second class and 154 inter class ; how many did it altogether carry ?

25. In the year 1932 A. D. there were 31 days in January, 29 in February, 31 in March, 30 in April, 31 in May, 30 in June, 31 in July, 31 in August, 30 in September, 31 in October, 30 in November and 31 in December. How many days are there in the whole year ?

26. A man had some money in a Bank ; he withdrew Rs. 476 and then again Rs. 294 and then found that there was a balance of Rs. 300 left. How much had he in the Bank ?

27. In a certain town, there are 948230 Mohammadans, 8704213 Hindus, and 25107 of other castes. What is the total population of the town ?

28. In one of the divisions of a province, there are 4031490 Hindus, in another 3920076, in a third 521076 and in the fourth 1234060. What is the total Hindu population in the whole province ?

29. *A* has Rs. 125 ; *B* has Rs. 37 more than *A* ; *C* has Rs. 20 more than what *A* and *B* together have and *D* has as much as the three others together have. What is the total sum that they have ?

30. In the High Department of a school there are 592 boys ; in the middle 705 and in the Primary Department 1704. What is the total numerical strength of the school ?

SUBTRACTION

§5. **Subtraction** is the method of finding what number is left, when a smaller number is taken from a greater one.

The number thus left is called the **remainder,** the number to be subtracted is called the **subtrahend** and that from which it is subtracted, the **minuend.**

§6. **The sign of minus.** The sign '—' is read minus. The sign when placed between two numbers signifies that the second number is to be subtracted from the first. Thus 8—5, means that 5 is to be subtracted from 8. It is read as eight *minus* five,

The symbol ⸗ is called the sign of difference. It indicates that the smaller number is to be subtracted from the greater one.

§7, Like addition table, to effect the operation of subtraction the students should commit the following **subtraction table** also to memory.

1 from	2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
2 = 1	3 = 1	4 = 1	5 = 1	6 = 1	7 = 1	8 = 1	9 = 1	10 = 1
3 ,, 2	4 ,, 2	5 ,, 2	6 ,, 2	7 ,, 2	8 ,, 2	9 ,, 2	10 ,, 2	11 ,, 2
4 ,, 3	5 ,, 3	6 ,, 3	7 ,, 3	8 ,, 3	9 ,, 3	10 ,, 3	11 ,, 3	12 ,, 3
5 ,, 4	6 ,, 4	7 ,, 4	8 ,, 4	9 ,, 4	10 ,, 4	11 ,, 4	12 ,, 4	13 ,, 4
6 ,, 5	7 ,, 5	8 ,, 5	9 ,, 5	10 ,, 5	11 ,, 5	12 ,, 5	13 ,, 5	14 ,, 5
7 ,, 6	8 ,, 6	9 ,, 6	10 ,, 6	11 ,, 6	12 ,, 6	13 ,, 6	14 ,, 6	15 ,, 6
8 ,, 7	9 ,, 7	10 ,, 7	11 ,, 7	12 ,, 7	13 ,, 7	14 ,, 7	15 ,, 7	16 ,,
9 ,, 8	10 ,, 8	11 ,, 8	12 ,, 8	13 ,, 8	14 ,, 8	15 ,, 8	16 ,, 8	17 ,, 8
10 ,, 9	11 ,, 9	12 ,, 9	13 ,, 9	14 ,, 9	15 ,, 9	16 ,, 9	17 ,, 9	18 ,, 9

§8. A question of subtraction can be put in many different ways. Thus, "What is the difference between 9 and 4"? may be put in the following ways also :—

1. Find the difference between 9 and 4.
2. By how much is 4 less than 9 ?
3. By how much is 9 greater than 4 ?
4. What must be added to 4 to make 9 ?
5. What must be the remainder when 4 is taken from 9 ?
6. By how much does 9 exceed 4 ?
7. The sum of two numbers is 9. One number is 4, find the other.

§9. The following is the rule for subtraction :-

Rule. *Write down the smaller number under the greater as in the process of addition and draw a line underneath. Begin at the right hand and subtract each digit of the subtrahend from the corresponding digit of the minuend and put down the result below the line, but if any lower digit be greater than the upper, add ten to the upper digit and then take the lower digit from the upper number thus formed ; put down the remainder as before. In this case add 1 to the next lower digit before carrying on the process of subtraction further.*

Example 1. Subtract 2135 from 9849.

Sol. Arrange the numbers according to the rule given above and proceed thus :—

	Explanation
9849 minuend 2135 subtrahend ; 7714 rem. **Ans.**	9−5=4, put down 4 ; 4−3=1, put down 1 8−1=7 ,, 7 ; 9−2=7, ,, 7

Example 2. Subtract 5639 from 8254.

Sol.

	Explanation
8254 minuend ; 5639 subtrahend 2615 remainder ; **Ans.**	9 cannot be taken from 4, we add 10 to the 4 making it 14 ; 14−9=5 ; put down 5. Now add 1 to the next lower figure 3 ;making it 4 ; 5−4=1, put down 1. Similarly add 10 to the 2 making it 12 ; 12−6 =6, put down 6. Now add 1 to the next lower figure 5 making it 6 ; 8−6=2, put down 2.

Test of correctness. Add subtrahend and the remainder, the sum will correspond with the minuend. **Or**.

Cast out nines as in addition. The difference of the remainders of the minuend and the subtrahend will correspond with the remainder of the *remainder*.

Observe the test of the second example.

8254 } 8+2+5+4=19=18+1 ; 1 is the remainder.
5639 } 5+6+3+9=23=18+5 ; 5 is the remainder.
2615 } 2+6+1+5=14=9+5 ; 5 is the remainder.

Here add 9 to the remainder of the minuend because it is less than the remainder of the subtrahend. $9+1=10$; $10-5=5$ which corresponds with the remainder of the *remainder*.

§10. A number preceded by the sign + is called a **positive** number and that preceded by the sign − is called a **negative** number. When no sign is affixed to a number it is considered as positive.

§11. **Expressions and terms.** An expression is one in which two or more numbers are connected by the sign + or − and the numbers thus connected are called **terms**. Thus $8+6-4-2$ is an *expression* and 8, 6, −4, −2 are its terms ; 8, 6 are *positive* 4, 2 are *negative*.

Example 3. Find the value of $1329-576+504-302-29$.

Sol. Sum of positive terms $=1329+504 \quad =1833$.

Sum of negative terms $=576+302+29=907$.

Value required $=1833-907=926$. **Ans.**

Hence the following

Rule. *Subtract the sum of all the negative terms from the sum of all the positive terms.*

EXERCISE 3.

Perform the following subtractions :—

1.	9534 2313	2.	8956 3524	3.	6359 2048	4.	8937 2513
5.	56289 49393	6.	34956 21099	7.	83569 64397	8.	20001 17354
9.	201319 189352	10.	301001 89599	11.	71935 49296	12.	900090 499938

Simplify :—

13. $265302-189543$.
14. $169356-89354$.
15. $290093-109989$.
16. $793590-189056$.
17. $825+576+328+21-675$.
18. $1817-936-1021+676-75$.

19. $15012+21-21025+80023-65233$.
20. $1002+2512-3409+89+185$.

Complete the following sums of addition by filling in the digits whose places are marked by asterisks :—

21. $\begin{array}{r} 5132 \\ 3{*}{*}2 \\ \hline {*}894 \end{array}$ 22. $\begin{array}{r} 3{*}256 \\ 29{*}{*}5 \\ \hline {*}4731 \end{array}$ 23. $\begin{array}{r} 83256 \\ {*}2{*}3{*} \\ \hline {*}758{*}5 \end{array}$ 24. $\begin{array}{r} 19516 \\ {*}{*}3{*} \\ \hline {*}8753 \end{array}$

Complete the following sums of subtraction by filling in the digits whose places are marked by asterisks :—

25. $\begin{array}{r} 7895 \\ 3{*}2{*} \\ \hline 4568 \end{array}$ 26. $\begin{array}{r} 2835 \\ {*}{*}9{*} \\ \hline 14{*}9 \end{array}$ 27. $\begin{array}{r} 25329 \\ {*}{*}98 \\ \hline {*}62{*}{*} \end{array}$ 28. $\begin{array}{r} 10000 \\ {*}{*}35 \\ \hline {*}{*} \end{array}$

29. The sum of two numbers is 1893609 and one of them is 930994 ; find the other.

30. Minuend is 803563 and the remainder 15102. Find the subtrahend.

31. *A* deposited Rs. 576 in the bank on Monday, Rs. 708 on Tuesday, Rs. 2105 on Thursday. *B* deposited Rs 376 in another bank on Monday, Rs. 728 on Tuesday, Rs. 1689 on Thursday. By how much does *A*'s money exceed *B*'s money in the bank ?

32. By how much does 1002490 exceed 704050.
33. What is the excess of 9497605 above 8888888 ?
34. What must be added to the sum of 432, 507, 78, 710 and 803 in order to get the number 3000 ?
35. A man earns Rs. 4000 in a month and he spends Rs. 3702. What is his savings in a month ?
36. In the year 1932 A.D. I was 57 years old ; when was I born ?
37. Hari was born in 1875 A. D., what was his age in 1932 ?
38. The population of a town is 17254120 out of which there are 8624022 males. Find the number of females.
39. I bought a house at Rs. 8249 and spent Rs. 1200 on its repairs. I then sold it for Rs. 9000. How much did I lose or gain ?

40. My son was born in the year 1902. When would he have been as old as I was in the year 1909, my year of birth being 1875? How old would I then have been?

41. One mountain is 13572 ft. high and another is 9024 ft. high. How much is the former higher than the latter?

42. I had Rs. 682 with me. I gave Rs. 205 to a friend, Rs. 82 in charity and spent Rs. 349 on my own account. Find how much money is left with me.

43. I spent Rs. 13562 for a horse and some land; the land alone was worth Rs. 8024; what was the value of the house?

44. A man had one lac of rupees at the time of his death. He gave Rs. 6120 to his son; Rs. 2090 to his daughter and the remainder to his wife. How much did the wife get?

MULTIPLICATION

§12. **Multiplication** is the short method of finding the sum of a given number of repetitions of a certain number. Thus to find the sum of six times 12, it will be

$$12+12+12+12+12+12=72.$$

The number to be repeated is called the **multiplicand** and the other which indicates the number of repetitions is called the **multiplier**, and the sum obtained the **product**. Thus in the above solution 12 is the multiplicand, 6 is the multiplier and 72 is the product.

§13. **The sign of multiplication** is $\times$. It is read as into. It is used to indicate that the numbers between which it is placed are to be multiplied together. Thus 12×6 which is read twelve into six indicates that 12 is to be multiplied by 6.

Each of the numbers multiplied together is called a **factor**. Thus 12 and 6 are the factors of 72.

Note. Sometimes the operation of multiplication is expressed by a dot. Thus 12×6 may be written as 12.6.

§14. Rapidity and accuracy in multiplication depends upon the complete familiarity with **multiplication tables**. For the sake of convenience we subjoin the table here.

Multiplication Table

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400

§15. To prove that four times six=six times four.

Four times six=6+6+6+6=24.

Six times four=4+4+4+4+4+4=24.

Or,

Let 24 counters be arraged as in the figure.

Now, No. of counters=No. of horizontal rows×No. of counters in each row=4×6

Also, No. of counters=No. of vertical rows×No. of counters in each row=6×4

* * * * * *
* * * * * *
* * * * * *
* * * * * *

∴ four times six=six times four.

It follows therefore that the *factors of a product may be taken in any order. Or the multiplicand and the multiplier may be interchanged without altering the product.*

§16. Short Multiplication.

When the multiplier does not exceed 20, the multiplication is called **short multiplication.**

Rule. *Write down the multiplier under the multiplicand and draw a line underneath. Begin at the unit's figure of the multiplicand and multiply each figure by the multiplier, putting down and carrying precisely as in addition.*

Example 1. Multiply 45321 by 9.

Exp.

(*i*) 9 times 1=9 ; put down 9.
(*ii*) 9 times 2=18 ; put down 8 and carry 1.
(*iii*) 9 times 3=27 ; 27+1=28 ; put down 8 and carry 2.
(*iv*) 9 times 5=45 ; 45+2=47 ; put down 7 and carry 4.
(*v*) 9 times 4=36 ; 36+4=40 ; put down 40.

The product=407889. **Ans.**

45321
9
407889

§17. Multiplication by a simple number followed by zeroes.

Rule. *Multiply the multiplicand by the simple number and add as many zeroes to the right of the product as there are zeroes to the right of multiplier.*

Example 2. Multiply 5374 by 900.

Sol. $5374 \times 900 = 5374 \times 9$ hundreds.
$= 48366$ hundreds.
$= 4836600$. **Ans.**

Note. In multiplication when one factor is **0**, the product is also **0**. Thus $165 \times 0 = 0$ and $0 \times 165 = 0$.

EXERCISE 4.

Multiply :—

1.	892 by 7.	**2.**	5603 by 8.	**3.**	5302	by	12.
4.	5023 by 13.	**5.**	7089 by 14.	**6.**	8972	by	15.
7.	6305 by 16.	**8.**	8925 by 17.	**9.**	12315	by	18.
10.	5605 by 10.	**11.**	6325 by 20.	**12.**	7321	by	100.
13.	6329 by 500.	**14.**	8935 by 6000.	**15.**	21526	by	8000.

---

§18. Long Multiplication.

When the multiplier exceeds 20, the multiplication is called **long multiplication.**

Rule. *Write the multiplier below the multiplicand and draw a line underneath. First multiply the multiplicand by the unit's figure of the multiplier and then multiply the multiplicand by ten's figure of the multiplier and put the product under the first product so that the unit's figure of this product may fall under the ten's figure of the first. Repeat this operation with hundred's, thousand's, etc.*

Example 3. Multiply 8556 by 237.

Sol. Proceeding by the rule given above

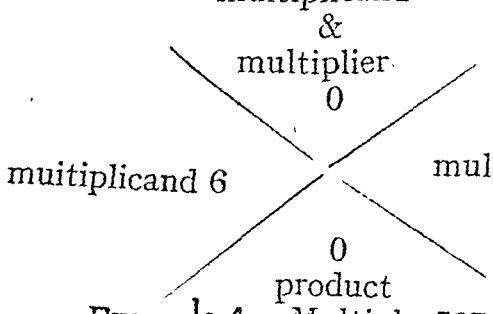
	8556
	237
$8556 \times 7 = 59892$	59892
$8556 \times 3 = 25668$	25668
$8556 \times 2 = 17112$	17112
	2027772 **Ans.**

Test of correctness. (*i*) Interchange the multiplicand and the multiplier, the product will be the same.

(*ii*) Cast out nines from the multiplicand and the multiplier ; multiply the remainders and again cast out nines. The remainder now obtained will correspond with the remainder which remains after casting out nines from the product.

8556 | 8+5+5+6=24=18+6 ; 6 is the remainder.
237 | 2+3+7 =12= 9+3 ; 3 is the remainder.
59892 | 6×3 =18=18+0 ; 0 is the remainder.
25668
17112
2027772

2+0+2+7+7+7+2=27=27+0 ; 0 is the rem.



Or

237
8556
1422
1185
1185
1896
2027772

Example 4. Multiply 53728 by 2005.

Sol. Proceeding by the rule given above we have

53728
2005
53728×5 =268640 268640
53728×2 thousands =107456 thousands 107456000
107724640. **Ans.**

§19. If there are zeroes to the right of the multiplicand or multiplier, or both, the zeroes may be neglected and the remaining figures may be multiplied together and the neglected zeroes may be annexed in the product.

Example 5. Multiply 85300 by 19200.

Sol.

853
192
1706
7677
853
163776

Neglecting zeroes, the product=163776

∴ the req. product=1637760000. **Ans.**

§20. **Multiplication by factors.**

Example 6. Multiply 728 by 192.

Sol. 192=16×12

728×12=8736

8736×16=139776

728
12
8736
16
139776 **Ans.**

§21. **Continued product.**

Multiplication of more than two numbers is called their **continued product.** Thus $728\times12\times16$ is an example of continued product, which has been solved above.

§22. When a number is multiplied by itself once, twice, thrice, etc. the product is called the second, third, fourth, etc. power of that number respectively. Thus :

6×6 is called the **second power** of 6,
$6\times6\times6$ is called the **third power** of 6,
$6\times6\times6\times6$ is called the **fourth power** of 6,

and so on.

6×6 is written as 6^2 and is read as " 6 squared "
$6\times6\times6$,, 6^3 ,, ,, ,, ,, " 6 cubed "
$6\times6\times6\times6$,, 6^4 is read as " 6 to the fourth power. "
Similarly 6^5 is read as " 6 to the fifth power. "
and so on.

§23. **Index.** The small figure which indicates how many times the number is multiplied is called the **index** or **exponent** of the power. Thus in 6^5, 5 is the index.

§24. If the three signs $+$, $-$, $\times$ are used in an expression the operation of *multiplication* is to be performed first and then that of *Addition* or *Subtraction.*

Thus $5\times2-3\times4+2\times4=10-12+8$

$=18-12=6.$ Ans.

EXERCISE 5.

Multiply by factors :—

1. 5635 by 156.
2. 5621 by 108.
3. 8551 by 117.
4. 25301 by 256.
5. 36205 by 208.
6. 21935 by 132.

Find the continued product of :—

7. $65\times18\times16$.
8. $56\times25\times36$.
9. $76\times21\times49$.
10. $45\times35\times25$.
11. $28\times38\times48$.
12. $55\times45\times65$.

Multiply and test the correctness of the following :—

13. 8935 by 1890.
14. 56389 by 15600.
15. 56000 by 27000.
16. 51800 by 45000.
17. 50009 by 25005.
18. 29359 by 250025.
19. 189725 by 41935.
20. 283561 by 283562.

Find the squares of :—

21. 45, 71, 121, 125 and 256.

Find the cubes of :—

22. 11, 21, 25, 36 and 40.

Find the values of :—

23. $9\times3-5\times5+7\times8+121$.

24. $25\times6+36\times4-28\times8+51$.

25. $37\times9-28\times12+35-17\times21+576$.

26. 192 pies make one rupee. How many pies are there in Rs. 374 ?

27. A book contains 579 pages and each page contains 3749 letters. How many letters are there in the whole book ?

28. The price of one Bigha of land is Rs. 904. Find the price of 98 Bighas.

29. 8634 men pass over the Ravi Bridge every day. How many men will pass over it in 706 days ?

30. The cost of one elephant is Rs. 3479 and that of one horse is Rs. 765. How much will 6 elephants and 16 horses cost ?

31. There is a tank of water with a hole in it. 78 gallons of water leak out by the hole per hour. When the tank is full of water, it is emptied in 48 hours. Find the capacity of the tank.

32. The population of a town is 127600. It increases at the rate of 320 persons each year. What will be the population 10 years hence ?

33. A railway train runs at an average speed of 22 miles an hour. It takes 42 hours to run from Calcutta to Delhi. Find the distance between Calcutta and Delhi.

34. The rays of light come from the sun to the earth in 448 seconds. Light moves at the rate of 186000 miles per second. Find the distance of the earth from the sun.

DIVISION.

§25. **Division** is the method of finding how many times one number is contained in another. The former of these numbers is called the **divisor**, the latter the **dividend** and the number of times found the **quotient**. That which is left after the operation is finished is the **remainder**.

Suppose we want to know how many times 6 is contained in 25. By the help of the multiplication table, we can at once tell "4 times, with a remainder 1."

The operation at the side also shows the same result.

Here 25 is the *dividend*, 6 is the *divisor*, 4 is the *quotient* and 1 is the *remainder*.

25
6...once,
19
6...twice,
13
6...thrice,
7
6...4times
1

From above it follows that

Divisor × quotient + remainder = dividend.

§26. The **sign of division is ÷**. It is read **'divided by'**. 25÷6 denotes 25 divided by 6.

§27. **Short Division.**

When the divisor does not exeed 20, it is called **short division**. The operation in this case is done mentally. The accuracy depends upon the complete familiarity with the multiplication table. (See page 17)

Example 1. Divide 8972 by 12.

Sol. Here 12 goes no time in 8, but 7 times in 89 and 5 over. Put down 7 under the 9. Then 12 goes 4 times in 57 and 9 over. Put down 4 under the 7. Lastly 12 in 92 goes 7 times and 8 over.

12)8972
747— 8 rem.

The quotient is 747 and the remainder 8. **Ans.**

§28. **Long Division.**

When the divisor exceeds 20, it is called **long division**.

The rule for operation is as under:—

Rule. *Write the divisor and the dividend thus ;*

divisor) *dividend*(

Now, try to find how many times the one or two figures on the left hand of the divisor contain in the one or two figures on the left hand of the dividend and write the result as the first figure (on the left) of the quotient. Multiply this partial quotient by the divisor and put down the product under the dividend on the left hand side. Subtract it and write down the remainder. Annex to the right of this remainder the next figure of the dividend to form the next partial dividend. Proceed as before and continue the process till all the figures of the dividend have been brought down. If at any stage the divisor is found greater than the partial dividend, put a zero to the quotient and bring down the next figure of the dividend.

Example 2. Divide 537089 by 328.

Solution.	*Explanation.*
328)537089(1637 328 —— 2090 1968 —— 1228 984 —— 2449 2296 —— 153	Here 3 goes once in 5. Put 1 as the first figure of the quotient. Multiply this 1 by 328 and put down the product under the dividend as shown in the process. The remainder is 209. Bring down the next figure 0 to form the partial dividend 2090. Now 3 goes 6 times in 20 . Put 6 as the second figure of the quotient. Multiply this 6 by 328 and put the product under the partial dividend. The next remainder is 122. Bring down the next figure 8 to form the next partial dividend 1228. Again 3 goes 4 times in 12, but if we put 4 as the next figure of

the quotient and multiply this 4 by 328 the product will exceed the partial dividend 1228, therefore put 3 as the next figure of the quotient. Multiply this 3 by 328 and put down the product under 1228. The remainder is 244. Bring down the next figure 9 to form the last partial dividend 2449. 3 goes 8 times in 24, but the product will exceed the dividend, therefore put 7 as the last figure of the quotient. Multiply this 7 by 328 and put the product under the last partial dividend. The remainder is 153

∴ the quotient is 1537 with a remainder 153. **Ans.**

Test of correctness. (*i*) Multiply the divisor by the quotient and add the remainder to the product. If the result corresponds with the dividend the answer is correct. Thus $637 \times 328 = 536936$. Adding the remainder to it ($536936 + 53 = 537089$) the result corresponds with the dividend. Hence the answer is correct.

(*ii*) Cast out nines from the divisor and the quotient, and write down the remainders. Multiply these remainders and again cast out nines. The remainder now left will correspond with the remainder obtained by casting out nines from the difference of the *dividend* and the *remainder*.

328 537089(1637
Rem. 153

Diffrence of the Dividend and the Rem. = 536936

$3+2+8 = 13 = 9+4$; 4 is the remainder of the Divisor.
$1+6+3+7 = 17 = 9+8$; 8 is the remainder of the Quotient.
$4 \times 8 = 32 = 27+5$; 5 is the final remainder.
$5+3+6+9+3+6 = 32 = 27+5$; is the remainder of difference.

∴ the final remainder corresponds with the remainder of the difference, the answer is correct.

Example 3. Divide 368002 by 892.

Sol.

892)368002(412
3568
1120
892
2282
1784
498

Explanation

Here 8 in 3 goes no time. Therefore take two figures on the left hand of the dividend. Now 8 in 36 goes 4 times.
Now proceed as explained in Ex. 2.
The quotient is 412 and the remainder 498. **Ans.**

§ 29. Division by 10, 100, 1000, and so on.

Rule. *To divide a number by 10, 100, 1000 and so on, cut off as many figures from the right of the dividend as there are zeroes in the divisor. The figures thus cut off will be the remainder and the remaining figures will form the quotient.*

Example 4. $18693 \div 10$ = quotient 1869, rem. 3.

Example 5. $18693 \div 100$ = quotient 186, rem. 93.

Example 6. $18693 \div 1000$ = quotient 18, rem. 693.

Example 7. $18693 \div 10000$ = quotient 1, rem. 8693.

§30. Division by factors.

This method is very important. The student is advised to note the solution very carefully and learn how to find out the complete remainder.

Example 8. Divide 85357 by 120, using factors $4 \times 5 \times 6$.

Sol.

4	85357	
5	21339, 1 unit	= 1
6	4267, 4 groups of 4 units	= 16
	711, 1 group of 4×5 units each	= 20
	complete remainder	= 37

$\therefore$ Quotient = 711 ; Remainder = 37. **Ans.**

From the above solution we deduce the following

Rule. *Complete remainder = 1st rem. + (2nd rem. × 1st divisor) + (3rd rem. × 1st divisor × 2nd divisor) + etc.*

Example 9. Divide 415635 by 180 using factors $2 \times 3 \times 5 \times 6$.

Sol.

2	415635	
3	207817, 1 unit	= 1
5	69272, 1 group of 2 units	= 2
6	13854, 2 groups of 2×3 units each	= 12
	2309, no group of $2 \times 3 \times 5$ units	= 0
	complete remainder	= 15

$\therefore$ Quotient = 2309 ; Remainder = 15 **Ans.**

EXERCISE 6.

Divide (1—12) :—

1. 56352 by 12.
2. 36432 by 18.
3. 106799 by 19.
4. 118456 by 221.
5. 105795 by 521.
6. 535001 by 625.

7. 285359 by 824. 8. 910025 by 728.
9. 565902 by 756. 10. 889956 by 1625.
11. 215623 by 9535. 12. 853699 by 9999.
13. Divide 56535698 separately by 10, 100, 1000, 10000.
14. Divide 89300022 separately by 10, 100, 1000, 10000.
15. Divide 48300987 separately by 100, 1000, 10000, 100000.

Divide by factors and find the complete remainders in (16—19) :—

16. 1563557 by 144. 17. 4800935 by 288.
18. 56359281 by 256. 19. 3790289 by 132.

20. Divide 5630298 by 5, 6, 7 in succession and find the complete remainder.

21. Divide 21908956 by 7, 9, 6 in succession and find the complete remainder.

22. A number is divided by 5, 6 and 7 in succession and the remainders are 3, 2 and 5 respectively. Find the remainder had the number been divided by 210.

23. A number is divided by 9, 8 and 12 in succession and the remainders are 7, 4 and 2 respectively. Find the remainder had the number been divided by 864.

24. A number is divided by 5, 6, 9 in succession and the remainders are 2, 3 and 4 respectively. Find the remainder had the number been divided by 270.

---

§31. If all the four signs $\times$, $-$, $+$, $\div$ occur in an expression, the operation of *Division* is to be performed first, and then of *Multiplication* and next those of *Addition* and *Subtraction*.

Thus $9\div3+2\times12\div4+5-3\times2$.

$=3+2\times3+5-3\times2$

$=3+6+5-6=8$. **Ans.**

EXERCISE 7.

Simplify the following expressions:—

1. $12\times6\div3+2$.
2. $15\div5\times4-1$.
3. $21\div7\times3-4$.
4. $25\times15\div3-25$.
5. $28-25\div5\times2$.
6. $45+51\div17\times3$.
7. $56\times81\div27-168$.
8. $55\div11-88\div44$.
9. $45\times15-121\div11$.
10. $144\div13\times3-14$.
11. $196\div28\times3-45\times15\div5+125$.
12. $196\div28\times5+56\times21\div7-150$.
13. $169\div13+168\div14+144\div18\times3$.
14. $226\times16\div8+5\times38-324\div81$
15. $117\div13\times9-81+104\div26\times4$.

Abbreviated Methods of Multiplication.

§32. To multiply a number by 10, 100, 1000, etc.

Rule. *Add as many zeroes to the multiplicand as there are zeroes in the multiplier.*

Example 1. Multiply 852 separately by 10 and 1000.
Sol. $852\times10=8520$; $852\times1000=852000$. **Ans.**

§33. To multiply by 20, 30, 40, etc.

Rule. *Add as many zeroes to the multiplicand as there are in the multiplier and then multiply the resulting number by the remaining digit or digits in the multiplier.*

Example 2. Multiply 5025 separately by 30 and 700.
Sol. $5025\times30=50250\times3=150750$
$5025\times700=502500\times7=3517500$ } **Ans.**

§34. To multiply a number by 11, 101, 1001, etc.

Rule. *Add 1, 2, 3 zeroes respectively to the multiplicand and then add the multiplicand to the resulting number.*

Example 3. Multiply 5023 separately by 11 and 1001.
Sol. $5023\times11=50230+5023=55253$
$5023\times1001=5023000+5023=5028023$ } **Ans**

§35. **To multiply a number by 5.**

Rule. *Annex a zero to the right of the multiplicand and then divide it by 2,* $\because 5 = 10 \div 2$.

Example 4. Multiply 89356 by 5.

Sol. $89356 \times 5 = 893560 \div 2$

$\therefore$ the reqd. product $= 446780$. **Ans.**

§36. **To multiply a number by 15.**

Rule. *Annex a zero to the right of the multiplicand and to it add its half.*

Example 5. Multiply 56359 by 15.

Sol. $56359 \times 15 = 563590 + 281795$.

$\therefore$ the reqd. product $= 845385$. **Ans.**

§37. **To multiply a number by 25.**

Rule. *Annex two zeroes to the right of the multiplicand and then divide it by 4,* $\because 25 = 100 \div 4$.

Example 6. Multiply 890023 by 25.

Sol. $890023 \times 25 = 89002300 \div 4$

$\therefore$ the reqd. product $= 22250575$. **Ans.**

§38. **To multiply a number by 125.**

Rule. *Affix three zeroes to the right of the multiplicand and then divide it by 8,* $\because 1000 \div 8 = 125$.

Example 7. Multiply 5670809 by 125.

Sol. $5670809 \times 125 = 5670809000 \div 8$.

$\therefore$ the reqd. product $= 708851125$. **Ans.**

§39. **To multiply a number by 625.**

Rule. *Affix four zeroes to the right of the multiplicand and then divide it by 16,* $\because 625 = 10000 \div 16$.

Example 8. Multiply 2102563 by 625.

Sol. $2102563 \times 625 = 21025630000 \div 16$

$\therefore$ the reqd. product $= 1314101875$. **Ans.**

§40. **To multiply a number by 15, 35, 45, 55 and 65.**

Rule. *Multiply the multiplicand by 30, 70, 90, 110, 130 [Art. 33] and divide the result by 2.*

Example 9. Multiply 5625 separately by 15, 45, 65.

Sol. $5625\times15=(5625\times30)\div2=168750\div2=84375.$
$5625\times45=(5625\times90)\div2=506250\div2=253125.$ } **Ans.**
$5625\times65=(5625\times130)\div2=731250\div2=365625.$

§41. **To multiply a number by 75, 175, 275, etc.**

Rule. *Multiply the multiplicand by 300, 700, 1100, and then divide the result by 4(Art. 40).*

§42. **To multiply a number by a number all the figures of which are nines, *i.e.*, by 9, 99, 999, etc.**

$\because$ $9=10-1$; $99=100-1$; $999=1000-1$ and so on
$\therefore$ We have the following

Rule. *Place as many zeroes to the right of the multiplicand as there are nines in the multiplier, and from the result subtract the multiplicand.*

Example 10. Multiply 895023 by 999.

Sol. $895023\times999=895023000-895023.$
$\therefore$ the reqd. product$=894127977.$ **Ans.**

§43. **To multiply a number by a number all the figures of which are ones, *i.e.*, 111, 1111, etc.**

Rule. *First put down the unit's digit of the multiplicand as the unit's digit of the product and then add each figure of the multiplicand to its next digit up to as many figures on its left as the number of digits in the multiplier, to get the other digit of the product carrying the number if any. The number thus formed is the required product.*

Example 11. Multiply 8972 by 111.

Sol.
8972
111
8972
8972
8972
995892 **Ans.**

(1) Put down 2 as the unit's figure of the product.
(2) $2+7=9$; put down 9 as the next figure of the product.
(3) $2+7+9=18$; put down 8 and carry 1. Since the multiplier consists of 3 digits we shall now omit 2, and add 7, 9, 8 then omit 7 and add $9+8+$...and so on.

§44. **To multiply a number by a number a little less or greater than any multiple of 10.**

Example 12. Multiply 89536 by 9995 and 10008.

Sol. (1) $\because$ $9995 = 10000 - 5$

$\therefore$ $89536 \times 9995 = 895360000 - 89536 \times 5.$

$= 895360000 - 447680.$

$\therefore$ the reqd. product $= 894912320.$ **Ans.**

(2) $\because$ $10008 = 10000 + 8$

$\therefore$ $89536 \times 10008 = 895360000 + 89536 \times 8$

$= 895360000 + 716288.$

$\therefore$ the reqd. product $= 896076288.$ **Ans.**

§45. **To multiply in a given number of lines.**

Sometimes the digits of the multiplier are so related to each other that the operation of multiplication may be considerably shortened by using some peculiar artifices.

The following three examples should be carefully studied :—

Example 13. Multiply 28536 by 9144 in two lines.

Sol.
```
  28536
   9144
------
256824***
 4109184
260933184
```

Explanation. First multiply by 9 leaving unit's, ten's and hundred's places blank ; then multiply the result by 16 to get the product of 144 at once. This product may be so arranged that units may come under units, tens under tens and so on.

Example 14. Multiply 856252 by 2096 in two lines.

Sol.
```
  856252
    2096
-------
 13700032
1781004160
----------
1794704192
```
Ans.

Explanation. Since $2096 = 2080 + 16$ therefore first multiply by 16, then multiply the result by 130 to get the product of 2080 at once and then add the results.

Example 15. Multiply 3025396 by 1441872 in three lines.

Sol.
```
  3025396
  1441872
---------
 54457128
217828512
435657024
-----------
4362233781312
```
Ans.

Explanation. First multiply by 18 then multiply the result by 4 to get the product of 72 at once and then multiply the product of 72 by 2 to get the product of 144. Arrange these products correctly and add.

EXERCISE 8.

Using the shortest method multiply :—

1. 59334 by 100, 1000.
2. 75624 by 30, 110.
3. 42502 by 1001, 10001.
4. 263391 by 5.
5. 263895 by 15.
6. 369538 by 25.
7. 378951 by 35.
8. 498359 by 75.
9. 361575 by 125.
10. 219357 by 125.
11. 279583 by 625.
12. 359387 by 625.
13. 569908 by 99.
14. 379002 by 999.
15. 453001 by 9999.
16. 378350 by 9999.
17. 56735 by 111.
18. 863542 by 1111.
19. 163508 by 9992.
20. 470025 by 9993.
21. 350259 by 10004.
22. 480902 by 10009.

Multiply in two lines (Imp.)

23. 53902 by 488.
24. 370892 by 726.
25. 163095 by 1449.
26. 359302 by 1043.
27. 219035 by 8128.
28. 459092 by 9135.
29. 569287 by 18012.
30. 193086 by 13256.
31. 453095 by 13195.
32. 319034 by 38019.
33. 897564 by 1936.
34. 763542 by 2115.

Multiply in three lines. (Imp.)

35. 479356 by 192216.
36. 289209 by 2981018.
37. 519410 by 15810135.
38. 219083 by 12801[illegible]256.
39. 483502 by 219189014.
40. 352092 by 180[illegible]8012.

Abbreviated Method of Division.

§43. To divide a number by 10, 100, 1000, etc.

Rule. *Cut off as many digits from the right side of the dividend as there are zeroes in the divisor. This will form the remainder and the remaining digits, the quotient.*

Example 1. Divide 85624 separately by 100 and 1000.

Sol. $85624 \div 100 = Q.\ 856,\ \text{rem.}\ 24.$
$85624 \div 1000 = Q.\ 85,\ \text{rem.}\ 624$ } **Ans.**

§47. **To divide a number by 30, 700, 11000, etc.**

Rule. [Apply Arts. 46, 30.]

§48. **To divide a number by 5.**

Rule. *Multiply the dividend by 2 and divide the product by 10. Now omit the last figure; the omitted figure ÷ 2 will be the remainder.*

Example 2. Divide 89356 by 5.

Sol. $89356 \div 5 = (89356 \times 2) \div 10.$

$= 178712 \div 10.$

$\therefore$ the reqd. quotient $= 17871$; remainder $= 2 \div 2 = 1$. **Ans.**

§49. **To divide a number by 25.**

Rule. *Multiply the dividend by 4, and divide the product by 100. Now omit the last two figures; the omitted figures ÷ 4 will be the remainder.*

Example 3. Divide 562359 by 25.

Sol. $562359 \div 25 = (562359 \times 4) \div 1000.$

$= 2249436 \div 100.$

$\therefore$ the reqd. quotient $= 22494$; remainder $= 36 \div 4 = 9$. **Ans.**

§50. **To divide a number by 125.**

Rule. *Multiply the dividend by 8, and divide the product by 1000. Now omit the last three figures; the omitted figures ÷ 8 will be the remainder.*

Example 4. Divide 159356 by 125.

Sol. $159356 \div 125 = (159356 \times 8) \div 1000$

$= 1274848 \div 1000$

$\therefore$ the reqd. quotient $= 1274$; rem. $= 848 \div 8 = 106$. **Ans.**

§51. **To divide a number by 625.**

Rule. *Multiply the dividend by 16 and divide the product by 10000. Now omit the last four figures; the omitted figures ÷16 will be the remainder.*

Example 5. Divide 219356 by 625.

Sol. $219356 \div 625 = (219356 \times 16) \div 10000$

$= 3509696 \div 10000.$

$\therefore$ the reqd. quotient $= 350$; rem. $= 9696 \div 16 = 606$. **Ans.**

§52. To divide a number by 15, 35, 45 etc.

Rule. *Multiply the dividend by 2 and divide the product separately by 30, 70 or 90, etc. as the case may be. The remainder÷2 is the real remainder.*

Example 6. Divide 8935 by 15.

Sol. $8935 \div 15 = (8935 \times 2) \div 30$

$= 17870 \div 30$

$= 595$; 20 rem. [Art. 30]

$\therefore$ the reqd. quotient $= 595$; rem. $= 20 \div 2 = 10$. **Ans.**

Example 7. Divide 8356 by 35.

Sol. $8356 \div 35 = (8356 \times 2) \div 70$

$= 16712 \div 70$

$= 238$; 52 rem.

$\therefore$ the reqd. quotient $= 238$; remainder $= 52 \div 2 = 26$. **Ans**

Example 8. Divide 15938 by 45.

Sol. $15938 \div 45 = (15938 \times 2) \div 90$.

$= 31876 \div 90$

$= 354$; 16 rem.

$\therefore$ the reqd. quotient $= 354$; remainder $= 16 \div 2 = 8$. **Ans.**

§53. To divide a number by 75, 175, 275, etc.

Rule. *Multiply the dividend by 4 and divide the product separately by 300, 700 and 1100 etc. as the case may be. The remainder÷4 is the real remainder.*

Example 9. Divide 15936 separately by 75, 175, 275.

Sol. (1) $15936 \div 75 = (15936 \times 4) \div 300$

$= 63744 \div 300$

$= 212$; 144 rem.

$\therefore$ the reqd. quotient $= 212$; rem. $= 144 \div 4 = 36$. **Ans.**

(2) $15936 \div 175 = (15936 \times 4) \div 700$

$= 63744 \div 700$

$= 91$; rem. 44.

$\therefore$ the reqd. quotient $= 91$; rem. $= 44 \div 4 = 11$. **Ans.**

(3) $15936 \div 275 = (15936 \times 4) \div 1100$
$= 63744 \div 1100$
$= 57$, rem. 1044
$\therefore$ the reqd. quotient $= 57$; rem. $= 1044 \div 4 = 261$. **Ans.**

§54. **To divide a number by 375 and 875.**

Rule. *Multiply the dividend by 8 and divide the product separately by 3000 and 7000. The remainders ÷ 8 are the real remainders.*

Example 10. Divide 897563 separately by 375 and 875.

Sol. (1) $897563 \div 375 = (897563 \times 8) \div 3000$,
$= 7180504 \div 3000$
$= 2393$; 1504 rem.
$\therefore$ the reqd. quotient $= 2393$, rem. $= 1504 \div 8 = 188$ **Ans.**

(2) $897563 \div 875 = (897563 \times 8) \div 7000$
$= 7180504 \div 7000$
$= 1025$; 5504 rem.
$\therefore$ the reqd. quotient $= 1025$, rem. $= 5504 \div 8 = 688$. **Ans.**

§55. **To divide a number by a number the figures of which are all nines, *i. e.* by 99, 999, 9999, etc.**

Rule. *Add one to the divisor to get a new divisor and divide the dividend by the divisor thus obtained; divide the quotient again by the same divisor and thus proceed as long as possible. Now add together all the quotients and all the remainders, carrying (if any) from the sum of the remainders to the sum of the quotient and adding the same to the sum of the remainders. The results thus obtained will be the required quotient and remainder.*

Example 11. Divide 129893568 by 99.

Sol. Since $99 + 1 = 100$

	quotient.		rem.
$\therefore$ $129893568 \div 100 =$	1298935	;	68
$1298935 \div 100 =$	12989	;	35
$12989 \div 100 =$	129	;	89
$129 \div 100 =$	1	;	29
$1 \div 100 =$	0	;	1
Total $=$	1312054	;	22
	2	;	2 [The carried No.]
	1312056	;	24 . Ans.

The process may be abbreviated thus :—

quotients	rem.	
1298935	68	*Explanation*
12989	35	2 was carried from remainder to quotient
129	89	and adding the same to remainder the reqd.
1	29	quotient=1312056 and remainder=24 **Ans.**
0	1	*Note the solution very carefully.*
1312056	22	

§56. **To divide a number by 998, 9997, etc.**

Rule. *If the divisor is less than the number made of all nines by 1, 2, 3, etc., then add 2, 3, 4, etc., as the case may be to get ths new divisor a number consisting of 1 and zeroes. Divide the dividend by the new divisor, multiply the quotient by the number added and then divide the result again by the new divisor. Thus proceed as long as possible. Now add together all the remainders and quotients carrying, if any, from the sum of the remainders to the sum of the quotients and also adding the added number* to *the remainder.*

Example 12. Divide 563425 by 997.

Sol. ∵ 997+3=1000

	Q.	rem.
∴ 563425÷1000	=563,	425
Now (563×3)÷1000	= 1,	689
again (1×3)÷1000	= 0,	3
	565,	117

∴ Q=565, rem.=117+3 or 120. **Ans.**

EXERCISE 9.

Find the value of the following by using the shortest method :—

1.	153956÷5	**2.**	895634÷5.
3.	9593898÷25	**4.**	2195639÷25.
5.	8893596÷15	**6.**	8893854÷15.
7.	2195683÷35.	**8.**	9935610÷55.
9.	49900256÷45.	**10.**	31935682÷65.
11.	569002863÷125	**12.**	389356881÷125.
13.	563789502÷75.	**14.**	889021566÷375.
15.	9635002056÷625.	**16.**	8935683561÷875.
17.	5689356954÷997.	**18.**	5156380242÷995.
19.	78502349835÷999.	**20.**	3190805621854÷9999.

§57. Brackets.

( ), { }, [ ] are called **brackets**. They denote that quantities enclosed within them are to be treated as one antity. Thus $6\times(3+2)$ means that we first add 2 to 3 and en multiply the result by 6. Hence $6\times(3+2)=6\times5=30$. ie above brackets are named thus :—

( ) is called a **circular** bracket.
{ } is called a **curly** bracket.
[ ] is called a **square** bracket.

Sometimes a line is placed above the terms as in $\overline{5+7}\times4$. is called **Vinculum**. It has also the force of a bracket. ence $\overline{5+7}\times4$ means 12×4 and *not* $5+28$.

§58. When a pair of brackets is used within another pair brackets, the expression within the innermost bracket is st simplified, then the expression of the next innermost ackets, and so on.

Example 1. simplify :—

$15+[25+\{35-8+(7-\overline{2+3})\}]$.

Sol. The expression $=15+[25+\{35-8+(7-5)\}]$
$=15+[25+\{35-8+2\}]$
$=15+[25+29]$
$=15+54=69$. **Ans.**

EXERCISE 10.

Simplify :—

1. $144\div(7+9)$.
2. $1872\div(18+8)$.
3. $728-(20+8)\times25$.
4. $[1580-(21+19)]\div140$.
5. $256-(70-20)\div50$.
6. $521-\overline{325+171}\div16$.
7. $15^2+(5+11)(8+4)$.
8. $(21)^2-(16-9)(21-10)$.
9. $15(26+14)-21-16$
10. $122-\overline{36+16}\div13$
11. $35-\{27-[28-21]+10)\}$.
12. $84-\{28+[27-(15-10)]\}$.
13. $182-\{73+[27-(21+\overline{4-3})\}]$.
14. $58-[58-\{58-(58-\overline{58-18})\}]$.
15. $225\div[164-\{75+(86-\overline{28-16})\}]$.
16. $(194+65)\times7+(352-220)\div11-952\div(91-35)$.
17. $6+8[3\times6+\{3+7-(8+3-6)-(2\times6\div3+3-2)\}]$.

§59. Some Very Important Typical Examples.

Example 1. What number is that which if I divide by 5, to the quotient I add 21, from the sum I take 36 and multiply the remainder by 6 then the product is 72 ?

Sol. The required number $=(72\div 6+36-21)\times 5$.

$=(12+36-21)\times 5$

$=27\times 5=135$. **Ans.**

This is a **backward process.** In this process addition is changed into subtraction and *vice versa.* Multiplication is changed into division and *vice versa.*

Example 2. What least number must be subtracted from 893564 to make it exactly divisible by 144 ?

Sol.

144)893564(6205
864
295
288
764
720
44

Evidently the number to be subtracted is 44. **Ans.**

Example 3. What least number must be added to 5638956 to make it divisible by 256 ?

Sol. Rem.$=44$

$\because 44+212=256$

$\therefore$ Evidently the number to be added is $256-44=212$ **Ans.**

256)5638956(22027
512
518
512
695
512
1836
1792
44

Note.---The new quotient will evidently be 22028.

Example 4. Find the greatest number of 6 digits which is exactly divisible by 1224.

Sol. The greatest number of 6 digits is 999999. On dividing this by 1224, the remainder is 1215.

Evidently the reqd. No. is $999999-1215=998784$. **Ans.**

Example 5. Find the least number of six digits which is exactly divisible by 1224.

Sol.

$$\begin{array}{r|l} 1224 & 100000\,(81 \\ & 9792 \\ \hline & 2080 \\ & 1224 \\ \hline & 856 \end{array}$$

Explanation

The least number of 6 digits is 100000. On dividing this by 1224, the remainder is 856. If we add $1224-856=368$ to 100000 we shall get the required number of 6 digits.

$\therefore$ the reqd. No. $=100368$. **Ans.**

Example 6. In a division sum, the divisor is 125, the quotient is 24 and the remainder is 61 ; find the dividend.

Sol. Dividend = divisor × quotient + remainder
$=125\times24+61$
$=3061$. **Ans.**

Example 7. In a division sum, the dividend is 1976, the quotient is 79 and the remainder is 1, find the divisor.

Sol. $\because$ Dividend = divisor × quotient + remainder
$\therefore$ dividend − remainder = divisor × quotient
Or $1976-1=1975=$ divisor × quotient
$\therefore$ the divisor is $1975\div79=25$. **Ans.**

Example 8. A number when divided successively by 7, 8, 9 leaves 2, 5 and 3 as remainders respectively. What are the remainders if the order of divisors is reversed ?

Sol. Complete remainder is equal to $2+7\times5+7\times8\times3=205$

Now suppose the final quotient is x, therefore the dividend $=7\times8\times9\times x+205$.

Now divide this dividend by 9, 8, 7 successively.

The remainders are 7, 6 and 2. **Ans.**

7	
8	, rem. 2
9	, rem. 5
Q $=x$	, rem. 3

9	$7\times8\times9\times x+205$
8	$7\times8\times x+22$, rem. 7
7	$7\times x+2$ rem. 6
	x, rem. 2

Test of correctness. The complete remainder in this case is also equal to $7+9\times6+9\times8\times2=205$.

Example 9. A boy multiplied 623 by a certain number and obtained 78852 as his answer. He was told that both the eights in the product were wrong ; find the multiplier and the correct product.

Sol.

623
124
2492
1246
623
77252

Explanation

(*i*) Since the unit's digit of the product is 2, therefore the unit's digit of the multiplier must be 4, multiplying 623 by 4 we get 2492.

(*ii*) Since the ten's digit of the product is 5, therefore, there must be 6 under 9 of the first row of the product. To get 6 we must multiply 623 by 2 ; $623 \times 2 = 1246$.

(*iii*) Since the last digit 7 in the product is also correct, therefore to get 7 we must multiply 623 by 1. Hence the multiplier is 124, the product is 77252. **Ans.**

Example 10. The sum of two numbers is 98 and their difference is 32 ; find the numbers.

Sol. The greater number $= (98+32) \div 2 = 130 \div 2 = 65$.
The smallest number $= (98-32) \div 2 = 66 \div 2 = 33$.
The reqd. numbers are 65 and 33. **Ans.**

Example 11. A man spending at the rate of Rs. 60 p. m. finds after 8 months that he is exceeding his income, he then reduces his expenditure to Rs. 45 p.m. ; at the end of 4 months he finds that he is just out of debt. Find the income.

Sol. Expenses for 8 months $=$ Rs. $60 \times 8 =$ Rs. 480.
Expenses for 4 months $=$ Rs. $45 \times 4 =$ Rs. 180,
Income of 1 year $= 480 + 180 =$ Rs. 660.
$\therefore$ Income per mensem $= 660 \div 12$.
$=$ Rs. 55. **Ans.**

EXERCISE 11.

1. What number is that which if I divide by 7, if to this quotient I add 14, from the sum if I take 20 and multiply the remainder by 8 then the product is 80 ?

2. What number is that which if I divide by 16, if to the quotient I add 10, if from the sum I take 12 and then multiply the remainder by 13, the product is 52 ?

3. What least number must be subtracted from 3593809 to make it exactly divisible by 256 ?

4. What is the least number which when subtracted from 8935063 makes the remainder divisible by 196 ?

5. What least number must be added to 210359 to make it divisible by 117 ?

6. What is the least number which when added to 4809356 makes the sum exactly divisible by 245 ?

7. Find the greatest number of 5 digits which is exactly divisible by 265.

8. Find the greatest number of 6 digits which is exactly divisible by 315.

9. Find the least number of 6 digits which is exactly divisible by 216.

10. Find the least number of 5 digits which is exactly divisible by 520.

11. Find the nearest number to 21935 which is exactly divisible by 516.

12. Find the nearest number to 26572 which is exactly divisible by 320.

13. In a division sum the divisor is 124, the quotient is 21 and the remainder is 51 ; find the dividend.

14. In a division sum the dividend is 2523, the quotient is 21 and the remainder is 24, find the divisor.

15. A number is divided by 541 and the successive partial dividends are 1239, 1573 and 4914. Find the dividend, quotient and remainder.

16. A number is divided by 321 and the successive partial dividends are 359, 383, 624 and 3038. Find the dividend, quotient and remainder.

17. The sum of two numbers is 854 and their difference is 198 ; find the numbers.

18. The difference of two numbers is 1547 and the largest number exceeds the smaller by 105 ; find the numbers.

19. The difference of two numbers is 415 and the greater is 956 ; what is the smaller number ?

20. In a division sum the quotient is 5 times the divisor and 15 times the remainder. If the remainder be 6, find the dividend.

21. A certain number when divided successively by 5, 4, 6 leaves 1, 3, 3 as remainders respectively. What are the remainders if the order of divisors be reversed ?

22. A number is successively divided by 5, 6, 8 leaving remainders 3, 4, 7 respectively. What will be remainders if the same number is divided by 8, 6, 5 ?

23. A number is divided thus :—

x) * * * *

y) * * * *, remainder 1

* * *, remainder 3

The complete remainder is 19 ; find the value of x.

24. A number is divided thus :—

) * * * * * *

7) * * * * * *, remainder 3

1 0 1 8 2, remainder 4

If the true remainder be 23, find the dividend.

25. A number is successively divided by 5, 6 and 8 leaving remainders 2, 3 and 4 respectively. If the true quotient be 21 find the dividend.

26. A number is successively divided by 7, 5 and 4 leaving remainders 3, 1 and 2 respectively. If the true quotient be 24, find the dividend.

27. Find the missing figures (denoted by stars) in the following examples on multiplication :—

(a)

```
      8 9 7 2
        * * *
    ---------
      * * * * *
    * * * * *
  * * * * *
---------------
* * * * 4 3 2
```

(b)

```
      5 9 3 8
        * * *
    ---------
    * * * * * *
  * * * * *
* * * * *
---------------
* * * * 3 3 4
```

28. A boy multiplies a number by 213 and obtains the product 1933036. If both the first *threes* are wrong but the other figures are right, find the correct answer and also find the multiplicand.

29. A boy multiplies 1526 by a certain number and obtains the product 477424. If both the sevens are wrong but the other figures are right, find the correct answer and also the multiplier.

30. At a game of cricket, A and B together score 32 runs ; B and C together score 37 runs ; and A and C together score 25 runs ; find the number of runs scored by each of them.

31. A and B together earn Rs. 130 in a month; B and C together earn Rs. 170; and A and C together earn Rs. 150; find the amount of rupees earned by each of them.

32. Divide Rs. 98 among A, B and C so that A may receive Rs. 11 more than B and B Rs. 6 more than C.

33. Divide Rs. 86 among A, B and C so that A may receive Rs. 7 more than B and B may receive Rs. 5 more than C.

34. A man spending at the rate of Rs. 820 a year finds after 5 years that he is exceeding his income, he then reduces his expenditure to Rs. 540 a year. After 2 years he clears off his debt; find his income.

35. A clerk living at the rate of Rs. 75 p.m. for a year finds that he is running into debt; he then reduces his expenditure to Rs. 40 p.m. for a year and then finds that he has cleared off his debt and saved Rs. 60; find his salary.

36. A farmer exchanged 25 cows for 125 sheep worth Rs. 10 per head; what did he receive for a cow?

37. A father left his eldest son Rs. 400 more than his second son; and he left his second son Rs. 300 more than his third; his whole estate was worth Rs. 6,400; what did each son receive?

38. Divide 4,680 rupees, after giving away 180 rupees to the poor, between A, B and C, giving B 216 rupees more than A, and C 336 rupees more than B.

39. The quotient = 5 times divisor=7 times remainder =105; find the dividend.

40. The Duke of Wellington died in the year 1852 aged 83; Napoleon was born in the year as the Duke and died in 1821; what was Napoleon's age at the time of his death?

41. Divide Rs. 1,800 among A, B and C, so that for every 2 rupees A gets, B shall get 3 and C 4.

42. Divide 5248 rupees among A, B and C, so that for every 5 rupees given to A, B may get 11 and C 16.

43. The price of a carriage with horse is 1920 rupees and the price of the carriage is 5 times that of the horse. Find the price of the horse.

44. If 23 men earn Rs. 1610 in a month, how many men will earn Rs. 1750 in the same time?

45. A gentleman left Rs. 112500 to be divided amongst his 4 sons and 3 daughters in such a way that each son might get three times as much as each daughter. How much did each son receive ?

46. The sum of two numbers is 15678 and the larger number exceeds the smaller by 1234. Find the numbers.

47. What number multiplied by 1256 will give the same product as (*i*) 314 by 476 ; (*ii*) 7536 by 378 ?

48. What least number must be added to 40398, so that the sum may be divisible by 2073 ?

49. The quotient which is obtained by dividing 39875365 by a certain number is 4607 and the remainder is 6387. Find the divisor.

50. The product of two numbers is 20405426 and half of one of them is 1739 ; find the other number.

51. A man bought 81 seers of ghee at Rs. 3 a seer and 127 seers of an inferior type at Rs. 2 a seer. He mixed the two and sold the whole for Rs. 500. How much did he gain or lose ?

52. *A* is 21 years older than *B* and 8 years younger than *C* who is 60 years of age. *D* is as old as the ages of *A* and *B* together. Is *C* older or younger than *D* ? and by how much ?

53. A man divided his property worth Rs. 12765 amongst his 4 sons in such a manner that the eldest received Rs. 145 more than the second, the second Rs. 130 more than the third and the third Rs. 120 more than the fourth. How much did each receive ?

54. A gentleman divided Rs. 123600 among his two sons, four daughters and one sister in such a way that each daughter received twice as much as the sister and each son one-half of what the three daughters received. How much did the sister get ?

55. Two men walk towards each other at the rate of 8 and 9 miles per hour. If the distance between them be 153 miles, when will they meet ?

CHAPTER III

COMPOUND QUANTITIES

§1. **Tables.** The following tables should be committe to memory by a student of Arithmetic :—

British Indian Money.

3 Pies (*p.*)	make	1	Pice
4 Pice or 12 Pies	,,	1	Anna (*a.*)
16 Annas	,,	1	Rupee (Re.)
15 Rupees	,,	1	Pound or Sovereign [£)

English Money.

4 Farthings (f. or q.)	make	1	Penny (*d*)
12 Pence	,,	1	Shilling (*s.*)
20 Shillings	,,	1	Pound or Sovereign (£)

---

2 Shillings	make	1	Florin
2 Shillings 6 Pence	,,	1	Half Crown
5 Shillings	,,	1	Crown
10 Shillings	,,	1	Half Sovereign
21 Shillings	,,	1	Guinea

Indian Weight.

8 Chawals	make	1	Rati
8 Ratis	,,	1	Masha
12 Mashas	,,	1	Tola
5 Tolas	,,	1	Chatak (ch.)
16 Chataks	,,	1	Seer (sr.)
40 Seers	,,	1	Maund (md.)

Avoirdupois Weight

(For weighing common goods.)

16 Drams (dr.)	make	1	Ounce (oz.)
16 Ounces	,,	1	Pound (lb.)
28 Pounds	,,	1	Quarter (qr.]
4 Quarters	,,	1	Hundred weight (cwt.)
20 Hundred weights	,,	1	Ton (ton.]

Note:-14 lbs, make 1 ston (st·)

Troy Weight.

(For weighing gold, silver, jewellery.)

24 Grains (gr.)	make	1 Penny weight (dwt.)
20 Penny weights	,,	1 Ounce (oz.)
12 Ounces	,,	1 Pound (lb.)

Apothecary's weight.

(Used in mixing medicines.)

20 grains	make	1 Scruple (scr.)
3 Scruples	,,	1 Dram (dr.)
8 Drams	,,	1 Ounce (oz.)
12 Ounces	,,	1 Pound (lb.)

Linear Measures

12 Inches (in.)	make	1 Foot (ft.)
3 Feet	,,	1 Yard (yd.)
$5\frac{1}{2}$ Yards	,,	1 Pole (po.)
40 Poles or 220 Yards	,,	1 Furlong (fur.)
8 Furlongs or 1760 yards	,,	1 Mile (mi.)
3 Miles	,,	1 League (lea.)

Note—Gunter's chain is used in land surveying.
1 chain=200 links=22 yds.
80 chains =1 mile.

Cloth Measures.

$2\frac{1}{4}$ Inches (in.)	make	1 Nail (nl.)
3 Nails	,,	1 Quarter (qr.) or span.
2 Quarters or spans	,,	1 Cubit.
4 Quarters or 2 cubits	,,	1 Yard.
3 Quarters	,,	1 Flemish ell
5 Quarters	,,	1 English ell.
6 Quarters	,,	1 French ell.

British Square Measures.

144 Square Inches (sq. in.)	make	1 Square Foot (sq. ft.)
9 Square Feet	,,	1 Square Yard (sq. yd.)
$30\frac{1}{4}$ Square Yards	,,	1 Square Pole (sq. po.)
40 Square Poles	,,	1 Rood (ro.)
4 Roods or 4840 sq. yds.	,,	1 Acre (a.)
640 Acres	,,	1 Square Mile.

Note. 10 Square chains or 4840 sq. yds. make 1 acre.

Punjabi Square Measures.

1 Square Karm	makes	1 Sersai
9 Sersais	make	1 Marla
20 Marlas	,,	1 Kanal
4 Kanals	,,	1 Bigha
2 Bighas	,,	1 Ghumaon
324 Square Yards	,,	1 Kanal.

Cubic Measures.

1728 Cubic Inches	make	1 Cubic Foot (cub. ft.)
27 Cubic Feet	,,	1 Cubic Yard (c. yd.)

Measures of time

60 Seconds (sec.)	make	1 Minute (m.)
60 Minutes	,,	1 Hour (hr.)
24 hours	,,	1 Day
7 days	,,	1 Week
4 Weeks	,,	1 month
12 Months	,,	1 Year
100 years	,,	1 Century.

Note. 365 days make 1 common year and 366 days make 1 Leap year. But for reducing days to the fraction of a year we always divide by 365.

Measure of Number

12 Units	make	1 Dozen
12 Dozen	,,	1 Gross
20 Units	,,	1 Score

Paper Measure of Capacity

24 Sheets of paper	make	1 Quire
20 Quires	,,	1 Ream
10 Reams	,,	1 Bale.

Measure of Capacity

20 Ounces	make	1 Pint (pt.)
4 Pints	,,	1 Quart (qt.)
4 Quarts	,,	1 Gallon (gal.)
2 Gallons	,,	1 Peck (pk.)
4 Pecks	,,	1 Bushel (bu.)
8 Bushels	,,	1 Quarter (qr.)

Angular Measures

60 Seconds (60″)	make	1 Minute (1′)
60 Minutes	,,	1 Degree (1°)
90 Degrees	,,	1 Right angle.

REDUCTION

§2. A quantity when expressed in one denomination is called a **simple quantity** and when expressed in more than one denomination, it is called a **compound quantity** ; as, Rs. 2 is a simple quantity and Rs. 2, 3*a*. 4*p*. is a compound quantity.

§3. **Reduction** is the process by which a compound quantity is expressed as a simple quantity or *vice versa*.

The following solved example will best illustrate the process :—

Descending Reduction

Example 1. Reduce Rs. 5, 6*a*. 9*p*. to pies.

Sol.

Rs.	*a*.	p.
5	6	4
16	:	:
80	:	:
6	:	:
86 *a*.		:
12		:
1032		:
9.....		:
1041. *p* **Ans.**		

Hence the following

Rule. *Multiply the number in the highest denomination by the number of units of the next inferior denomination contained in a unit of the higher. Add the number of the inferior denomination if any and continue this process for each succeeding denomination till the required result is obtained.*

Example 2. Reduce £ 15 10*s*. 8*d*. to pence.

Ascending Reduction

Example 1. Reduce 1041 pies to Rs. *a*.*p*.

Sol 12)1041 *p*.

16)86*a*.+9*p*.

Rs.5+6*a*.

Rs. 5, 6*a*. 9*p*. **Ans.**

Hence the following

Rule. *Divide the given number by the number of units which make a unit of the next higher denomination putting down the remainder if any. This remainder will be of the same denomination as its dividend. Continue this process till the required result is obtained.*

Example 2. Reduce 3728 pence to £ .*s*.*d*.

Sol.

£.	s.	d.
15	10	8
20		
300		
10		
310s.		
12		
3720		
8		
3728d.		

Ans.

Example 3. Reduce 15 mds. 37 sr. 8 ch. to chataks.

Sol.

mds.	sr.	ch.
15	37	8
40		
600		
37		
637 sr.		
16		
10192		
8		
10200 ch.		

Ans.

Example 4. Rednce 15 tons 8 cwt. 3 qr. 16 lbs. to lbs.

Sol.

tons	cwt.	qr.	lb.
15	8	3	16
20			
300			
8			
308 cwt.			
4			
1232			
3			
1235 qr.			
28			
9880			
2470			
34580			
16			

Sol. 12)3728d.

20)310s. +8d.

£15+10s.

∴ £ 15. 10s. 8d. **Ans.**

Example 3. Reduce 10200 chataks to maunds etc.

Sol. 16)10200 ch.

40) 637 sr.+8ch.

15 mds.+37 sr.

∴ 15 mds. 37 sr. 8 ch. **Ans.**

Example 4. Reduce 34596 lbs. to tons etc.

Sol. 28 { 4)34596 lbs.
7) 8649

4) 1235qr.+16 lbs.

20) 308 cwt.+3 qr.

15 tons+8cwt.

∴ 15 tons 8 cwt. 3 qr. 16 lbs. **Ans.**

Example 5. Reduce 10 miles 116 yds. 2 ft. 3 in. to inches.

Sol.

Miles	yds.	ft.	in.
10	116	2	3
1760			
17600			
116			
17716 yds.			
3			
53148			
2			
53150 ft.			
12			
637800			
3			

637803 inches **Ans.**

Example 5. Reduce 637803 inches to miles etc.

Sol. 12)637803 inches.

3) 53150 ft. +3 in.

10) 17716 yds. +2 ft.

1760 { 11) 1771 +6 rem.

16) 161

10 mi. +1 rem.

Rem. $= 6 + 10 \times 0 + 10 \times 11 \times 1$

$= 6 + 110 = 116$ yds.

∴ 10 miles 116 yds. 2 ft. 3 in. **Ans.**

EXERCISE 12.

Reduce to annas :—

1. Rs. 15, 7*a*.
2. Rs. 21, 4*a*.
3. Rs. 37, 6*a*.
4. Rs. 35, 9*a*.
5. Rs. 47, 15*a*.
6. Rs. 55, 10*a*.

Reduce to pies :—

7. Rs. 21, 5*a*. 4*p*.
8. Rs. 23, 6*a*. 9*p*.
9. Rs. 35, 6*a* 9*p*.
10. Rs. 28, 13*a*. 4*p*.
11. Rs. 26, 5*a* .9*p*.
12. Rs. 28, 5*a*. 10*p*.

Reduce to pice :—

13. Rs. 31, 9*a*. 6*p*.
14. Rs. 31, 12*a* 9*p*.
15. Rs. 51, 9*a*. 3*p*.
16. Rs. 28, 10*a*. 9*p*.
17. Rs. 28, 13*a*. 3*p*.
18. Rs. 41, 15*a*, 9*p*.

Reduce to shillings :—

19. £ 21, 9*s*.
20. £ 48, 15*s*.
21. £ 31, 18*s*.
22. £ 45, 16*s*.
23. £ 56, 19*s*.
24. £ 37, 6*s*.

Reduce to pence :—

25. £ 27, 15*s*. 6*d*.
26. £ 99, 15*s*. 9*d*.
27. £ 21, 9*s*. 7*d*.
28. £ 45, 13*s*. 3*d*.
29. £ 31, 14*s*. 10*d*.
30. £ 137, 10*s*. 6*d*.

Reduce to chataks :—

31. 45 mds. 8 sr. 9 ch.
32. 48 mds. 9 sr. 15 ch.
33. 35 mds. 15 sr. 10 ch.
34. 85 mds. 37 sr. 10 ch.
35. 89 mds. 21 sr. 15 ch.
36. 78 mds. 35 sr. 12 ch.

Reduce to ratis :—

37. 8 tolas, 9 mashas 6 ratis.
38. 9 tola 10 mashas 7 ratis.
39. 18 tolas 10 mashas 7 ratis.
40. 21 tolas 11 mashas 3 ratis.
41. 98 tolas 11 mashas 4 ratis.
42. 85 tolas 7 mashas 7 ratis.

Reduce to inches :—

43. 38 yds. 2 ft. 9 in.
44. 31 yards 2 ft. 5 in.
45. 46 yards 1 ft. 8 in.
46. 84 yds. 1 ft. 11 in.
47. 87 yds. 2 ft. 11 in.
48. 21 yds. 2 ft. 4 in.

Reduce to seconds :—

49. 5 days 3 hrs. 5 min. 35 sec.
50. 26 days 18 hrs. 8 min. 40 sec.
51. 21 days 20 hrs. 35 min. 30 sec.
52. 47 days 12 hrs. 27 min. 35 sec.
53. 45 days 22 hrs. 18 min. 45 sec.
54. 25 days 18 hrs. 45 min. 36 sec.

Reduce to lbs :—

55. 7 tons 15 cwt. 2 qr. 21 lb.
56. 22 tons 18 cwt. 3 qr. 14 lb.
57. 22 tons 16 cwt. 3 qr. 7 lb.
58. 16 tons 12 cwt. 2 qr. 11 lb.
59. 35 tons 12 cwt. 1 qr. 26 lb.
60. 22 tons 13 cwt. 3 qr. 24 lb.

Inverse operations.

Reduce to Rs. *a. p.* :—

61. 28972 pies.
62. 23575 pies.
63. 13787 pies.
64. 378356 pies.
65. 383092 pies.
66. 483569 pies.
67. 383542 pice.
68. 318302 pice.
69. 359352 pice.
70. 110001 pice.

Reduce to £. *s. d.* :—

71. 215356 pence.
72. 350204 pence.
73. 893596 pence.
74. 832029 pence.
75. 310983 pence.
76. 889354 pence.

77. 123859 farthings. 78. 8356 guineas.
79. 5635 guineas. 80. 1608 crowns.
81. 11508 half crowns. 82. 1823 florins.

Reduce to mds. chks. etc. :—

83. 283059 chks. 84. 350093 chks.
85. 478309 chks. 86. 2683505 tolas.
87. 3108050 tolas. 88. 4780506 tolas.
89. 2185028 mashas. 90. 4780291 mashas.

Reduce to tons etc.

91. 835935 qr. 92. 3783594 lb.
93. 1835694 lb. 94. 3802564 lb.
95. 2100093 lb. 96. 4702356 lb.

Reduce to weeks, days, etc. :—

97. 893020842 minutes. 98. 450208354 minutes.
99. 435635942 sec. 100. 485283591 seconds.

Reduce to miles yards etc.

101. 563056935 inches. 102. 856380354 inches.
103. 583015925 inches. 104. 410205935 inches.

Reduce to right angles, degrees etc :—

105. 853428′. 106. 215028′.
107. 473590″. 108. 990259″.

Reduce to acres, sq. yards, etc :—

109. 7856937 sq. inches. 110. 9999999 sq. inches.
111. 7777777 sq, inches. 112. 87000001 sq. inches.

COMPOUND ADDITION

§4. **Compound Addition** is the method of finding the sum of several quantities (of the same kind) expressed in more than one denomination.

Rule. *Arrange the quantities so that units of the same denomination may be under one another, then draw a line underneath, add the number of the lowest denomination and reduce the sum to the next higher denomination, put down the remainder and carry the quotient. Repeat this operation in all other subsequent denominations.*

Example. Add together Rs. 15, 8*a*. 9*p*., Rs. 21, 9*a*. 10*p*. and Rs. 45, 7*a*. 6*p*.

Sol.

Rs.	*a.*	*p.*
15	8	9
21	9	10
45	7	6
8	10	1

$9p.+10p.+6p.=25p.=2a.\ 1p.$; put down $1p.$ and carry $2a.$; $2a+8a.+9a.+7a.=26a.=$ Re. $1,10a.$; put down $10a.$ and carry Re. 1. Now add Rs. and put down the sum.

EXERCISE 13.

Add the following :—

1.

Rs.	*a.*	*p.*
21	7	8
35	8	9
45	6	3
21	4	2

2.

Rs.	*a.*	*p.*
35	9	8
56	14	4
39	15	7
21	4	9

3.

Rs.	*a.*	*p*
17	5	4
21	9	7
35	13	4
47	15	9

4.

Rs.	*a.*	*p.*
501	7	3
288	12	6
85	9	11
37	10	9

5.

Rs.	*a.*	*p.*
201	11	3
88	7	2
45	12	9
113	13	8

6.

Rs.	*a.*	*p.*
805	14	7
87	11	8
203	10	5
86	8	9

7.

£	*s.*	*d.*
121	15	9
2035	8	11
335	17	10
556	12	8
4356	19	9
437	8	9

8.

£	*s.*	*d.*
1139	7	8
521	9	7
326	14	2
8972	18	7
25	13	11
7	14	9

9.

£	*s.*	*d.*
1625	7	4
838	11	4
325	13	9
35	7	8
16	18	9
321	9	8

10.

£	*s.*	*d*
728	15	8
156	12	9
27	16	7
8356	7	11
356	19	8
28	16	7

11.

£	*s.*	*d.*
87	10	3
523	9	7
1124	4	11
534	7	2
302	9	8
27	19	11

12.

£	*s.*	*d.*
256	8	3
57	12	7
1359	7	11
326	4	8
256	9	10
28	16	6

13.

mds.	srs.	chs.
88	37	9
47	15	15
137	18	14
8356	28	12
431	9	9
126	35	10

14.

mds.	srs.	chs.
47	21	9
521	8	15
3569	26	11
728	37	7
3526	25	9
76	18	11

15.

mds.	srs.	chs.
38	14	12
219	12	9
5139	7	15
374	29	8
88	37	15
12	17	10

16.

yds.	ft.	in.
21	2	8
576	1	9
728	2	11
135	1	10
325	2	10
266	1	9

17.

yds.	ft.	in
21	1	5
35	2	9
356	2	7
156	1	11
28	2	9
27	1	7

18.

yds.	ft.	in.
17	2	9
85	1	11
382	2	9
83	1	10
295	2	9
85	1	7

19.

tolas	mashas	ratis
7	8	4
12	9	7
8	11	6
85	7	4
32	10	2

20.

tolas	mashas	ratis
85	7	5
21	9	4
121	4	6
86	10	7
46	5	4

21.

tolas	mashas	ratis
55	4	3
88	9	7
55	7	6
44	4	2
21	5	9

22.

mi.	fur.	po.
19	6	15
28	7	35
35	4	21
39	2	28
47	5	39

23.

mi.	fur.	po.
21	7	4
39	4	15
235	5	35
37	2	18
45	3	29

24.

mi.	fur.	po.
49	6	18
21	2	23
56	4	18
23	2	21
328	7	34

25.

hr.	min.	sec.
15	35	40
21	29	30
18	45	35
19	25	21
24	28	45

26.

hr.	min.	sec.
18	21	9
21	18	24
45	35	49
56	28	48
48	52	28

27.

hr.	min.	sec.
99	25	45
45	36	25
75	21	49
28	35	55
27	26	48

28.

sq. yds.	sq. ft.	sq. in.
45	7	121
36	8	135
40	6	134
121	4	79
18	2	16

29.

sq. yds.	sq. ft.	sq. in.
145	4	128
116	7	115
156	4	75
28	7	28
45	6	16

30.

sq. yds.	sq. ft.	sq. in.
156	4	29
134	6	99
21	3	121
128	2	142
75	8	17

31.

ac.	r.	sq.po.
17	2	18
28	3	26
121	1	36
32	0	39
16	2	7

32.

ac.	r.	sq.po.
156	0	35
117	1	34
85	2	5
56	3	27
8	2	9

33.

ac.	r.	sq.po.
156	2	17
77	1	39
18	3	7
78	2	29
7	3	35

34.

wk.	da.	hr.
7	6	23
9	4	17
17	3	12
14	2	21
7	1	8

35.

wk.	da.	hr.
9	6	15
15	3	8
17	4	3
5	2	22
4	3	16

36.

wk.	da.	hr.
21	4	16
15	6	4
8	5	21
85	2	22
6	3	18

37.

deg.	min.	sec.
28	21	35
17	35	45
21	9	35
17	28	9
8	36	16

38.

deg.	min.	sec.
45	16	35
29	8	27
21	15	9
17	8	47
25	19	28

39.

rt. ang. deg.	min.
45	70
24	21
28	17
9	8
21	25

COMPOUND SUBTRACTION

§5. **Compound Subtraction** is the method of finding the difference of two quantities (of the same kind) expressed in more than one denomination.

Rule. *Write down the smaller quantity below the greater so that the units of the same denomination may be under one another and draw a line underneath. Begin from the right and subtract each number of the lower line from the corresponding number of the upper line. If the number of the lower line be greater than the corresponding number of the upper line, then add the upper number as many numbers of the same denomination as make a unit of the next higher denomination, also add 1 to the next preceding number of the lower line.*

Example. Subtract Rs. 8, 3*a*. 4*p*. from Rs. 15, 1*a*. 2*p*.

Sol.

Rs.	*a*.	*p*.
15	1	2
8	3	4
6	13	10

Explanation. Add 12*p*. to 2*p*. making 14*p*. ; 14*p*.—4*p*. =10*p*. ; put down 10*p*. Now add 1 to 3 making 4. Again 4*a*. cannot be subtracted from 1*a*. add 16*a*. to 1*a*. making 17*a*. ; 17*a*.—4*a*.=13*a*. ; put down 13*a*. and add 1 to 8 making Rs. 9 ; Rs. 15—Rs. 9=Rs. 6 put down Rs. 6.

∴ the difference is Rs. 6, 13*a*. 10*p*. **Ans.**

EXERCISE 14.

Perform the operation of the subtraction in the following :—

	Rs.	*a.*	*p.*		Rs.	*a.*	*p.*	
1.	28	7	9	2.	21	9	7	
	24	6	5		16	8	5	
	Rs.	*a.*	*p.*		Rs.	*a.*	*p.*	
4.	35	11	7	5.	126	12	8	
	29	15	11		87	13	9	
	£	*s.*	*d.*		£	*s.*	*d.*	
7.	128	5	9	8.	156	17	7	
	57	14	10		148	9	11	
	£	*s.*	*d.*		£	*s.*	*d.*	
10.	129	11	4	11.	123	6	7	
	128	13	7		119	12	9	
	ton	cwt.	qr.		ton	cwt.	qr.	
13.	21	16	2	14.	124	4	1	
	18	19	3		56	18	2	
	mds.	srs.	ch.		mds.	srs.	ch.	
16.	33	27	8	17.	45	17	3	
	17	37	9		29	39	5	
	yds.	ft.	in.		yds.	ft.	in.	
19.	35	2	4	20.	41	1	8	
	29	1	11		37	2	9	

	Rs.	*a.*	*p.*	
3.	56	7	2	
	29	9	8	
	Rs.	*a.*	*p.*	
6.	218	4	3	
	159	9	7	
	£	*s.*	*d.*	
9.	111	2	4	
	59	14	11	
	£	*s.*	*d.*	
12.	118	2	3	
	117	11	7	
	ton	cwt.	qr.	
15.	121	2	3	
	89	16	2	
	mds.	srs.	ch.	
18.	127	19	12	
	89	38	15	
	yds.	ft.	in.	
21.	36	1	7	
	35	2	11	

Subtract (22—30) :—

22. Rs. 1585, 6*a.* 8*p.* from Rs. 2912, 2*a.* 3*p.*

23. Rs. 2109, 2*a.* 7*p.* from Rs. 3509, 1*a.* 2*p.*

24. £1385, 8*s.* 9*d.* from £ 1936, 7*s.* 5*d.*

25. £ 2139, 17*s.* 10*d.* from £ 4521, 12*s.* 7*d.*

26. 136 mds. 28 srs. 15 chks. from 247 mds. 21 srs. 12 chks.

27. 128 mds. 29 srs. 12 chks. from 318 mds. 28 srs. 9 chks.

28. 121 yds. 1 ft. 9 in. from 156 yds. 2 ft. 3 in.

29. 86 tons 12 cwt. 2 qr. from 151 tons 7 cwt. 1 qr.

30. 128 tons 18 cwt. 3 qr. from 159 tons 6 cwt. 2 qr.

31. What must be added to Rs. 128, 3*a*. 9*p*. to make Rs. 156, 1*a*. 2*p*. ?

32. What must be subtracted from £ 121, 2*s*. 6*d*. to get the remainder £ 89, 17*s*. 9*d*. ?

Find the value of the following :—

33. Rs. 41, 3*a*. 4*p*.+Rs. 21, 2*a*. 9*p*.—Rs. 55, 11*a*. 3*p*. —Rs. 17, 12*a*. 2*p*.+Rs. 16, 8*a*. 3*p*.

34. £ 41, 17*s*. 6*d*.+£17, 15*s*. 2*d*.—£ 121, 8*s*. 2*d*.—£ 2, 3*s*. 11*d*. +£78, 2*s*. 7*d*.

35. 21 mds. 28 srs. 10 chks.+27 mds. 24 srs. 12 chks.+ 28 mds. 12 srs. 13 chks.—22 mds. 3 srs. 9 chks.—29 mds. 29 srs. 3 chks.

COMPOUND MULTIPLICATION

§6. **Compound Multiplication** is the method by which the sum of a given number of repetitions of a compound quantity is found.

The multiplicand in compound multiplication is a compound quantity and the multiplier an abstract number.

§7. When the multiplier is a number not greater than 20, we have the following rule:—

Rule. *Write the multiplier under the lowest denomination of the multiplicand and draw a line underneath. Multiply the lowest denomination by the multiplier and reduce the product to the next higher denomination, put down the remainder and carry the quotient to the next product. Repeat the process till all the denominations are multiplied.*

Example 1. Multiply Rs. 5, 6*a*. 8*p*. by 9.

Sol.

Rs.	*a*.	*p*.
5	6	8
		9
48	12	0

Ans.

Explanation. (*i*) 8*p*.×9=72*p*.=6*a*. 0*p*. carry 6*a*. (*ii*) 6*a*.×9=54*a*. ; 54*a*.+6*a*. =60*a*. =Rs. 3, 12*a*. ; put down 12*a*. and carry Rs. 3. (*iii*) Rs. 5×9=Rs. 45 ; Rs. 45 +Rs. 3=Rs. 48 ; put down Rs. 48.

§8. When the multiplier is a number greater than 20 but can be split up into factors, each factor being less than 20, we have the following rule :—

Rule. *Multiply the multiplicand by one of the factors as in Art. 7 and then multiply the product by another and so on.*

Example 2. Multiply £ 15, 13*s.* 4*d.* by 132.

Sol.

£	*s.*	*d.*
15	13	4
		12
188	0	0
		11
£ 2068	0	0 **Ans.**

Explanation. ∵ $132 = 12 \times 11$ therefore, first multiply by 12 and then by 11. The result is the required product.

§9. When the multiplier cannot be split up into convenient factors, the process is a combination of multiplication and addition or multiplication and subtraction; thus :—

Example 3. Multiply Rs. 5, 8*a.* 9*p.* by 109 and 83.

Sol. $109 = 12 \times 9 + 1$; $83 = 12 \times 7 - 1$;

(*i*)

	Rs.	*a.*	*p.*
	5	8	9
			12
	66	9	0
			9
	599	1	0
Add	5	8	9
	604	9	9 **Ans.**

	Rs.	*a.*	*p.*
	5	8	9
			12
	66	9	0
			7
	465	15	0
Subtract	5	8	9
	460	6	3 **Ans.**

Aliter.

Rs.	*a.*	*p.*
5	8	9
		109
604	9	9

Explanation.

(*i*) $9p. \times 109 = 981p. = 81a.\ 9p.$ put down 9*p.* and carry 81*a.*

(*ii*) $8a. \times 109 = 872a.$; $872a. + 81a. = 953a. =$ Rs. 59, 9*a.* ; put down 9*a.* and carry Rs. 59. (*iii*) Rs. $5 \times 109 =$ Rs. 545, ; Rs. 545 + Rs. 59 = Rs. 604 ; put down Rs. 604.

∴ the reqd. product is Rs. 604, 9*a.* 9*p.* **Ans.**

Similarly we can solve the (*ii*) part.

EXERCISE 15

Multiply (1—20) :—

1. Rs. 8, 5*a*. 4*p*. by 12.
2. Rs. 21, 10*a*. 8*p*. by 15.
3. Rs. 12, 2*a*. 8*p*. by 18.
4. Rs. 15, 1*a*. 4*p*. by 16.
5. Rs. 21, 5*a*. 6*p*. by 21.
6. Rs. 26, 4*a*. 8*p*. by 25.
7. £ 17, 13*s*. 4*d*. by 24.
8. £ 12, 17*s*. 6*d*. by 36.
9. £ 24, 17*s*. 8*d*. by 40.
10. £ 16, 19*s*. 10*d*. by 42.
11. £ 42, 18*s*. 6*d*. by 56.
12. £ 58, 12*s*. 8*d*. by 64.
13. £ 156, 17*s*. 8*d*. by 133.
14. £ 148, 19*s*. 6*d*. by 139.
15. 15 mds. 37 srs. 8 chks. separately by 117, 119.
16. 21 mds. 28 srs. 12 chks. ,, by 128, 113.
17. 15 tons 16 cwt. 2 qr. 12 lb. ,, by 104, 93.
18. 45 tons 12 cwt. 3 qr. 21 lb. ,, by 135, 149.
19. 35 sq. yards 8 sq. ft. 116 sq. in. separately by 121, 129.
20. 36 sq. yds. 7 sq. ft. 120 sq. inch separately by 124, 141.
21. What is the cost of 145 bags of wheat at Rs. 25, 3*a*. 8*p*. per bag ?
22. Find the value of 156 bags of rice at Rs. 38, 11*a*. 9*p*. per bag.
23. Find the value of 1156 maunds of wheat at Rs. 5, 10*a*. 6*p*. per maund.
24. Find the wages of 97 workmen for 15 days at Re. 1, 2*a*. 6*p*. per day for each man.
25. A sum of money was divided among 257 men ; each man got £ 27, 13*s*. 9*d*. ; find the sum divided.

COMPOUND DIVISION

§10. **Compound Division** is the method of dividing a compound quantity by a given number and to find the value of one of the parts. It is also the method of finding how often one compound quantity is contained in another compound quantity of the same kind.

Thus in compound division we consider two cases. In the first case the dividend is a compound quantity, the divisor is an abstract number and quotient is a compound quantity. This method is called **Partition.**

In the second case the dividend and the divisor are both compound quantities of the same kind and the quotient is an abstract number. The method is called **Quotition.**

§11. **Case I—Partition.**

Rule. *First find out how often the divisor is contained in the highest denomination of the given quantity; put down the quotient and reduce the remainder to the next inferior denomination. Add to it the number of the same denomination in the dividend and repeat the operation till all the denominations are divided by the divisor.*

Example 1. Divide Rs. 547, 4*a*. 3*p*. by 15.

Sol.

	Rs.	*a*.	*p*.
15)	547	4	3
	36	7	9

Exlpanation.

(*i*) Rs. 547÷15=Q. Rs. 36 Rem. Rs. 7. Put down Rs. 36 and reduce Rs. 7 to annas.

(*ii*) Rs. 7×16=112*a*.112*a*.+4*a*.=116*a*. ; 116*a*.÷15=Q 7*a*. Rem. 11*a*. Put down 7*a*. and reduce 11*a*. to pies.

(*iii*)11*a*.×12=132*p*. 132*p*.+3*p*.=135*p*. ; 135*p*÷15=9*p*. Put down 9*p*. ∴ Rs. 36, 7*a*. 9*p*. **Ans.**

Example 2. Divide Rs. 1451, 10*a*. 3*p*. by 51and 75.

Sol.

	Rs.	*a*.	*p*.	
51)	1451	10	3	(Rs. 28
	102			
	431			
	408			
	23			
	16			
	368			
	10			
51)	378			(7*a*.
	357			
	21			
	12			
	252			
	3			
51)	255			(5*p*.
	255			

	Rs.	*a*.	*p*.	
75)	1451	10	3	(Rs19.
	75			
	701			
	675			
	26			
	16			
	416			
	10			
75)	426			(5*a*.
	375			
	51			
	12			
	612			
	3			
75)	615			(8*p*.
	600			
	15			

∴ the reqd. quotients are :— (*i*) Rs. 28, 7*a*. 5*p*. (*ii*) Rs. 19, 5*a*. 8*p*. and 15*p*. over. **Ans.**

§12. **To divide by 10, 100, 1000, etc.**

Rule. *Cut off from the right of each dividend as many*

figures as there are zeroes in the divisor. The figures on the left will denote the quotient and figures to the right the remainder.

Example 3. Divide £3371, 13s. 4d. by 100 and 700.

Sol.

£	s.	d.
100)33,71	13	4
20	:	:
1420	:	:
13	:	:
14, 33		:
12		:
96		:
4		:
4,00		:

£ 33, 14s. 4d. **Ans.**

£	s.	d
7)3371	13	4
100)4,81	13	4
20	:	:
1620	:	:
13		:
16,33		:
12		:
396		:
4		:
4,00		

£ 4, 16s. 4d. **Ans.**

§14. **Quotient to the nearest pie or penny.**

Rule. *Add 1p. or 1d. to the quotient if the remainder is half or greater than half the divisor, otherwise neglect it.*

Example 4. Divide Rs. 990, 1a. 6p. by 35 and 41.

Sol.

Rs.	a.	p.	
35)990	1	6	(Rs. 28
70	:	:	
290	:	:	
280	:	:	
10	:	:	
16	:	:	
160	:	:	
1	:	:	
35)161(4.		:	
140		:	
21		:	
12		:	
252		:	
6		:	
35)258(7p.			
245			
13			

Rs.	a.	p.	
41)990	1	6	(Rs. 24
82	:	:	
170	:	:	
164	:	:	
6	:	:	
16	:	:	
96	:	:	
1	:	:	
41)97(2a		:	
82		:	
15		:	
12		:	
180		:	
6		:	
41)186(4p.			
164			
22			

In the first case the remainder is 13*p*. which is less than half the divisor, therefore neglect it. The required quotient, in this case, is Rs. 28, 4*a*. 7*p*. **Ans.**

In the second case remainder is 22*p*. which is greater than half the divisor, therefore add 1*p*. to the quotient. The reqd. quotient in this case is Rs. 24, 2*a*. 5*p*. **Ans.**

§14. **Case II—Quotition.**

Rule. *Reduce the dividend and the divisor to the same denomination and then divide.*

Example 5. How many times is Rs. 5, 7*a*. 7*p*. contained in Rs. 147, 12*a*. 9*p*. ?

Sol.

Rs.	*a*.	*p*.
147	12	9
16	:	:
2352	:	:
12 ...	:	:
2364*a*.		:
12		:
28368		:
9.....		:
28377p.		

Rs.	*a*.	*p*.
5	7	7
16	:	:
80	:	:
7 ...	:	:
87*a*.		:
12		:
1044		:
7		:
1051*p*.		

∴ the reqd. quotient is 28377 ÷ 1051 = 27. **Ans.**

EXERCISE 16.

Divide :—

1. Rs. 525, 6*a*. 4*p*. by 11.
2. Rs. 738, 10*a*. 1*p*. by 13.
3. Rs. 1129, 12*a*. 4*p*. by 14.
4. Rs. 1375, 13*a*. 4*p*. by 16.
5. £ 2028, 12*s*. 6*d*. by 15.
6. £ 1945, 10*s*. 8*d*. by 16.
7. £ 1929; 3*s*. 5*d*. by 11.
8. £ 1821, 7*s*. 1*d*. by 13.
9. Rs. 1576, 5*a*. 8*p*. by 10.
10. Rs. 2178, 6*a*. 2*p*. by 10.
11. £ 1753, 6*s*. 8*d*. by 10.
12. £ 2156, 8*s*. 4*d*. by 10.

Divide to the nearest pie or penny :

13. Rs. 2156, 7*a*. 8*p*. by 15.

14. Rs. 5156, 9*a*. 7*p*. by 16.
15. £ 2948, 13*s*. 7*d*. by 13.
16. £ 8895, 15*s*. 4*p*. by 14.
17. Rs. 3856, 15*a*. 5*p*. by 100.
18. Rs. 5159, 14*a*. 10*p*. by 100.
19. £ 4839, 19*s*. 7*d*. by 100.
20. £ 5139, 17*s*. 3*d*. by 100.
21. £ 15389, 17*s*. 11*d*. by 1000.
22. £ 49556, 18*s*. 4*d*. by 1000.

Divide by factors and find the complete remainder:—

23. Rs. 2157, 13*a*. 5*p*. by 56.
24. Rs. 8917, 12*a*. 9*p*. by 72.
25. Rs. 5675, 11*a*. 7*p*. by 108.
26. Rs. 5815, 7*a*. 4*p*. by 132.
27. £ 5159, 4*s*. 11*d*. by 156.
28. £ 7728, 12*s*. 9*d*. by 160.
29. £ 2756, 12*s*. 2*d*. by 90.
30. £ 5159, 2*s*. 10*d*. by 140.
31. £ 5785, 17*s*. 11*d*. by 88.
32. £ 6939, 15*s*. 7*d*. by 256.

Divide each of the following as nearly as possible into the given number of equal parts :—

33. Rs. 563893, 3*a*. 8*p*. into 728 parts.
34. Rs. 383028, 15*a*. 6*p*. into 531 parts.
35. £ 193502, 17*s*. 8*d*. into 551 parts.
36. £ 280291, 18*s*. 3*d*. into 929 parts.
37. 51356 mds. 38 srs. 15 chks. into 123 parts.
38. 48501 mds. 27 srs. 11 chks. into 129 parts.
39. 15481 tons 17 cwt. 2 qr. 21 lb. into 161 parts.
40. 65321 tons 12 cwt. 3 qr. 24 lb. into 181 parts.

How many times is

41. Rs. 7, 8*a*. 9*p*. contained in Rs. 113, 3*a*. 3*p*. ?
42. Rs. 28, 13*a*. 7*p*. contained in Rs. 461, 9*a*. 4*p*. ?
43. £ 17, 14*s*. 5*d*. contained in £ 372, 2*s*. 9*d*. ?
44. £ 16, 17*s*. 11*d*. contained in £ 760, 6*s*. 3*d*. ?
45. 121 mds. 28 srs. 13 chks. contained in 3043 mds 5 chks. ?
46. 156 mds. 37 srs. 8 chks. contained in 6434 maund 17 srs. 8 chks. ?
47. 56 tons 7 cwt. 2 qr. 8 lb. contained in 281 tons 17 cwt 3 qr. 12 lbs.

48. 41 tons 3 cwt. 1 qr. 21 lb. contained in 329 tons 7 cwt. 2 qr.

49. The cost of 156 tables is Rs. 835, 4*a*. What is the cost of each table ?

50. How many chairs each worth Rs. 5, 8*a*. 6*p*. can I buy for Rs. 71, 14*a*. 6*p*. ?

51. How many horses at Rs. 95, 7*a*. each can I buy for Rs. 1336, 2*a*. ?

52. How many days must a labourer work at 14*a*. 6*p*. per day to earn Rs. 19, 6*p*. ?

53. A man pays Rs. 318, 4*a*. 6*p*. as income tax which is 5*p*. for every rupee in his income ; what is his income ?

54. A person completes a journey of 298 miles 2 fur. 28 poles in Feb. 1944. What distance does he travel per day ?

55. How many lengths each equal to 7 fur. 22 pole 4 yds. will make up 52 miles 10 poles ?

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§15. Some Important Problems in Compound Rules.

Example 1. *A*, *B*, *C* together have Rs. 1012, 6*a*. between them, *B* and *C* have together Rs. 810, 12*a*. and *A* and *C* have together Rs. 708, 2*a*. How much has *C* ?

Sol. *A*, *B* and *C* together have Rs. 1012, 6*a*.
and *B*, *C* together have Rs. 810, 12*a*.
∴ *A* alone has Rs. 1012, 6*a*.—Rs. 810, 12*a*.
=Rs. 201, 10*a*.

Also *A* and *C* together have Rs. 708, 2*a*.
∴ *C* alone has Rs. 708, 2*a*.—Rs. 201, 10*a*.
=Rs. 506, 8*a*. **Ans.**

Example 2. Divide Rs. 151, 14*a*. among *A*, *B* and *C* so that for every Rs. 3, 2*a*. that *A* gets, *B* gets Rs. 4, 6*a*. and *C* gets Rs. Rs. 2, 10*a*.

Sol. Rs. 3, 2*a*. + Rs. 4, 6*a*. + Rs. 2, 10*a*. = Rs. 10, 2*a*.
If Rs. 10, 2*a*. are divided *A* gets Rs. 3, 2*a*., *B* gets Rs. 4 6*a*., *C* gets Rs. 2, 10*a*., but Rs. 10, 2*a*. is contained 15 times in Rs. 151, 14*a*.

∴ *A* gets Rs. 3, 2*a*. × 15 = Rs. 46, 14*a*.
B gets Rs. 4, 6*a*. × 15 = Rs. 65, 10*a*.
C gets Rs. 2 10*a*. × 15 = Rs. 39, 6*a*.
} **Ans.**

Example 3. A labourer was employed on the condition that he would get Re. 1, 4*a*. on every day he is present and he would pay a fine of 5*a*. on every day he is absent. After a month he received Rs. 29, 11*a*. How many days was he absent ?

Sol. His remuneration for 30 days = Re. 1, 4*a*. × 30
= Rs. 37, 8*a*.
But he was paid = Rs. 29, 11*a*.

∴ he lost his remuneration and paid as fine for the days he was absent Rs. 37, 8*a*. − Rs. 29. 11*a*. = Rs. 7, 13*a*.

∵ he lost Re. 1, 4*a*. + 5*a*. = Re. 1, 9*a*. daily in case he is absent

∴ he lost Rs. 7, 13*a*. in 5 days. **Ans.**

Example 4. The total expenses of a family when wheat is at Rs. 5, 2*a*. per maund are Rs. 96, 2*a*. ; when wheat is at Rs. 5, 4*a*. per maund they are Rs. 97, 4*a*. (other expenses remaining the same). Find the total expenses of the family when wheat is at Rs. 4, 14*a*. per maund.

Sol. An increase of (Rs. 5, 4*a*. − Rs. 5, 2*a*.) *i.e.*, 2*a*. per maund in the price of wheat makes an increase of Rs. 97, 4*a*. − Rs. 96, 2*a*.) = 18*a*. in the family expenses.

It follows therefore, that the quantity of wheat consumed by the family = 18 ÷ 2 = 9 maunds.
∴ expenses on wheat = Rs. 5, 2*a*. × 9 = Rs. 46, 2*a*.
and the other expenses = Rs. 96, 2*a*. − Rs. 46, 2*a*.
= Rs. 50.
Now the price of 9 mds. of wheat = Rs. 4, 14*a*. × 9.
= Rs. 43, 14*a*.
Hence the reqd. expenses = Rs. 43, 14*a*. + Rs. 50
= Rs. 93, 14*a*. **Ans.**

Example 5. A purse contains an equal number of pounds, shillings and pence. If the total amount be £ 26, 7*s*. 1*d*, how many coins of each kind are there ?

Sol. 1 pound + 1 shilling + 1 penny = 253*d*.

∴ number of each kind of coin = £26, 7*s*. 1*d*. ÷ 253*d*.
= 6325*d*. ÷ 253*d*.
= 25. **Ans.**

Note. In problems on coins the number of coins and the value of coins should be carefully distinguished.

Example 6. A man having lived at the rate of £ 1253 a year for 4 years finds himself in debt, and then reduced his expenditure to £ 1120 a year, he is just out of debt in 3 years What is his income?

Sol. Clearly 4 yrs'. expenditure +3 yrs'. expenditure =7 yrs'. income

$\therefore$ 7 yrs' income $= (1253 \times 4 + 1120 \times 3)$ or £ 8372

$\therefore$ Annual income $=$ £ $8372 \div 7 =$ £ 1196. **Ans.**

EXERCISE 17.

1. *A*, *B* and *C* have Rs. 1058, 5*a*. between them, *A* and *B* have together Rs. 809, 12*a*. and *A* and *C* have together Rs. 749. 13*a*. How much has *A*?

2. *A*, *B* and *C* have Rs. 1030, 6*a*. between them, *B* and *C* have together Rs. 818, 1*a*. and *A* and *C* have together Rs. 707 14*a*. How much has *C*?

3. *A*, *B* and *C* were partners in a business. After a year they distributed the profit amounting to Rs. 1296, 14*a*. between them. *A* and *B* got together Rs. 784, 5*a*.; *B* and *C* got together Rs. 940, 11*a*. How much did each get?

4. *A* and *B* have together Rs. 1265, 5*a*. 3*p*., *B* and *C* have together Rs. 857, 15*a*. 9*p*. and *A* and *C* have together Rs. 1050, 4*a*. How much has each?

5. Divide Rs. 117 among *A*, *B* and *C* so that for every Rs. 3, 5*a*. that *A* gets *B* gets Rs. 4, 6*a*. and *C* gets Rs. 2, 1*a*.

6. Divide Rs. 183, 6*a*. among *A*, *B* and *C* so that for every Rs. 2, 12*a*. that *A* gets, *B* gets Rs. 3, 4*a*. and *C* gets Rs. 4, 3*a*.

7. How many yards of velvet at Rs. 5, 2*a*. a yard must a cloth merchant give in exchange for 41 mds. of wheat at Rs. 4, 10*a*. per maund?

8. How many sheep at Rs. 9, 14*a*. a head must a farmer give in exchange for 16 bullocks at Rs. 24, 11*a*. per bullock?

9. How many times can you give away 2 florins and a farthing from 45 guineas and 4 crowns ?

10. A carriage and a horse were purchased for Rs. 321, 4*a*. What did each cost if the horse was worth 4 times as much as the carriage ?

11. A house and its furniture were purchased for Rs. 3891, 15*a*. What did each cost if the price of the house was 8 times as much as the furniture ?

12. Divide Rs. 57, 12*a*. among *A*, *B* and *C* so that for every rupee that *A* gets *B* gets an eight-anna piece and *C* gets a four-anna piece.

13. Divide Rs. 530, 4*a*. among *A*, *B* and *C* so that for every anna that *A* gets, *B* gets 4 annas and *C* gets 1 rupee.

14. What is the least sum which should be added to Rs. 556, 7*a*. 8*p*. to divide it equally amongst 35 persons ?

15. What is the least sum which should be subtracted from Rs. 728, 7*a*. 9*p*. to divide it equally amongst 24 persons ?

16. How often would a cart wheel which is 4 yards 2 ft. round, revolve in going over 4 miles 3 furlongs 8 yards 6 inches ?

17. The fore-wheel of a carriage is 8 ft. 6 inches in circumference and the hind-wheel 4 ft. 10 inches more. How many more turns will the former make than the latter in a distance of 4 miles 2 furlongs ?

18. A person bought 121 sheep at Rs. 7, 8*a*. 9*p*. per sheep, 21 of them died. At what price per sheep must he sell now so as to gain Rs. 25, 14*a*. 3*p*. on the whole ?

19. A person bought 21 bags of wheat each containing 2 mds. 20 srs. at Rs. 8, 8*a*. per md. Two of them were stolen. At what price per bag must he sell the rest so as to gain Rs. 4, 12*a*. 10*p*. on the whole ?

20. A man gained Rs. 37, 15*a*. 6*p*. by selling 15 cows at Rs. 566, 11*a*. 6*p*. ; what was the cost price of each cow ?

21. A man gained Rs. 29, 6*a*. by selling 20 chairs at Rs. 10 per chair ; what was the cost price of each chair ?

22. A labourer was engaged for a month at 12*a*. 6*p*. per day on the condition that he would pay a fine of 2*a*. 6*p*. in case he will come late. After the stipulated time he received Rs. 21, 1*a*. 6*p*. How often was he late ?

23. A person whose pay for a week is Rs. 2, 13*a*. 4*p*. is fined 1*a*. 3*p*. if he comes late. At the end of 21 weeks he received Rs. 57, 15*a*. How often was he late ?

24. A labourer was employed on the condition that he would get 14*a*. 6*p*. for every day he is present and would lose 4*a*. 6*p*. for every day he is absent. After 25 days he received Rs. 14, 5*a*. 6*p*. How many days was he absent ?

25. A servant entered into an agreement with a gentleman that he would get Re. 1, 2*a*, for each day he is present and would pay a fine of 5*a*. 6*p*. for each day he is absent. After a month he received Rs. 19, 1*a*. only. How many days was he present ?

26. A certain number of sovereigns, twice as many crowns, 5 times as many half crowns, 8 times as many shillings and 12 times as many six-pences together amount to £ 28, 5*s*.; find the number of each coin.

27. A certain number of two-anna pieces, four times as many four-anna pieces, 6 times as many eight-anna pieces and 8 times as many rupees together amount to Rs. 121, 4*a*. Find the number of rupees.

28. If 5 balls cost as much as 21 bundles of pencils worth 14*a*. a bundle, how many balls can be bought for Rs. 165, 6*a*. ?

29. If 10 tables cost as much as 18 chairs worth Rs. 5, 8*a*. each, how many tables can be bought for Rs. 297 ?

30. A grocer mixes 20 lbs. of tea costing 1*s*. 2*d*. per lb. with 12 lbs. costing 2*s*. 1*d*. per lb. ; find the cost to the nearest penny of the mixture per lb.

31. A shopkeeper mixes 15 mds. of sugar at Rs. 11, 4*a*. a maund with 8 mds. at Rs. 15, 8*a*. a maund. Find to the nearest pie the cost of the mixture per maund.

32. How much water must be mixed with 20 seers of milk worth 3*a*. per seer in order to reduce its price to 2*a*. 6*p*. per seer ?

33. How much water must be mixed with 25 gallons of wine worth Rs. 4, 8*a*. per gallon to reduce its price to Rs. 3, 2*a*. per gallon ?

34. Divide Rs. 1989, 8*a*. into three such parts that the first part shall be Rs. 44, 2*a*. more than the second and the second part Rs. 107, 15*a*. more than the third.

35. Divide Rs. 1537, 11*a*. into three such parts that the first part shall be Rs. 127, 3*a*. more than the sum of the second and third, and the second Rs. 7, 14*a*. more than the third.

36. I divide £50, 3*s*. 4*d*. among 4 men, 5 women and 6 children, such that each man receives thrice as much as each child and each woman twice as much as each child. Find how much each man, woman and child receives.

37. An equal number of men, women, and boys earn Rs. 56, 4*a*. in 5 days. A man earns 7*a*., a woman 6*a*. and a boy 5*a*. daily. Find the number of men.

38. A number of boys, twice as many women and thrice as many men earn Rs. 255 in 8 days. A boy earns 5*a*., a woman 8*a*. and a man 10*a*. daily ; find the number of boys.

39. The total expenses of a family when wheat is at Rs. 3 12*a*. per maund are Rs. 53, 12*a*. when wheat is at Rs. 4. 2*a*. per maund they are Rs. 55, 10*a*. (other expenses remaining the same,) find the total expenses of the family when wheat is at Rs. 4, 6*a*. per maund.

40. The total expenses of a family when rice is at Rs. 4, 2*a*. per maund are Rs.64, 12*a*. and when rice is at Rs. 4 per maund they are Rs. 64, (other expenses are not changing) ; find the total expenses of the family when rice is at Rs. 3, 12*a*. per maund.

41. A person having lived at the rate of Rs. 2526 a year for six years finds himself in debt, and then reduces his expenditure to Rs. 2016 a year ; he is just out of debt in 4 years. Find his annual income.

42. The Calcutta rupee is worth 1*s*. 11*d*. 3*f*. each ; how many must be given for £ 9895, 16*s*. 8*d* ?

CHAPTER IV
FACTORS AND PRIME NUMBERS

§1. **Factors.**

$8 \times 3 = 24$.

Here 24 is the product of 8 and 3
and 8 and 3 are the factors of 24.
But 24 has other factors also *i.e.*, $6 \times 4 = 24$, and
$12 \times 2 = 24$

Definition. A number which can divide a given number without a remainder is called a **factor** or **measure** of that number.

EXERCISE 18.

Split up into factors :—

1. 15.	2. 25.	3. 39.	4. 42.
5. 55.	6. 65.	7. 78.	8. 85.
9. 49	10. 70.		

Can you split the following into factors :—

11. 13.	12. 19.	13. 21.	14. 35.
15. 41.	16. 23.	17. 34.	18. 29.
19. 31.	20. 32.	21. 37.	22. 72.

§2. **Prime numbers.**

In the previous exercises, you have noticed that some of the numbers could not be split up into factors, they could only be divided by themselves and unity. Such numbers are called **prime numbers.**

Definition. A number which is divisible only by itself and unity is called a **prime number.**

§3. A number which can be split up into factors is named a **composite number.**

§4. It may also be noted that numbers are either **even** or **odd.** A number which can be divided by 2 exactly is called an *even* number and a number which cannot be so divided is called an *odd* number. Thus 2, 4, 6, 8 etc. are even and 1, 3, 5, 7, etc. are odd numbers.

§5. Two numbers are prime to each other, when their only factor is 1, thus 5 and 7 are prime to each other.

§6. One number is *divisible* by another when it can be divided by that number exactly *i.e.*, without any remainder. Thus 20 is divisible by 5 because when 20 is divided by 5, there is no remainder left.

§7. **Criteria of divisibility.**

The following facts will help us to resolve a composite number into its elementary factors :—

(*i*) A number is divisible by 2 if its first right hand digit is zero or divisible by 2. Thus 120 and 156 are divisible by 2.

(*ii*) A number is divisible by 3 if the sum of its digits is divisible by 3. Thus 2562 is divisible by 3 ∵ 2+5+6+2 =15 is divisible by 3.

(*iii*) A number is divisible by 4 if its last two digits are zeroes or are divisible by 4. Thus 2500, 5736 are divisible by 4.

(*iv*) A number is divisible by 5 if its last digit is 0 or 5. Thus 1620 and 15925 are divisible by 5.

(*v*) A number is divisible by 6 if it is divisible by both 2 and 3. Thus 516 and 354 are divisible by 6.

(*vi*) A number is divisible by 8 if its last three digits are zeroes or are divisible by 8. Thus 51000 and 125256 are divisible by 8.

(*vii*) A number is divisible by 9 if the sum of its digits is divisible by 9. Thus 12375 is divisible by 9 ∵ 1+2+3+7+5=18 is divisible by 9.

Note. But if a number is not divisible by 9 the remainder in that case is the same as when the sum of its digits is divided by 9.

Let the number be 563425.

∵ 100000−1 is divisible by 9 ∴ 500000−5 is so too
10000−1 ,, 9 ∴ 60000−6 ,, ,, ,,
1000−1 ,, 9 ∴ 3000−3 ,, ,, ,,
100−1 ,, 9 ∴ 400−4 ,, ,, ,,
10−1 ,, 9 ∴ 20−2 ,, ,, ,,

Hence 563425−(5+6+3+4+2+5) is divisible by 9.

Hence 563425 when divided by 9 gives the same remainder as 5+6+3+4+2+5, i. e , 25 when divided by 9.

∵ 25÷9 gives rem. 7 563425÷9 will also give rem. 7.

(*viii*) A number is divisible by 10 if its last digit is zero. Thus 1260, 1200 are divisible by 10.

(*ix*) A number is divisible by 11 if the difference between the sum of its digits in the *odd* and in the *even* places is 0 or is divisible by 11. Thus 4950 and 9185 are divisible by 11.

$\because$ $(4+5)-(9+0)$ and $(9+8)-(1+5)$ are 0 and 11 respectively.

Note. But if a number is not divisible by 11, and also if the sum of the digits in the odd places is greater than the sum of the digits in the even places, then the remainder, when the number is divided by 11 is the same as when this difference is divided by 11. But if the sum of the digits in the even places is greater than the sum of the digits in the odd places, then the remainder obtained by dividing this difference by 11 is subtracted from 11 to get the real remainder.

Let the number be (i) 895632 and (ii) 985632

(i) The sum of the digits in the odd places $=2+6+9=17$
and the sum of the ,, ,, even ,, $=3+5+8=16$
difference $=17-16=1$

the remainder by dividing this 1 by 11=1
$\therefore$ the remainder by dividing 895632 by 11=1

(ii) Ths sum of the digits in the even places $=3+5+9=17$
and the ,, ,, ,, odd ,, $=2+6+8=16$
difference $=17-16=1$
the remainder by dividing this 1 by 11=1
$\therefore$ the remainder by dividing 985632 by 11=11−1=10

Proof

100000+1 or 99990+11	is divisible by	11,	$\therefore$	800000+8 is so too
10000−1 or 9999	,,	,, 11,	$\therefore$	90000−9 ,,
1000+1 or 990+11	,,	,, 11,	$\therefore$	5000+5 ,,
100−1 or 99	,,	,, 11,	$\therefore$	600−6 ,,
10+1 or 11	,,	,, 11,	$\therefore$	30+3 ,,

Hence 895632+8−9+5−6+3−2 is divisible by 11.

Hence 895632 when divided by 11 gives the same remainder as 8−9+5−6+3−2 when divided by 11.

8−9+5−6+3−2 means that the sum of the digits in the odd places is greater than the sum of the digits in the even places by 1.

$\therefore$ The remainder by dividing this 1 by 11 is 1.
$\therefore$ the remainder in 895632 is also 1.
Similarly we can prove the second part.

(*x*) A number is divisible by 12 if it is divisible by both 3 and 4. Thus 1608 and 2904 are divisible by 12.

(*xi*) To divide a number by 7, 11 and 13 we have the following rule :—

Rule.—*Divide the figures of the number into groups containing three each, as far as possible; counting from right to left. Add the alternate groups, and find their difference. If the difference is zero or is divisible by 7 or 11 or 13, the number itself is also divisible by them.*

Thus 89563796 is divisible by 7, 48566254 is divisible by 11, and 563569357 is divisible by 13.

For (*i*) 89563796 has 3 groups counting from right to left *i. e.*, 89,563,796, and the difference of alternate groups = (796 + 89) − 563 = 322.

∵ the difference 322 is divisible by 7.

∴ the number is also divisible by 7.

Similarly 48566254 and 563569357 are divisible by 11 and 13 respectively.

Note But if a number is not divisible by 7 or 11 or 13 and also if the sum of the groups in the odd places is greater than the sum of the groups, in the even places then the remainder, when the number is divided by 7 or 11 or 13 is the same as when the difference of the groups is divided by 7 or 11 or 13. But if the sum of the groups in the even places is greater than the sum of the groups in the odd places then the remainder obtained by dividing the difference of the groups by 7 or 11 or 13 is subtracted from 7 or 11 or 13 as the case may be, to get the real remainder.

Let the number be (i) 56325732569 and (ii)56325932569.

The numbers after dividing into groups (from right to left) are 56,325,732, 569 and 56, 325,932,569.

(i) The sum of the groups in the odd places = 569 + 325 = 894
The sum of the groups in the even places = 732 + 56 = 788
Difference = 894 − 788 = 106

∵ the rems. by dividing 106 by 7, 11 and 13 are 1, 7, 2 respectively

∴ the remainders by dividing 56325732569 separately by 7, 11 and 13 are 1, 7 and 2 respectively.

(ii) In 56, 325, 932, 569 :–

The sum of the groups in the even places = 932 + 56 = 988
The sum of the groups of the odd places = 569 + 325 = 894
Difference = 988 − 894 = 94.

∵ the rems. by dividing 94 by 7, 11 and 13 are 3, 6, 3 respectively

∴ the remainders by dividing 56325932569 separately by 7, 11, and 13 are 7 − 3 = 4, 11 − 6 = 5 and 13 − 3 = 10 respectively.

§8. The following facts also may be noted carefully :—

1 A number of two digits made of the same figures as 11, 22 33, etc, is always divisible by 11.

2. A number of three digits made of the same figures as 111,222 333, etc. is always divisible by 3, 37.

3, A number of 2 figures once repeated as 6767, 8989 is always divisible by 101.

4. A number of 3 figures once repeated as 256256, 754754, is. lways divisible by 7, 11 and 13.

5. A number of 4 figures once repeated as 23452345 is always divisible by 73 and 137.

6. A number of 5 figures once repeated as 2563425634 is always divisible l y 11.

7. A number of 6 figures once repeated as 256342256342 is always divisibls by 101.

8. A number of 7 figures once repeated as 25603492560349 is always divisible by 11.

9. A number of 8 figures once repeated as 1502345615023456 is always divisible by 17.

10 A number of 9 figures once rspeated as 156034239156034239 always divisible by 7, 11, 13, and 19.

EXERCISE 19.

Without actual division find which of the following numbers are divisible by 2 :—

1. 576.	2. 7280.	3. 859.	4. 725.
5. 963.	6. 524.	7. 832.	8. 941.

Without actual division find which of the following numbers are divisible by 4 :—

9. 936.	10. 842.	11. 1228.	12. 1556.
13. 822.	14. 914.	15. 1518.	16. 2196.

Without actual division find which of the following numbers are divisible by 8 :—

17. 1528.	18. 2176.	19. 2514.	20. 3572.
21. 15424.	22. 21596.	23. 16536.	24. 15932.

Without actual division find which of the following numbers are divisible by 3 :—

25. 8972.	26. 15234.	27. 22632.	28. 16103.
29. 25836.	30. 21537.	31. 19552.	32. 25932.

Without actual division find which of the following numbers are divisible by 9 :—

33. 18936. 34. 25654. 35. 56934. 36. 38952.
37. 103569. 38. 123043. 39. 220563. 40. 185409

Without actual division find which of the following numbers are divisible by 11 :—

41. 189567. 42. 663564. 43. 285835.
44. 2156389. 45. 5685669. 46. 839382918.

Supply the missing digits (denoted by stars) so that the following numbers may be divisible by 8 :—

47. 185362*. 48. 215635*. 49 219355*.
50. 2856*2. 51. 56856*6. 52. 3858*4.

Supply the missing digits (denoted by stars) so that the following numbers may be divisible by 9 :—

53. 26534*5. 54. 2*86358. 55. 21563*89.
56. 218*7356. 57. 563*8935. 58. 359356*6.

Supply the missing digits (denoted by stars) so that the following numbers may be divisible by 11 :—

59. 152*5 60. 9835*5. 61. 5060*1.
62. 25*351. 63. 78542*. 64. 56*502.

Without actual division write down the remainders when the following numbers are divided by 9 and 11 :—

65. 85638. 66. 363596. 67. 5935685.
68. 4335652. 69. 5730052. 70. 8125005.

---

§9. There is no direct method of determining primes. We give below a list of the prime numbers from 1 to 227.

1	11	29	47	71	97	113	149	173	197
2	13	31	53	73	101	127	151	179	199
3	17	37	59	79	103	131	157	181	211
5	19	41	61	83	107	137	163	191	223
7	23	43	67	89	109	139	167	193	227

§ 10. **To ascertain which numbers are prime.**

(*i*) A number whose last digit is 0, 2, 4, 6, 8, *i.e.*, an even number is divisible by 2 and hence every even number except 2 is not a prime. A number whose last digit is 0 or 5

is divisible by 5 and hence every such number except 5 is not a prime. Hence the last digit of every prime number except 2 and 5 must be 1, 3, 7, 9.

(*ii*) If the last digit be 1, 3, 7 or 9, try as divisors the primes 3, 7, 11, 13, etc., if there is a remainder in each case, the number is a prime. It is not necessary to try a divisor whose square is greater than the given number.

§ 11. **To resolve or decompose** a composite number into its prime factors is to find those prime numbers whose product is equal to the given number. Thus.

$$280=2\times2\times2\times5\times7=2^3\times5\times7.$$

When the factors obtained are all primes, the number is said to be resolved into prime or elementary factors.

Note. No number can be resolved into prime factors in more than one way.

§12. To resolve a composite number into its prime factors.

Rule. *Divide the given number by a prime number which will divide it exactly. Then divide the quotient by another and so on till you obtain a quotient which is a prime number. The divisors and the last quotient are the prime factors of the given composite number.*

Example 1. Resolve 41580 into its elementary factors.

Sol.

Explanation.

2)41580	(*i*) ∵ the last figure is 0,
2)20790	∴ the No. is divisible by 2 and 5
3)10395	(*ii*) ∵ $4+1+5+8=18=3\times6$,
3) 3465	∴ the No. is divisible by 3,
3) 1155	(*iii*) ∵ $(4+5+0)-(1+8)=0$,
5) 385	∴ the No. is divisible by 11.
11) 77	Now divide the number successively
7	by 2, 3, 5 and 11,

∴ 2, 2, 3, 3, 3, 5, 7, 11 are prime factors. **Ans.**

Example 2. Divide 95040 into its prime factors.

Sol.

10	95040	ten	test applies
11	9504	eleven	test applies
8	864	eight	test applies
9	108	nine	test applies
	12		

$\therefore$ number $= 10 \times 11 \times 8 \times 9 \times 12$

$= 2 \times 5 \times 11 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 2 \times 2$

$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 11$. **Ans.**

Example 3. A number $1568x35y$ is divisible by 88; find x and y.

$\because$ the number is divisible by 8×11

$\therefore$ the last three digits of the number must be divisible by 8.

$\therefore 35y$ must be 352 or $y = 2$.

Substituting 2 for y the number $= 1568x352$.

To be divisible by 11 the difference of the sum of its digits in the even and odd places must be either 0 or a multiple of 11; therefore add the digits in the even and odd places;

$2+3+8+5=18$ and $5+x+6+1=12+x$

To make the difference 0, x is evidently $=6$

$\therefore x=6$ and $y=2$. **Ans.**

§13. To find the number of different divisors of any composite number.

Rule. *Find the prime factors of the given number and add 1 to the index of each factor. The product of the increased indices is the required result, unity and the number itself being included.*

Example 4. How many different integers besides unity will divide 37800 without remainder?

Sol. $37800 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$

$= 2^3 \times 3^3 \times 5^2 \times 7$.

No. of divisors $= (3+1)\ (3+1)\ (2+1)\ (1+1) = 96$.

$\therefore$ the number of divisors besides unity $= 96 - 1 = 95$. **Ans.**

Note. The index of 7 is 1

EXERCISE 20

Resolve into elementary or Prime Factors :—

1. 728	2. 536.	3. 327.
4. 5145.	5. 3892.	6. 5148.
7. 5445.	8. 5436.	9. 6754.
10. 8790.	11. 5841.	12. 7865.
13. 7680.	14. 8430.	15. 7315.
16. 27720.	17. 999999.	18. 725760.
19. 1666665.	20. 2393160.	21. 11918907.

22. Write down all the prime numbers lying between 15 and 81.

23. Find the digits x and y when the number $15x8351y$ is divisible by 72.

24. Find the digits x and y when the number $1565x562y$ is divisible by 88.

25. Find the digits x and y when the number $2856354xy$ is divisible by 99.

26. What is the smallest number of which the alternate figures are zeroes that is divisible both by 9 and 11 ?

27. By what prime numbers may 1157 be divided so that the remainder may be 2 in each case ?

28. By what prime numbers may 21946 be divided so that the remainder may be 1 in each case ?

29. There are four prime numbers : the product of the first three is 1683 and that of the last three is 3553; find them.

30. Show that 3001 has no divisor.

31. Find how many different integers, besides unity, divide 14175.

32. Arrange 1, 2, 5, 5, 6, 8, so as to form a number divisible by 2475. Enunciate the principle on which you proceed.

[*Hint.* Find the prime factors of 2475]

33. How many different integers, besides unity, will divide 42336 without remainder ?

34. The product of 495 with a certain other number gave $2875ab$ find ; a, b.

35. Without performing the operation of division prove that 19434492 is divisible by 99.

CHAPTER V

GREATEST COMMON MEASURE (G. C. M.)

§ 1. **Definition.** If one number divides another number without remainder, it is called a **measure** ; as, 2 is a measure of 8.

If that *measure* divides more than one number without remainder that measure is called the **Common Measure** ; as, 2 is the common measure of 8, 12 and 16.

But 8, 12 and 16 are also divisible by another common measure 4 and there is no other common measure greater than 4 which can divide 8, 12 and 16 exactly. 4 is, therefore, called the **Greatest Common Measure** (G. C. M.) Hence *the Greatest Common Measure is the greatest measure which divides two or more given numbers without a remainder.*

The following are the methods of finding the G. C. M. of the given numbers :—

Example. Find the G. C. M. of 176, 286 and 88.

Sol. $176 = 11 \times 2 \times 2 \times 2 \times 2$
$286 = 11 \times 13 \times 2$
$88 = 11 \times 2 \times 2 \times 2$
} $\because$ 11 and 2 are the common measures
$\therefore$ the G.C.M. $= 11 \times 2 = 22$. **Ans.**

Rule. *Find out the prime factors of the given numbers, select and multiply together all the common measures. The product will be the G. C. M. of those numbers.*

EXERCISE 21.

Write down the G. C. M. of :—

1.	15, 25.	2.	16, 24.	3.	25, 30.	4.	21, 27.
5.	28, 35.	6.	30, 40.	7.	48, 64.	8.	26, 65.
9.	28, 84.	10.	16, 72.	11.	42, 48.	12.	16, 36.
13.	64, 84.	14.	35, 45.	15.	51, 85.		
16.	28, 35, 49.	17.	96, 108, 132.	18.	144, 176, 208.		
19.	192, 216, 360.	20.	525, 300, 625.	21.	576, 432, 720.		
22.	1500, 1600, 450.			23.	336, 420, 630.		
24.	370, 666, 962.			25.	5120, 7200, 9600.		
26.	12500, 57000, 3040.						

§3. **Second Method: To find the G. C. M. by division.**

Take two numbers 45 and 25
G. C. M. of 45 and 25=5.
Difference of 45 and 25=20

45 and 25 are divisible by 5 and you also observe that their difference is also divisible by 5. Hence *if two numbers are divisible by a number the difference is also divisible by that number.* This method depends upon this law. Hence the following rule :—

Rule. *Divide the greater number by the less and bring down the remainder. Put the remainder as the divisor and the original divisor as the dividend. Continue the process till there is no remainder left. The last divisor would be the required G. C. M.*

Example. Find the G. C. M. of 285 and 465.

Sol.

285)465(1
285
180)285(1
180
105)180(1
105
75)105(1
75
30)75(2
60
15)30(2
30

The required G. C. M.=15. **Ans.**

Explanation. Here 285 is less than 465. Therefore divide 465 by 285. 180 is the remainder ; according to the law stated above the G. C. M. of 465 and 285 would be the same as the G. C. M. of 285 and 180. Therefore make 180 the divisor and the original divisor 285 the dividend and continue the process till there is no remainder left. 15 is the last divisor. Therefore 15 is the G. C. M. required.

Note. If the G. C. M. of two numbers be the unity the numbers are prime to one another.

§4. **To find the G. C. M. of more than two numbers.**

Rule. *First find the G. C. M. of any two numbers. Then find the G. C. M of the G. C. M. thus obtained and of another of the given numbers and so on. The last G. C. M. will be the G. C. M. required.*

Example. Find the G. C. M. of 585, 819, 1404 and 702.

Sol.

585)819 (1	117)1404(12	117)702(6
585	117	702
234)585(2	234	×
468	234	
117)234(2	×	
234		

Exp. First find the G.C.M. of 585 and 819 which is 117: then find the G.C.M. of 117 and 1404 which is 117. Again, find the G. C. M. of 117 and 702.

∴ the reqd. G. C. M. is 117. **Ans.**

EXERCISE 22.

Find the G.C.M. of:—

1. 54, 126.	2. 272,304.	3. 275,450.
4. 364, 576.	5. 381, 891.	6. 715, 871.
7. 689, 1573.	8. 872, 986.	9. 225, 625.
10. 2145, 2235.	11. 1625, 2625.	12. 1875, 2425.
13. 1348, 1564.	14. 1568, 1897.	15. 3015, 5040.
16. 56340, 75348.	17. 62100, 59040.	
18. 525, 625, 375.	19. 576, 600, 384.	
20. 6435, 8970, 7235.	21. 6161, 2440, 3111.	
22. 71500, 51300, 89720.	23. 10353, 8925, 9180.	
24. 56700, 89730, 56730,	25. 18750, 16950, 19320,	

☞ **Some Important Typical Examples on G. C. M.**

Example 1. What is the largest sum of money which will divide Rs. 7. 4*a*. 8*p*. and Rs. 6. 11*a*. 4*p*. exactly ?

Sol. Rs. 7. 4*a*. 8*p*. = 1400*p*.

Rs. 6. 11*a*. 4*p*. = 1288*p*.

G.C.M. of 1400*p*. and 1288*p*. will divide both the sums.

∵ their G.C.M. = 56*p*. ∴ the reqd. sum = 4*a*. 8*p*. **Ans.**

Example 2. Find the greatest number that will divide 2629 and 2483 leaving remainders 4 and 8 respectively.

Sol ∵ on dividing 2629, the remainder is 4,

∴ 2629−4, *i.e.*, 2625 is exactly divisible by that number.

Similarly, 2483−8=2475 is exactly divisible by that number.

Hence the greatest number is the G. C. M. of 2625 and 2475 which is 75.

∴ the reqd. number=75. **Ans.**

Example 3. Find the greatest number that will divide 578, 503, and 528 leaving the same remainder in each case.

Sol. Let r be the remainder.

Then $578-r$, $503-r$, $528-r$ are exactly divisible by that number.

If two numbers are divisible by a number, their difference is also divisible by that number. Art. 3

∴ $(578-r)-(503-r)$, $(528-r)-(503-r)$, and $(578-r)-(528-r)$

or $(578-r-503+r)$, $(528-r-503+r)$, $(578-r-528+r)$

* or (578−503), (528−503) and (578−528)

or 75, 25 and 50 are also divisible by that number.

G. C. M. of 75, 25 and 50=25.

Hence the reqd. number=25. **Ans.**

Note the line asterisked and deduce a rule from it.

Example 4. The sum of two numbers is 144 and their G. C. M. is 24 ; find them.

Sol. Let the quotient by dividing the first No. by 24=x and second 24=y

then the first number $=24x$

and the second number $=24y$.

$\therefore \quad 24x+24y=144$

or $\quad 24(x+y)=144$

or $\quad x+y=6$.

The numbers whose sum is 6 are (1, 5) ; (2, 4) ; (3, 3).

We reject the 2nd and 3rd pairs because they are not prime to each other.

∴ only possible pair is 1, 5.

∵ multiplying by 24 the numbers are 24×1 and 24×5.

∴ 24 and 120 are the required numbers. **Ans.**

Example 5. The product of two numbers is 2560 an their G. C. M. is 16 ; find all the possible pairs of such numbers.

Sol. The first number $=16x$ [Ex 4.]
and the second number $=16y$
$\therefore\ 16x \times 16y = 2560$
or $256xy = 2560$
or $xy = 10$

The possible pairs whose product is 10 are 1, 10 and 2, 5.
$\therefore$ multiplying by 16 the numbers are either 16, 160 or 32, 80. **Ans.**

Example 6. What number is that which when divided by 5 and the quotient again by 6 and the quotient again by 8, will give the G. C. M. of 121 and 143 ?

Sol. The G. C. M. of 121 and 143 $=$ 11.

Now the question is what number is that which when divided by 5 and the quotient again by 6 and that quotient again by 8, will give 11 ?

$\because$ 5, 6 and 8 are the three divisors and 11 the last quotient, therefore the reqd. No. $=(11\times5\times6\times8)=2640$. **Ans.**

Example 7. 5635 and 3915 are divided by a certain number of three digits and the remainder is the same in both cases. Find the number and the remainder.

Sol. Let d be the divisor and q' and q the quotients.

5635 = divisor $\times$ quotient + remainder
3915 = divisor $\times$ quotient + remainder
$\therefore\ 1720 = d\,(q - q')$ [*by subtraction*]
$\therefore$ d is a factor of 1720
but $1720 = 172\times10$
$\because$ the number 'd' is of three digits.
$\therefore$ it is 172 and the remainder 131. **Ans.**

$$
\begin{array}{r}
32 \\
172\overline{)5635} \\
516 \\
\hline
475 \\
344 \\
\hline
131
\end{array}
$$

Note. $\because$ $1720 = 344\times5$ and 860×2
$\therefore$ 344 and 860 can also be the reqd. numbers.

Example 8. In solving a division sum, the dividend is 158728 and the successive remainders are 351, 216 and 108. Find the divisor and the quotient.

Sol. On subtracting the remainders 351,216 and 108 from the partial dividends the successive partial products will be 1587 −351=1236, 3512−216=3296 and 2168 −108=2060, Hence the divisor must be a common factor of these products.
G. C. M. of 1236, 3296 and 2060 is 412
∴ the divisor is 412 and the quotient is
(158728−108)÷412=385. **Ans.**

```
   ***
)158728
  ****
  3512
  ****
  2168
  ****
   108
```

Example 9. In working out a question on the G. C. M. of two numbers the quotients are 1, 3, 5. The last divisor is 132 ; find the numbers.

Sol ∵ the last divisor is 132 and quotient 5.
∴ dividend =132 ×5=660.
Now divisor =660, quotient= 3 and remainder=132
∴ dividend =660×3+132=2112.

```
2112)2772(1
 2112
 ----
 660)2112(3
 1280
 ----
 132)660(5
  660
  ---
  660
```

Now again divisor is 2112, quotient 1 and rem. 660
∴ dividend =2112 ×1 +660=2772.
Hence the numbers are 2112 and 2772. **Ans.**

Example 10. Find the numbers between 1800 and 2700 that have 323 as their G. C. M.

Sol. Evidently the numbers required are the multiples of 323.

The smallest possible number is 323×6 since 323×5 is less than 1800 and the largest number is 323×8 since 323×9 is greater than 2700.

∴ Nos. must be 323×6, 323×7, 323×8
or 1938, 2261, 2584. **Ans.**

EXERCISE 23.

1. Find the greatest number which will exactly divide 1561 and 2453.

2. What is the largest sum of money which will divide both Rs. 8, 12*a*. and Rs. 15, 4*a*. exactly ?

3. Find the largest sum of money which will divide both Rs. 10, 10*a*. 8*p*. and Rs. 16, 5*a*. 4*p*. exactly.

4. Two masses of silver weighing 85 and 95 tolas respectively are each to be made into coins of the same weight. Find the weight of the largest possible coin.

5. Two masses of gold weighing 2825 and 2275 tolas respectively are each to be made into ornaments of the same weight. What is the weight of the largest possible ornament ?

6. Two bills of Rs. 6075 and Rs. 8505 respectively are each to be paid by cheques of the same amount. What is the largest possible amount of each cheque ?

7. Find the greatest number that will divide 213, 241 and 297, leaving remainder 3 in each case.

8. Find the greatest number that will divide 365, 512 and 323 so as to leave a remainder 8 in each case.

9. Find the greatest number that will divide 364 and 532 leaving remainders 4 and 7 respectively.

10. Find the greatest number that will divide 653 and 679 leaving remainders 5 and 4 respectively.

11. Find the greatest number which will divide 362, 633 and 310 leaving 2, 3 and 4 as remainders respectively.

12. Find the greatest number which will divide 1742, 3723 and 1843 leaving 4,5 and 6 as remainders respectively.

13. Is there any number that will divide 520 and 728 leaving remainder 7 in each case ?

14. Is there any number that will divide 719 and 524 leaving remainders 3 and 5 respectively ?

15. Is there any number that will divide 515 and 321 leaving remainders 3 and 7 respectively ?

16. Find the greatest number that will divide 151, 175 and 235 leaving the same remainder in each case.

17. Find the greatest number that will divide 221, 263 and 326 leaving the same remainder in each case.

18. The sum of two numbers is 180 and their G. C. M. is 15. Find the numbers.

19. The sum of two numbers is 192 and their G. C. M. is 24. Find all the possible pairs of such numbers.

20. The sum of two numbers is 162 and their G. C. M. is 18. Find all the possible pairs of such numbers.

21. The product of two numbers is 2700 and their G. C. M. is 15. Find all the possible pairs of such numbers.

22. The product of two numbers is 8640 and their G. C. M. is 24. Find all the possible pairs of such numbers.

23. The two topmost classes of a school are to be divided into sections having equal number of boys in each. Find the maximum number of boys in each section, there being 225 and 315 boys in the classes.

24. Find the greatest number that will divide 964, 1238 and 1400 leaving remainders 41, 31 and 51 respectively.

25. Find the greatest number that will divide 295, 436 and 622 leaving remainders 15, 16 and 27 respectively.

26. A labourer was engaged for a certain number of days for Rs. 12, 8*a*. But being absent on some of those days he was paid only Rs. 9, 1*a*. ; prove that his daily wages could not be more than 5 annas.

27. A person bought a certain number of oranges for Re. 1, 8*a*. 6*p*. and sold some of them without profit for 12*a*. 6*p*. ; show that he had still left *at least* 24 oranges.

28. Find the two numbers nearest to 10000 that have 169 for their G. C. M.

29. Find the greatest number of 4 digits and the least number of 5 digits that have 144 for their G. C. M.

30. Find the greatest and the least numbers of five digitsd that have 85 for their common measure. What is their G. C. M. ?

31. What number is that which when divided by 6, the quotient again by 7, and the quotient again by 8, will give the G. C. M. of 144 and 156 ?

32. Two numbers 5824 and 7256, when divided by the least number of three digits give the same remainder ; findt he number and the remainder.

33. Two numbers 65636 and 78956 are divided by the greatest number of four digits and the remainder is the same in both cases. Find the number and the remainder.

34. In solving a division sum, the dividend is 28556 and the successive remainders are 142, 138 and 99. Find the divisor and the quotient.

35. In a division question, the dividend is 53529 and the successive remainders are 37, 108 and 33. Find the divisor and the quotient.

36. In working out a question, in the G. C. M. of two numbers, the qoutients are 3, 10 and 2. The last divisor is 45. Find the numbers.

37. In working out a question in the G. C. M. of two numbers the last divisor is 25 and the quotients in order are 10, 3, 2, 2, 1 and 2 ; find the numbers.

38. In solving a question in the G. C. M. of two numbers the different remainders were 315, 165, 150 and 15 and the first two quotients 1 and 40 ; find the numbers and last two quotients.

39. Find the greatest and the least numbers of 6 digits each so that they have 251 for their common measure. Also find their G. C. M.

40. Prove that 4157 and 24727 are prime to one another.

41. Show that 85242 and 656361 are not prime to one another.

42. Is it possible to divide 5239 into two parts such that their G. C. M. may by 26 ?

43. 264 oranges and 693 mangoes are to be distributed among some girls so that each girl may get as many mangoes and as many oranges as another girl. Find the largest possible number of girls and the least possible number of fruits of each kind which a girl gets.

CHAPTER VI

LOWEST COMMON MULTIPLE (L. C. M.)

§1. **Definition.** If a number is divisibe by another number without remainder the former is called a **multiple** of the latter, but if it is exactly divisible by several numbers it is called a **common multiple** of them. For instance, 15 is a multiple of 3 because it is divisible by 3 without remainder ; but 15 is also exactly divisible by 5 so it is a common multiple of both these numbers.

Besides 15 there are other numbers also as 30, 45, 60, 75, etc. etc., which are common multiples of 3 and 5. Since 15 is the least number, therefore it is called the **lowest common multiple** (L. C. M.)

Hence the lowest common multiple of the given numbers is the least number which is exactly divisible by each of these numbers.

Note. Every common multiple of the given numbers is also divisible by their Lowest Common Multiple (L C. M;)

The following are the methods for finding the L. C. M, of the given numbers :—

§2. **First method: By use of Multiplication Table.**

Example Find the L. C. M. of 8 and 12.

Sol. Multiples of 8 are 16, 24, 32, 40, 48, 56, 64, 72, and so on ;

Multiples of 12 are 24, 36, 48, 60, 72, and so on :

∵ 24, 48, 72, etc., are common multiples.

∴ 24 is the lowest common multiple (L. C. M.) **Ans.**

EXERCISE 24.

Find the L. C. M. of the following numbers by use of multiplication table :—

1.	4, 6.	2.	6, 9.	3.	9, 12,	4.	12, 15.
5.	12, 16.	6.	16, 20.	7.	15, 20.	8.	6, 15.
9.	12, 18.	10.	8, 10.	11.	10, 20.	12.	6, 8.

§3. **Second Method : By use of G. C. M.**

Let us take the first example once again and find out the G. C. M. and L. C. M. of these numbers.

G. C. M. of 8 and 12=4.
L. C. M. of 8 and 12=24.
Product of the numbers=12×8=96.
Product of the G. C. M. and L. C. M.=24×4=96.

The example shows that *the product of two numbers is equal to the product of their G. C. M. and L. C. M.*

Proof. Let A and B be any two numbers, P their G.C.M. and Q their L. C. M.

If we, therefore divide A and B by P, the quotients M and N (say) must be prime to each other ;

$\therefore A=M\times P$; $B=N\times P$;
$\therefore Q=M\times N\times P$
Now $A\times B=M\times P\times N\times P$
$=P\times M\times N\times P$
$=P\times Q$ and hence the theorem is proved.

Thus for finding the L. C. M. of two numbers we have the following rule :

Rule. *Divide the product of the given numbers by their G. C. M. The quotient is the L. C. M. required.*

Or, *Divide one of the numbers by their G. C. M. and multiply the quotient by the other. The result is the L. C. M. required.*

Example. Find the L. C. M. of 81 and 99.

Sol The G. C. M. of 81 and 99=9 ;
$\therefore$ L. C. M.= $(99\times81)\div9=891$. **Ans.**

Or, $99\div9=11$; $81\div9=9$.
$\therefore$ L. C. M.=11×81 or $99\times9=891$. **Ans.**

EXERCISE 25.

Find the L. C. M. of the following by use of G. C. M. :—

1.	36, 48.	2.	54, 72.	3.	75, 125.
4.	121, 165.	5.	176, 256.	6.	150, 225.
7.	256, 400,	8.	224, 336.	9.	222, 333,
10.	625, 500.	11.	528, 792.	12.	1210, 1430.
13.	448, 672.	14.	630, 840.	15.	690, 920.
16.	780, 910.	17.	620, 744.	18.	975, 1625.

§4 **Third Method: By use of Factors.**

The following is the rule for finding the L. C. M. of the given numbers when the numbers can be readily separated into Prime Factors:—

Rule. *Split the given numbers into Prime Factors and write down the continued product of all the different ones. If a Prime Factor occurs twice or thrice in any of them it must occur the same number of times in the product also.*

Example. Find the L. C. M. of 12, 15, 18 and 21.

Sol. Prime Factors of $12=2\times2\times3$

" " " $15=5\times3$

" " " $18=2\times3\times3$

" " " $21=3\times7$

$\therefore$ the reqd. L. C. M. $=2\times2\times3\times3\times5\times7=1260$. **Ans.**

Explanation. The L. C. M. must contain the factors 2, 3, 5 and 7. 2 occurs twice in 12; 3 occurs twice in 18; 5 and 7 occur only once. Therefore there must be *two* factors 2, *two* factors 3, *one* factor 5 and *one* factor 7 in the L. C. M.

EXERCISE 26.

Find the L. C. M. of the following by factors:—

1.	12, 18.	2.	16, 24.	3.	21, 27.
4.	9, 12, 15.	5.	12, 18, 24.	6.	21, 27, 36.
7.	27, 36, 45.	8.	45, 84, 90.	9.	72, 120, 108.
10.	33, 55, 121.	11.	51, 85, 120.	12.	45, 60, 75,
13.	54, 81, 63.	14.	36, 60, 72.	15.	64, 96, 112.
16.	121, 132, 143.	17.	77, 88, 176.	18.	132, 108, 72.
19.	21, 74, 27, 36.	20.	32, 36, 48,56.	21.	28, 56, 63, 84.

---

§5. **Fourth Method: By Ordinary Division.**

When several numbers are given, their L. C. M. may be more easily found by ordinary division. The following examples will illustrate the method:—

Example 1. Find the L. C. M. of 16, 24, 36 and 48.

Sol.

2)	16, 24, 36, 48
2)	8, 12, 18, 24
2)	4, 6, 9, 12
2)	2, 3, 9, 6
3)	1, 3, 9, 3
	1, 1, 3, 1

$\therefore$ the L. C. M.
$=2\times2\times2\times2\times3\times3$
$= 144$. **Ans.**

Rule. *Place all the given numbers in a line with a comma placed between each of them as shown on page 90. Divide the numbers by a factor which is common to at least two of them. Place the quotients and the undivided numbers in a line below. Continue the process till all numbers prime to one another are left. The continued product of all the divisors and the numbers in the last line will be the L. C. M. required.*

Note. The process may be shortened by rejecting at any stage of the work, any number which can exactly divide another number in the same line. As in the preceding example, 16 and 24 might be rejected at the beginning, since each of them is a sub-multipe of 48 in the same line.

Example 2. Find the L. C. M. of 4, 6, 8, 12, 16, 20, 30.

Sol.

2) 4, 6, 8, 12, 16, 20, 30

2) 6, 8, 10, 15

3, 4, 5, 15

$\therefore$ the L. C. M. $= 2 \times 2 \times 4 \times 15 = 240$. **Ans.**

EXERCISE 27:

Find the L. C. M. of the following by division:—

1. 6, 9, 12,	2. 8, 12, 20.	3. 4, 10, 15.
4. 15, 20, 25,	5. 15, 25, 40.	6. 16, 24, 32.
7. 6, 8, 12, 15.	8. 3, 4, 10, 12.	9. 5, 15, 20, 30.
10. 6, 9, 15, 21.	11. 24, 32, 20, 45,	
12. 12, 18, 36, 45, 48.	13. 8, 12, 15, 18, 21, 30.	
14. 5, 10, 15, 25, 35, 45.	15. 10, 20, 30, 40, 50, 60.	
16. 3, 4, 5, 6, 7, 8, 9, 10.	17. 4, 6, 8, 10, 12, 14, 16.	
18. 3, 5, 7, 9, 11, 13, 15.	19. 20, 25, 30, 40, 35, 50.	
20. 12, 24, 36, 72, 84, 144.	21. 9, 15, 18, 24, 30, 45.	
22. 21, 28, 35, 42, 49.	23. 55, 65, 88, 78, 143.	
24. 7, 8, 9, 14, 16, 18.	25. 88, 99, 132, 143, 130.	

Some Important Typical Examples on L. C. M.

Example 1. Find the least number which when divided by 8, 9, 12 and 15 will leave remainder 2 in each case.

Sol. L. C. M. of 8, 9, 12 and 15 = 360.

Since remainder is 3 in each case

$\therefore$ the required number $= 360 + 3 = 363$. **Ans.**

Example 2. Find the least number which when divided by 8, 12, 18 and 24 leaves the remainders 6, 10, 16 and 22 respectively.

Sol. Since the difference between the numbers and the corresponding remainders is 2 in each case

$\therefore$ if we add 2 to the required number, the sum will be exactly divisible by 8, 12, 18 and 24 ;

but L. C. M. of 8, 12, 18 and 24 = 72

$\therefore$ the required number = 72 — 2 = 70 **Ans.**

Example 3. Find the least number of 5 digits which is exactly divisible by 10, 15 and 25.

Sol. Least number of 5 digits = 10000.
L. C. M. of 10, 15 and 25 = 150.

Let us see if 10000 is divisible by 150.
By dividing we find that the remainder is 100.
If we subtract 100 from 10000 the remaining number will be exactly divisible by 150, *i. e.*, by 10, 15 and 25 but the number will reduce to one of 4 digits. Therefore we must add (150—100), *i. e.*, 50 to 10000.

$\therefore$ the required number = 10050. **Ans.**

```
150)10000(66
    900
   1000
    900
    100
```

Note. Students should cleary understand the last step.

Example 4. Find the greatest number of 6 digits which being divided by 8, 12, 16 and 20 leaves 5, 9, 13 and 17 as remainders respectively.

Sol. The L. C. M. of 8, 12, 16 and 20 = 240.

Dividing the greatest number of six digits 999999 by 240 we get 159 as remainder. Hence the number divisible by 240 is 999999—159 or 999840.

$\because$ the remainder in each case is less than the divisor by 3

$\therefore$ the required number = 999840—3 = 999837. **Ans.**

Example 5. What is the least number which is exactly divisible by 11, but when divided by 5, 6, 7, leaves the remainder 4 in each case ?

Sol. L. C. M. of 5, 6 and 7 = 210.

That is, 210 is exactly divisible by 5, 6 and 7 but when 210 is divided by 11 it gives a remainder 1. Now multiply this

remainder 1 by such a number that if we add 4 to the product the sum may be divisible by 11.

Now $1\times7+4$ is divisible by 11.

i.e., 7 is such a number.

$\therefore$ the required number $=210\times7+4=1474$. **Ans.**

Example 6. The L. C. M. of two numbers is 2880 and their G. C. M. is 25. How many pairs of such numbers can be formed ?

Sol. Suppose the numbers divided by their G. C. M. give x and y as quotients.

$\therefore$ the numbers are $24x$ and $24y$
$\therefore$ the L. C. M. is $24xy$
$\therefore$ $24xy=2880$
or $xy=120$. (See Note below).

Let (1) $x=1, y=120$	(5) $x=5, y=24$
(2) $x=2, y=60$	(6) $x=6, y=20$
(3) $x=3, y=40$	(7) $x=8, y=15$
(4) $x=4, y=30$	(8) $x=10, y=12$.

Of these pairs Nos. 2, 4, 6 and 8 are not prime to each other, hence they are rejected.

The admissible pairs are Nos. 1, 3, 5 and 7, *i.e.*,
1, 120 ; 3, 40 ; 5, 24 ; 8, 15.

Hence the required numbers by multiplying the pairs by 24 are 24, 2880 ; 72, 960; 120, 576 ; 192, 360. **Ans.**

Note. Since L. C. M. $\times$ G. C. M. $=$ the product of two numbers hence we can arrive at this result as in Example 5, page 83.

Example 7. The G. C. M. of three numbers is 15 and L. C. M. 450. If two of the numbers be 30 and 45, determine the value of the third number.

Sol. Dividing the first two numbers by the G. C. M. the quotients are 2 and 3.

Let the quotient by dividing the third number be x.
Then the L. C. M. $=15\times2\times3\times x=90x$
$\therefore$ $90x=450$ or $x=5$

$\therefore$ the third number $=5\times15=75$. **Ans.**

Note. This is the least value of the number, but other values are also possible, e. g.

$$75\times 2=150$$
$$75\times 3=225$$

Example 8. The G. C. M. of two numbers of 4 digits is 103 and their L. C. M. is 337840 ; determine the numbers.

Sol. By way of illustration take two numbers 75 and 125 and note their relation to each other.

L. C. M. of 75 and $125=25\times 5\times 3$ $\quad 25)\ 125,\ 75$

and their G. C. M. $=25$ $\quad\quad\quad

Hence the G. C. M. of reqd. numbers= the G. C. M, of 84, 144. *i.e.*, 12.

G. C. M. of the two numbers=12
and their sum=84

Now we can proceed as in Example 4, page 83, and find that the pairs of such numbers are 12, 72 ; 24, 60 ; 36, 48. Out of these pairs only the 3rd pair satisfies the given condition. Hence the reqd. numbers are 36, 48. **Ans.**

Example 10. The product of two numbers is 2688 and their L. C. M. is 336 ; find all pairs of such numbers.

Sol. Since the product of two numbers is equal to the product of their G. C. M. and L. C. M.

$\therefore$ G. C. M. $=2688 \div 336 = 8$.

G. C. M. $=8$ and product $=2688$

Now proceed as in Ex. 5, page 83.

The numbers are 8, 336; 16, 168 ; 24, 112; 48, 56. **Ans.**

EXERCISE 28.

1. Find the least number which is exactly divisible by 16, 24 and 36.

2. Find the least number which is exactly divisible by 14, 21, 24, 30 and 42.

3. Four bells toll after an interval of 8, 9, 12 and 15 seconds. When will they toll together ?

4. Find the least number which when divided by 18, 27 and 36 will leave remainder 7 in each case.

5. Find the least number which leaves a remainder 3 in each case when it is divided by 15, 21, 27 and 30.

6. Find the smallest number which being increased by 5 will be exactly divisible by 6, 15, 27, 35 and 45.

7. What is the smallest number which when diminished by 3 is exactly divisible by 9, 12, 15, 18 and 21 ?

8. Find the least number which when divided by 5, 6, 7, 8 and 9 leaves remainders 3, 4, 5, 6 and 7 respectively.

9. What is the smallest number which when divided by 12, 15, 18, 24 and 25 leaves remainders 8, 11, 14, 20 and 21 respectively ?

10. What is the least number which when divided by 39, 52 and 72 leaves remainders 36, 49 and 69 respectively ?

11. Find the least number of 5 digits which is exactly divisible by 5, 6, 7 and 8.

12. Find the least number of 4 digits which when divided by 6, 9, 12 and 15 leaves remainders 1, 4, 7 and 10 respectively.

13. Find the greatest number of 6 digits which being divided by 8, 12, 16, 20 and 24 leaves 6 as rem. in each case.

14. Find the greatest number of 5 digits which when added to 2312, may make the sum exactly divisible by 5, 10, 15 and 20.

15. Find the least number which when added to 791 and then divided separately by 6, 7, 8 and 9 may leave remainders 3, 4, 5 and 6 respectively.

16. What is the least number which when divided by 6, 7 and 9 leaves rem. 4 in each case but is exactly divisible by 11 ?

17. A number is exactly divisible by 7 ; but when divided by 2, 3, 4, 5 and 6 leaves remainders 1, 2, 3, 4 and 5 respectively. Find it.

18. Find the least multiple of 17 which leaves a remainder 2 when divided by any of the six *even* integers 4, 6, 8, 10, 12 and 14.

19. Three men journey 24, 30 and 42 miles a day on cycles round a circular field the circumference of which is 180 miles. After how many days will they meet again ?

20. Find the nearest number to 100000 that can be divided exactly by 2, 3, 4, 5, 6 and 7 respectively.

21. Find the nearest integer to 67281 which when divided by 3, 8, 11 and 16 leaves remainders 1, 6, 9 and 14 respectively.

22. A heap of stones can be made up exactly into groups of 25, but when made up into groups of 18, 27 and 32, there are always 11 left. Find the least number of stones that may be contained in such a heap.

23. I have a certain number of oranges numbering between 600 and 900. If 2 oranges are taken away the remainder

can be equally divided among 3, 4, 5, 6, 7, or 12 boys. Find the number of oranges I have.

24. Three ponies are running round a race course of 5280 yards. The first runs 440 yards a minute, the second 352 yards and the third 264 yards. Find the time that elapses between their once being together and then coming all together next time again.

25. The circumferences of the wheels of a carriage are 7 ft. 4 in. and 11 ft. What is the least distance in which both the wheels will make an exact number of revolutions ?

26. A vessel is to be exactly filled by any one of the measures 1 seer, 2 seers, 3 seers, 5 seers, 6 seers, 9 seers or 15 seers. Find the smallest possible vessel that will serve the purpose.

27. Find the numbers lying between 400 and 600 which are divisible by 5, 10, 15 and 20.

28. Find numbers lying between 300 and 400 which when divided by 8, 12, 16 and 24 will leave a remainder 6 in each case.

29. Find numbers lying between 500 and 800 which when divided by 12, 15, 18, 20, will leave remainders 5, 8, 11 and 13 respectively.

30. Find the greatest number and the least number which being subtracted from 3000 will make the result divisible by 7, 11 and 13.

31. Two numbers are 125 and 135 ; prove that their product is equal to the product of their G. C. M. and L. C. M.

32. The G. C. M. of two numbers is 32 and their L. C. M. is 7680 ; if one of them is 512, find the other.

33. The L. C. M. of two numbers is 6328 and their G. C. M. is 113, one of the numbers is 791 ; find the other.

34. The L. C. M. of two numbers is 1575 and their G. C. M. is 25 ; one of the numbers is between 200 and 300 ; find the numbers.

35. The product of two numbers is 34560 and their G. C. M. is 12. Find their L. C. M.

36. The product of two numbers is 8820 and their L. C. M. is 1260. Find their G. C . M.

37. Find the least number of 5 digits which can be divided exactly by all the *even* numbers up to 16 inclusive.

38. Find the greatest number of 5 digits which is exactly divisible by all the *odd* numbers up to 13 inclusive.

39. Find the least number of 6 digits which when divided by the *even* numbers up to 20 inclusive, may leave one as remainder in each sase.

40. The G. C. M. of three numbers is 12 and the L. C. M. 360. If two of the numbers be 24 and 36, determine the value of the third number.

41. 34, 51 and a third number have 17 as their G. C. M. and 510 as their L. C. M. What values can the third number have ?

42. The L. C. M. of two numbers is 1260 and their G. C. M. is 15. How many pairs of such numbers can be formed ?

43. The G. C. M. of two numbers is 21 and their L. C. M. is 2520. How many of such numbers can be formed ?

44. Find two numbers of *four* digits each such that their G. C. M. is 75 and their L. C. M. is 31500.

45. The G. C. M of two numbers of *five* digits each is 995 and their L. C. M. is 202980. Find the numbers.

46. Find all the numbers between 250 and 600 that have 1728 as their L. C. M.

47. The sum of two numbers is 60 and their L. C. M, is 72 ; find the numbers.

48. The sum of two numbers is 126 and their L. C. M. is 216. Determine the numbers.

49. The product of two numbers is 1728 and their L. C. M. is 144. Find all the pairs of such numbers.

50. The L. C. M. of two numbers is 168 and their product is 2352 ; find the numbers.

51. Find the three largest numbers such that their G. C. M. is 77 and L. C. M. is 1155.

MISCELLANEOUS EXERCISES SET 1.

I

1. Multiply 536025 by 1214496 by the shortest method known to you.

2. Find the number such that if it be added 16 times to 73921, the sum will be 75953.

3. A grocer buys 20 lbs. of tea at 10*a*. a lb. and 30 lbs. of tea at 12*a*. a lb. and having mixed them sells 35 lbs. of the mixture at 11*a*. a lb. At what price per lb. must he sell the remainder that he may neither gain nor lose ?

4. If a man started from New York at 9 o'clock this morning to walk to San Francisco, a distance of 300 miles and walked every day from 9 A. M. to 2 P. M. and from 3 P. M. to 7. P. M at an average speed of 5 feet per second, on what day and at what hour would he arrive at San Francisco ?

5. A boy had to divide 64287 by 123. He copied a figure wrong in the divisor and obtained as his quotient 502 and remainder 31. What mistake did he make ?

6. Find the least number of five digits which is exactly divisible by 7, 8, 9 and 12.

7. A bag contains £26, 5*s*. in half crowns, florins and shillings. There are three times as many florins and four times as many shillings as half crowns. Find how many coins of each kind the bag contains.

8. A man spends Rs. 1200 annually for 5 years and runs into debt. He then reduces his expenses to Rs. 650 a year, and in 6 years just clears off his debt ; what is his yearly income ?

II

1. Find the greatest number of 6 digits which is exactly divisible by 2037.

2. A man spends Rs. 750 a year for 6 years and saves some money ; he then raises his expenditure during the next 10 years to Rs. 1050 a year and finds all his savings spent. What is his yearly income ?

3. A man bought two heaps of mangoes, one for Rs. 10 5*a.* and the other for Rs. 18, 9*p*. If the price of each mango be the same and neither less than three annas, nor more than four annas, find the total number of mangoes he bought.

4. Divide Rs. 145, 12*a.* 6*p*. between two persons giving twice as much to one of them as to the other.

5. A number was divided by 7, 8 and 9 in succession and the remainders are 3, 5 and 4 respectively. Find the remainder had it been divided by 504.

6. What number multiplied by 238 gives the same result as 408 multiplied by 350 ?

7. Multiply 8350695 by 1921664 in three lines.

8. A purse contains a certain number of sovereigns, four times as many shillings and six times as many four-penny pieces ; and the whole sum is £ 13. Find how many sovereigns, shillings and four-penny pieces are there in the purse.

III

1. Find the least number that can be divided by all the *even* numbers up to 20 inclusive.

2. Multiply 3509856 by 1215672 in three lines.

3. Find the dividend and the remainder in

$$\begin{array}{l} 5)\ * * * * *\ , \\ \overline{7)\ * * * * *\ 1}\ \text{remainder.} \\ \overline{8)\quad * * * *\ ,}\ 0\ \text{remainder.} \\ \overline{\qquad 3\ 1\ 9\ ,}\ 1\ \text{remainder.} \end{array}$$

4. An estate of 1416 acres 2 roods 16 sq. poles was divided into allotments, each 4 acres 3 roods 27 sq. poles in area ; how many allotments were made ?

5. Three persons *A*, *B* and *C* are possessed of certain sums of money such that *A* and *B* together have 170 rupees ; *A* and *C* together have 200 rupees ; and *B* and *C* together have 210 rupees. What is the sum possessed by each ?

6. A horse and its saddle together cost Rs. 1250 and the horse is worth seven times the saddle. What is the value of each ?

7. A number being successively divided by 6, 8 and 9 leaves remainders 3, 2 and 5 respectively. Find the remainders if the order of divisors be reversed.

8. (*a*) Find the greatest number of 4 digits and the least number of 5 digits that have 124 for their G. C. M.

(*b*) Write in words 77665544332211.

IV

1. Find the least number which will, in each case, leave a remainder 4 when divided by 15 and 18.

2. Find the greatest and the least numbers of 6 digits that have 251 for their common measure. What is their G. C. M. ?

3. A plot of land, measuring 7 acres 2 roods 12 sq. yards, is sold for £907, 16*s*. What would a field of 8 acres 16 sq. poles fetch if sold at the same price per acre ?

4. A boy multiplies 533 by a certain number and obtains 70055 as his answer. If both the zeroes are wrong but the other figures are right, find the correct answer.

5. A number is divided by 235 and the successive partial dividends are 1532, 1229 and 546. Find the dividend, quotient and the remainder.

6. Divide Rs. 3954 between A, B and C so that A might get twice as much as B and C together and B one-third of C.

7. If 37 shares at £98, 17*s*. 6*d*. each and 9 shares at another price per share, altogether cost £4350, 2*s*. 9*d*., how much d d one of the latter shares cost ?

8. Find x in the following division question :—

$x)*****$

$y)*****$, 3 rem.

$****$, 4 rem.

} Complete remainder is 23.

V

1. What is the least number, which when divided by 16, 24, 36, 48 and 60 leaves remainders 9, 17, 29, 41 and 53 respectively ?

2. The sum of £ 363, 15*s*. is placed in three bags. One bag contains £ 78, 3*s*. and the other half as much again. How much is in the third ?

3. By what number must 825 be multiplied so that when the product is subtracted from one million, the result is 484375 ?

4. Find the prime factors of 27573.

5. Find the quotient in the following division question :—

*) 5 3 8 9 7 8
11) * * * * *, 2 rem.
* * * *, 8 rem.

} Complete remainder=66.

6. A man living at the rate of Rs. 350 a month for 10 months finds that his expenses are exceeding his income. He then reduces his expenditure to Rs. 275 a month. At the end of 5 months he finds that he is just out of debt. What is his income ?

7. A man bought 144 organges at the rate of 6 for four pice and exchanged them for mangoes at 8 pice a mango, how many mangoes did he receive ?

8. In solving a question in the G. C. M. of two numbers, the quotients are 5, 1, 18, 1, 3, 1, and 2. The last divisor is 15. Find the numbers.

IV

1. Find the greatest number which will divide 1028, 1629 and 2130 leaving remainders 3, 4 and 5 respectively.

2. I buy 500 oranges at 2 for three half-pence, and again 500 oranges at 2 for three pence, after which 60 of the better sort are eaten ; I then sell the remainder at five farthing each. How much do I gain or lose ?

3. A man living at the rate of Rs. 250 a month for 8 months finds that his expenses are exceeding his income, and reduces his expenditure to Rs. 160 a month ; at the end of 10 months he finds that he is not only out of debt but has saved Rs. 180 ; what is his income ?

4. A number being successively divided by 5, 6 and 7 leaves 2, 0 and 5 as remainders respectively. What are the remainders if it is divided by 7, 6 and 5 ?

5. What least number must be added to 8356935 to make it exactly divisible by 2356 ?

6. A, *B* and *C* have Rs. 973, 5*a*. between them ; *B* and *C* have together Rs. 442 and *A* and *C* have Rs. 687, 12*a*. together. How much has C ?

7. In a division sum the quotient is 6 times the divisor and thirty times the remainder. If the remainder be 3, find the dividend.

8. Find the multiplier in the following examples on multiplication and complete the operation :—

(*a*)

```
  8562
   ***
  ****
 *****
*****
-------
****822
```

(*b*)

```
  5623
   ***
  ****
 *****
*****
-------
***252
```

VII

1. Multiply 8356926 by 25616192 in three lines.

2. A number when divided by 775 gives a remainder 188, what will be the remainder if the same number be divided by 31 ?

[Hint. Number=775×Quotient +188
=31×25 ×Quotient +31×6+2]

3. How many chairs worth Rs. 2, 3*a*. 6*p*. each should be given in change for 71 dozen of pencils worth 7*a*. 6*p*. per dozen ?

4. How often is the sum of £19, 11*s*. 4*d*. contained in the eleventh part of £1291, 8*s* . ?

5. Divide £56, 13*s*. 4*d*. between *A*, *B* and *C* so that *A* may have £3, 6*s*. 8*d*. more than *B* and *B* £1, 3*s*. 4*d*. more than *C*.

6. A certain number when divided successively by 7, 8 and 9 leaves 2, 0, 6 as remainders respectively. What are the remainders if the order of the divisors be reversed ?

7. Find the greatest number which will divide 116, 221 and 356 leaving the same remainder in each case.

8. Find the greatest and the least numbers of 5 digits exactly divisible by 15, 20, 25 and 30.

VIII

1. A number is divided thus :—

*) * * * * * * * *
9) * * * * * * *, rem, 1.
* * * * *, rem, 3.

If the true remainder be 61, find the complete divisor.

2. Among how many persons can I distribute Rs. 103, 5*a*. 10*p*. if I give 15*a*. 9*p*. to each and how much will be left ?

3. *A* and *B* together earn Rs. 371, 2*a*. a month ; *B* and *C* together earn Rs. 466, 12*a*. and *A* and *C* together earn Rs. 536, 14*a*. ; find the amount earned by each of them.

4. Find the least number which is exactly divisible by 7, but when divided by 3, 5, 6, 12 may leave one as remainder in each case.

5. Five bells began to toll simultaneously and they toll at intervals of 4, 6, 8, 9 and 10 seconds. After what time will they again toll simultaneously ?

6. The product of two numbers is 864 and their quotient is 6, find the numbers.

[Hint. Product÷quotient=(smaleter number)^2]

7. The total expenses of a family when wheat is at Rs. 4 , 8*a*. per maund are Rs. 45 ; when wheat is at Rs. 5 per maund, they are Rs. 48 (other expenses remaining the same) ; find its total expenses when wheat is at Rs. 5, 12*a*. per maund.

8. Find the greatest number which will divide 53032, 118433 and 140349 leaving 7, 8 and 9, as remainders respectively.

IX

1. Simplify by the shortest method known to you :—

(*a*) 7535693×9999. (*b*) 2637535693÷9999.

2. Multiply 856935943 by 16218108 in three lines.

3. A man bought 50 cows at Rs. 45 each, 60 cows at Rs. 49 each and 75 cows at Rs. 52 each ; at what price per head must he sell the cows so as to gain Rs. 160 by this bargain ?

4. A purse contains a certain number of sovereigns, five times as many shillings, and eight times as many pence. If the whole sum is £6, 8*s*. 4*d*., find how many sovereigns, shillings and pence there are in the purse.

5 Find the least number which when divided by 36, 48, 60, 72 and 80 leaves remainders 20, 32, 44, 56 and 64 respectively.

6. The product of two numbers is 700 and their G.C.M. is 5 ; find all the possible pairs of such numbers.

7. Find the remainder and the dividend in the following question :—

7) * * * * * * * *
8) * * * * * * *, remainder 5,
4 3 5 6 9, remainder 2.

8. What quantity of water must I add to a pipe of wine which cost Rs. 900 to reduce its price to Rs. 5 a gallon ? (126 gallons make 1 pipe).

X

1. A man at his death directed in his will that his property worth Rs. 10800 is to be divided among his 4 sons, 3 daughters and 2 cousins such that each son would receive four times, and each daughter three times as much as each cousin ; how much does each son receive ?

2. Find the number which when divided by 25 gives a quotient which if diminished by 9 and the result multiplied by 12 gives a product 108.

3. A man's yearly income is £825, 13*s*. 4*d*. and his daily expenditure a guinea and a half . How much did he save in the year 1944 ?

4. The sum of two numbers is 135 and their G. C. M. is 15 ; find all the possible pairs of such numbers.

5. If the monthly expenses of a family be Rs. 62, 10*a*, when rice is at Rs. 3, 12*a*. per maund and Rs. 65, 4*a*. when rice is at Rs. 4, 2*a*. per maund, what would the expenses be when rice would be at Rs. 4, 5*a*. per maund, supposing other expenses remain the same ?

6. Find the sum of money which when multiplied by 12, Rs. 14 added to the product, the sum divided by 15, and Rs. 2, 10*a*. 8*p*. added to the quotient, the result is Rs. 16.

7. Complete the following examples on multiplication:—

(*a*)

```
  5 3 3 2
    * * *
  -------
  * * * * *
  * * * *
* * * * *
-----------
* * * * 1 8 0
```

(*b*)

```
  8 5 1 7
    * * *
  -------
  * * * * *
  * * * *
* * * * *
-----------
* * * * 6 9 1
```

8. (*a*) Supply the missing figures denoted by the stars in the following division sum :—

```
7 * *)5 2 * * 1( * 2
    * * 1 *
  -------
    1 * 1 1
    * * * *
  -------
      1 * 9
```

(*b*) A certain number when divided successively by 12, 15 and 8 leaves remainders 5, 3, 7 respectively. Find the remainder if it were divided by 1440.

CHAPTER VII

VULGAR FRACTIONS

§1. **Definition.** A **fraction** denotes part or parts of a unit.

Illustration. (*i*) Let the straight line AB indicate a unit.



Divide it into three equal parts as $AC=CD=DB$.

AC is one-third of the unit and AD two-thirds of it, AC and AD are therefore fractions of AB.

(*ii*) 8*a*. is a fraction of Re. 1, because it is one-half of a rupee. 10*a*. 8*p*. is a fraction of Re. 1, because it is two-thirds of a rupee. 12*a*. is a fraction of Re. 1, because it is three-fourths of a rupee.

§2. **Notation.** A fraction is expressed by two numbers placed one above the other with a horizontal line between them. The lower number is called the **denominator** and the upper number is called the **numerator**. The numerator and denominator are called the **terms** of a fraction. Thus one-half, two-thirds, three-fourths, etc., etc., are expressed as $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, in which the upper numbers 1, 2, 3 are the *numerators* and the lower numbers, 2, 3 and 4, the *denominators*.

§3. The denominator shows the number of equal parts into which the unit is divided and the numerator shows the number of times such equal parts are taken. Thus $\frac{3}{4}$ means that a unit is divided into four equal parts and three of these parts are taken.

$\frac{1}{2}$, $\frac{3}{4}$, etc., are also read as one over two, three over four, etc. Fractions expressed in the above notation are called **vulgar fractions**, *i.e.*, *common or ordinary* fractions.

§4. The following distinctions may be made in fractions :—

(1) **Proper Fraction** is one in which the numerator is less than the denominator as $\frac{2}{5}$, $\frac{5}{7}$, $\frac{11}{19}$, etc., etc.

(2) **Improper fraction** is one in which the numerator is equal to or greater than the denominator as $\frac{9}{5}$, $\frac{7}{3}$, $\frac{11}{4}$, etc.

(3) **Simple fraction** is one in which the numerator and denominator are both integers, as $\frac{5}{6}$, $\frac{9}{4}$, etc.

(4) **Compound fraction** is a fraction of a fraction as $\frac{3}{4}$ of $\frac{5}{7}$; $\frac{2}{3}$ of $\frac{4}{7}$ of $\frac{11}{9}$, etc.

(5) **Complex fraction** is one in which the numerator or the denominator or both are not integers, *i. e.*, they are themselves fractional as $\frac{\frac{5}{6}}{8}$; $\frac{9}{\frac{7}{8}}$; $\frac{\frac{3}{2}}{\frac{5}{6}}$; $\frac{2}{3\frac{1}{2}}$; $\frac{5\frac{1}{2}}{8\frac{1}{2}}$; $\frac{\frac{1}{2}\times\frac{3}{2}}{\frac{5}{6}\times\frac{7}{8}}$ etc.

In the example given here, we have used the whole numbers combined with fractions, as $3\frac{1}{2}$, $5\frac{1}{2}$, $8\frac{1}{2}$, etc. Such quantities are called **mixed numbers**.

EXERCISE 29 (Oral).

Express as fractions :—

1. Two-thirds. 2. Three-fifths. 3. Five-sevenths.
4. Nine-fourteenths. 5. Six-elevenths.
6. Twenty-five *over* one hundred and twenty-one.
7. One hundred and twenty-three *over* five hundred and six.

Express in words :—

8. $\frac{3}{11}$. 9. $\frac{8}{13}$. 10. $\frac{7}{2\overline{2}}$. 11. $\frac{25}{36}$.
12. $\frac{78}{139}$. 13. $\frac{75}{149}$. 14. $\frac{75}{419}$. 15. $\frac{2}{821}$.

Write down the values of :—

16. $\frac{1}{2}$ of Re. 1. 17. $\frac{1}{4}$ of Re. 1. 18. $\frac{3}{4}$ of Re. 1.
19. $\frac{7}{16}$ of Re. 1. 20. $\frac{5}{16}$ of 1 seer. 21. $\frac{5}{8}$ of 1 md.
22. $\frac{2}{3}$ of 1 yard. 23. $\frac{5}{36}$ of 1 yard. 24. $\frac{11}{1760}$ of 1 mile.
25. $\frac{3}{4}$ of Rs. 5, 12*a*.

REDUCTION OF FRACTIONS

§5. To reduce a fraction to its lowest terms.

When the numerator and the denominator of a fraction are prime to each other, it is said to be in its lowest terms. To reduce a fraction to its lowest terms depends on the following proposition :—

When the numerator and the denominator of a fraction are each divided by the same number, the value of the fraction is not altered. Thus:—

$$\frac{15}{6}=\frac{15\div 3}{6\div 3}=\frac{5}{2}.$$

There are two ways of reducing a fraction to its lowest terms. First by reducing the numerator and the denominator into their elementary factors and cancelling the common ones. Secondly by finding the G. C. M. of the numerator and the denominator and dividing both by this G. C. M.

Example. Reduce $\frac{1155}{5005}$ to its lowest terms.

Sol. First Method. $\frac{1155}{5005}=\frac{3\times 5\times 7\times 11}{13\times 5\times 7\times 11}=\frac{3}{13}$ **Ans.**

Note 1. It should be very carefully noted that when a factor is cancelled, it is replaced by 1 and not by 0.

Note 2. By cancelling the common factors we actually divide the numerator and the denominator by the same number. The test of divisibility are very useful in solving such questions:—

We shall solve the above question thus:—

$$\frac{1155}{5005}=\frac{231}{1001}\ \text{[by dividing the terms by 5.}$$
$$=\frac{33}{143}\ \text{[by ,, ,, ,, ,, 7.}$$
$$=\frac{3}{13}\ \text{[by ,, ,, ,, ,, 11.}$$

$$\begin{array}{r|l} & 3 \\ & 33 \\ & 231 \\ \frac{1155}{5005}= & \text{Ans,} \\ & 1001 \\ & 143 \\ & 13 \end{array}$$

Second Method. G. C. M. of 1155 and 5005=385.

$$\therefore \frac{1155}{5005}=\frac{1155\div 385}{5005\div 385}=\frac{3}{13}\ \text{Ans.}$$

EXERCISE 30.

Reduce to their lowest terms:—

1. $\frac{15}{21}, \frac{18}{24}$.
2. $\frac{24}{72}, \frac{21}{56}$.
3. $\frac{270}{180}, \frac{120}{150}$.
4. $\frac{18}{27}, \frac{51}{85}$.
5. $\frac{35}{45}, \frac{55}{121}$.
6. $\frac{5}{125}, \frac{112}{144}$.
7. $\frac{121}{165}, \frac{135}{195}$.
8. $\frac{169}{273}, \frac{240}{315}$.
9. $\frac{192}{208}, \frac{208}{240}$.

10. $\frac{256}{336}, \frac{300}{450}$. 11. $\frac{576}{600}, \frac{728}{819}$. 12. $\frac{640}{704}, \frac{576}{720}$.

13. $\frac{456}{627}, \frac{625}{750}$. 14. $\frac{952}{1224}, \frac{1728}{5760}$. 15. $\frac{1452}{1980}, \frac{1625}{2275}$.

16. $\frac{2184}{2912}, \frac{2880}{4160}$. 17. $\frac{2240}{3500}, \frac{6435}{7293}$. 18. $\frac{2466}{2603}, \frac{3366}{4488}$.

19. $\frac{2475}{2750}, \frac{2651}{7471}$. 20. $\frac{5841}{6435}, \frac{5005}{17017}$. 21. $\frac{5786}{21566}, \frac{11575}{12964}$.

22. $\frac{10584}{15624}, \frac{11479}{75055}$. 23. $\frac{13080}{13952}, \frac{29175}{33843}$. 24. $\frac{128352}{238368}, \frac{45576}{60768}$.

25. $\frac{3297641}{4316353}$. 26. $\frac{1345800}{3078520}$. 27. $\frac{1629858}{4777170}$.

Reduce by cancelling the following to their lowest terms:—

28. $\frac{16\times18}{27\times80}$ 29. $\frac{100\times49}{48\times175}$ 30. $\frac{21\times10\times16}{24\times18\times70}$

31. $\frac{19\times23\times26}{57\times78\times92}$ 32. $\frac{17\times29\times56\times65}{51\times49\times87\times39}$

---

§6. To reduce a mixed number to an improper fraction.

Rule. *Multiply the whole number by the denominator of the fractional part and then add the numerator to the product. Put the result as the new numerator and the denominator of fractional part as the denominator.*

Thus $5\frac{1}{3}=\frac{5\times3+1}{3}=\frac{16}{3}$

§7. To reduce an improper fraction as a whole or a mixed number.

Rule. *Divide the numerator by the denominator, express the quotient as a whole number and the remainder as the numerator and the denominator of the improper fraction as the denominator of the fractional part.*

Thus $^{502}_{21} = 502\div21=(23\times21+19)\div21=23\frac{19}{21}$.

EXERCISE 31.

Reduce the following mixed numbers to equivalent improper fractions:—

1. $3\frac{1}{2}$. 2. $3\frac{1}{4}$. 3. $5\frac{2}{3}$. 4. $7\frac{1}{8}$.

5. $9\frac{3}{4}$. 6. $12\frac{7}{8}$. 7. $11\frac{5}{6}$. 8. $16\frac{5}{6}$.

9. $16\frac{7}{8}$. 10. $21\frac{9}{11}$. 11. $24\frac{9}{16}$. 12. $28\frac{7}{8}$.

13. $121\frac{3}{5}$. 14. $124\frac{7}{8}$. 15. $324\frac{3}{20}$. 16. $521\frac{5}{12}$.

17. $981\frac{5}{21}$. 18. $892\frac{20}{21}$. 19. $8312\frac{24}{25}$. 20. $5625\frac{21}{25}$.

Reduce the following improper fractions to mixed numbers :—

21.	$\frac{8}{7}$.	22.	$\frac{15}{4}$.	23.	$\frac{21}{5}$.	24.	$\frac{28}{5}$.
25.	$\frac{77}{9}$.	26.	$\frac{101}{11}$.	27.	$\frac{124}{13}$.	28.	$\frac{137}{15}$.
29.	$\frac{302}{15}$.	30.	$\frac{931}{16}$.	31.	$\frac{1225}{18}$.	32.	$\frac{1625}{24}$.
33.	$\frac{7218}{29}$.	34.	$\frac{7625}{31}$.	35.	$\frac{12823}{35}$.	36.	$\frac{16784}{49}$.
37.	$\frac{18320}{45}$.	38.	$\frac{21529}{39}$.	39.	$\frac{16526}{43}$.	40.	$\frac{21730}{51}$.

§8. To reduce a whole number or a fraction to an equivalent fraction with any denominator.

To reduce a fraction to an equivalent fraction with any denominator depends on the following proposition:—

When a numerator and the denominator of a fraction are each multiplied by the same number, the value of the fraction is not altered, Thus :

$$\frac{3}{4}=\frac{3\times 3}{4\times 3}=\frac{9}{12}$$

Illustration. $\frac{3}{4}$ of Re. 1 $=12a.$; $\frac{12}{16}$ of Re. 1 $=12a.$

$\therefore \frac{3}{4}=\frac{12}{16}$ { since the numerator and the denominator of $\frac{3}{4}$ are each multiplied by 4.

Graphical Illustration. Take two pieces of squared paper each having equal numbers of squares of the same size as shown below:—

Fig. I

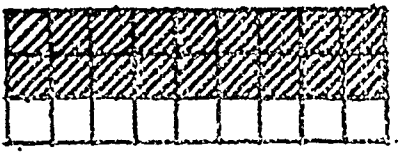
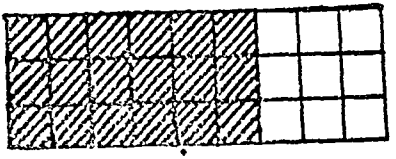


Fig. II



In the first figure 18 squares are shaded and they express $\frac{2}{3}$ of the whole figure.

In the second figure also 18 squares are shaded and they express $\frac{6}{9}$ of the whole figure..

$\therefore \frac{2}{3}=\frac{6}{9}$ { since the numerator and the denominator of $\frac{2}{3}$ are each multiplied by 3.

Example 1. Express 4 with denominator 12.

Sol. $4=\frac{4}{1}=\frac{4\times12}{1\times12}=\frac{48}{12}$ **Ans.**

Example 2. Express $\frac{5}{6}$ with denominator 96,

Sol. $\because\ 96=6\times16\ \ \therefore\ \frac{5}{6}=\frac{5\times16}{6\times16}=\frac{80}{96}$ **Ans.**

§9. **To reduce fractions to equivalent ones with their least common denominator (L. C. D.)**

Rule. *Find the L. C. M. of the denominators ; this will be the least common denominator, Then divide the L. C. M. so found by the denominator of each fraction and multiply the quotient so found by the numerator of the fraction which belongs to it for the new numerator of that fraction.*

Note. Before applying the above Rules reduce mixed numbers to improper fractions and compound fractions to simple ones. Also if the L. C. D. be required, the given fractions should first be reduced to their lowest terms.

Example 3. Reduce $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$ and $\frac{5}{12}$ to equivalent fractions with the least common denominator.

Sol. L. C. M. of the dedominators 3, 6, 8, and 12=24.

$\because\ 24\div3=8\ ;\ \ \therefore\ \frac{2}{3}=\frac{2\times8}{3\times8}=\frac{16}{24}$

$\because\ 24\div6=4\ ;\ \ \therefore\ \frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24}$

$\because\ 24\div8=3\ ;\ \ \therefore\ \frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}$

$\because\ 24\div12=2\ ;\ \ \therefore\ \frac{5}{12}=\frac{5\times2}{12\times2}=\frac{10}{24}$

$\therefore$ the required fractions are $\frac{16}{24}$, $\frac{20}{24}$, $\frac{21}{24}$, and $\frac{10}{24}$. **Ans.**

EXERCISE 32 (Oral).

Write down the missing numerators in the following fractions:—

1. $\frac{1}{2}=\frac{\ }{6}$.
2. $\frac{2}{3}=\frac{\ }{12}$.
3. $\frac{3}{4}=\frac{\ }{16}$.
4. $\frac{2}{5}=\frac{\ }{15}$.
5. $\frac{3}{7}=\frac{\ }{35}$.
6. $\frac{4}{9}=\frac{\ }{18}$.
7. $\frac{2}{13}=\frac{\ }{52}$.
8. $\frac{5}{12}=\frac{\ }{60}$.
9. $\frac{7}{16}=\frac{\ }{80}$.
10. $\frac{5}{18}=\frac{\ }{90}$.
11. $\frac{6}{19}=\frac{\ }{76}$.
12. $\frac{13}{16}=\frac{\ }{256}$.

Write down the missing denominators in the following fractions:—

13. $\frac{3}{4}=\frac{9}{\quad}$. 14. $\frac{5}{6}=\frac{30}{\quad}$. 15. $\frac{7}{8}=\frac{63}{\quad}$.

16. $\frac{12}{13}=\frac{60}{\quad}$. 17. $\frac{15}{16}=\frac{75}{\quad}$. 18. $\frac{16}{25}=\frac{64}{\quad}$.

19. $\frac{24}{25}=\frac{120}{\quad}$. 20. $\frac{11}{24}=\frac{99}{\quad}$. 21. $\frac{9}{14}=\frac{81}{\quad}$.

22. Express the whole numbers 12, 15, 16 and 20 as fractions with denominators 13.

23. Express the whole numbers 11, 24, 25 and 45 as fractions with denominators 20.

24. Express $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{9}{16}$ as equivalent fractions with denominators 96.

25. Express $\frac{5}{8}$, $\frac{7}{16}$, $\frac{9}{32}$ and $\frac{17}{64}$ as equivalent fractions with denominators 128.

Reduce to equivalent fractions having the least common denominator:—

26. $\frac{3}{4}$, $\frac{5}{6}$. 27. $\frac{5}{12}$, $\frac{7}{16}$. 28. $\frac{5}{9}$, $\frac{7}{12}$.

29. $\frac{7}{8}$, $\frac{11}{12}$. 30. $\frac{5}{14}$, $\frac{11}{21}$. 31. $\frac{6}{21}$, $\frac{9}{14}$.

32. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{8}$. 33. $\frac{3}{5}$, $\frac{9}{10}$, $\frac{7}{15}$. 34. $\frac{5}{6}$, $\frac{8}{15}$, $\frac{7}{30}$.

35. $\frac{1}{10}$, $\frac{1}{15}$, $\frac{5}{12}$, $\frac{7}{18}$. 36. $\frac{5}{19}$, $\frac{3}{38}$, $\frac{4}{57}$, $\frac{5}{76}$. 37. $\frac{5}{14}$, $\frac{9}{28}$, $\frac{6}{35}$, $\frac{9}{42}$.

38. $\frac{5}{12}$, $\frac{7}{16}$, $\frac{9}{32}$, $\frac{17}{24}$. 39. $\frac{3}{7}$, $1\frac{4}{9}$, $\frac{5}{14}$, $\frac{8}{27}$. 40. $\frac{2}{3}$, $1\frac{2}{5}$, $\frac{7}{8}$, $2\frac{1}{15}$.

---

§ 10. Comparison of fractions.

Example. Compare the fractions $\frac{5}{7}$ and $\frac{3}{7}$.

Explanation. A unit is divided into 7 equal parts in both cases ; but in the first case 5 parts are taken and in the second case 3 parts are taken. Therefore $\frac{5}{7}$ is greater than $\frac{3}{7}$.

Rule 1. *When two fractions have the same denominator, the greater is that which has the greater numerator.*

Example. Compare the fractions $\frac{7}{20}$, $\frac{7}{15}$.

Exp. In the first case a unit is divided into 20 equal parts and 7 parts are taken. In the second case a unit is divided into 15 equal parts and 7 parts are taken.

Now it is obvious that a part of the unit which is divided into 15 equal parts must be greater than the part of a unit which is divided into 20 equal parts. Therefore $\frac{7}{15}$ is greater than $\frac{7}{20}$.

Rule 2. *When two fractions have the same numerator, the greater is one which has the smaller denominator.*

Note. A fraction therefore increases either when its numerator increases or when its denominator diminishes and similarly it diminishes either when its numerator diminishes or when the denominator increases.

§11. But if neither the numerators nor the denominators of the given fractions are equal they must be reduced to equivalent fractions having the least common denominator and then apply Rule 1. Thus:

Compare the fractions $\frac{5}{12}$, $\frac{7}{16}$.

$\because$ $\frac{5}{12}, \frac{7}{16} = \frac{20}{48}, \frac{21}{48}$.

$\therefore$ $\frac{21}{48}$ *i.e.*, $\frac{7}{16}$ is greater than $\frac{20}{48}$, *i.e.*, $\frac{5}{12}$.

Example 1. Arrange the fractions $\frac{2}{11}, \frac{5}{11}, \frac{3}{11}$, in ascending order of magnitude.

Fractions after arranging in ascending order of magnitude are $\frac{2}{11}, \frac{3}{11}, \frac{5}{11}$. **Ans.** (Rule 1).

Example 2. Arrange the fractions $\frac{15}{16}, \frac{15}{31}, \frac{15}{28}$, in descending order of magnitude.

Fractions after arranging in descending order of magnitude are $\frac{15}{16}, \frac{15}{28}, \frac{15}{31}$. **Ans.** (Rule 2).

Example 3. Arrange in ascending and descending order of magnitude the fractions $\frac{5}{12}, \frac{9}{16}, \frac{7}{24}$.

Sol. First reduce the fractions to equivalent ones having the least common denominator.

$\therefore$ $\frac{5}{12}, \frac{9}{16}, \frac{7}{24} = \frac{20}{48}, \frac{27}{48}, \frac{14}{48}$.

and of these $\frac{14}{48}$ is the least and $\frac{27}{48}$ is the greatest.

$\therefore$ ascending order is $\frac{7}{24}, \frac{5}{12}, \frac{9}{16}$.
and descending order is $\frac{9}{16}, \frac{5}{12}, \frac{7}{24}$. } **Ans.**

§12. The defect of a fraction from 1 is called its **complement**, thus $\frac{1}{5}$ and $\frac{2}{7}$ are respectively the *complements* of $\frac{4}{5}$ and $\frac{5}{7}$.

Rule 3. *Fractions may sometimes be very conveniently*

compared by taking their complements, provided that each of the complements has in its numerator the same number, the greatest and least fractions will be those that have the least and the greatest complements.

☞ Study the following two examples carefully:—

Example 4. Find the *greatest* and the *least* of the fractions $\frac{5}{6}$, $\frac{24}{25}$ and $\frac{7}{8}$.

Sol. The complements of these fractions are $\frac{1}{6}$, $\frac{1}{25}$, $\frac{1}{8}$ respectively; of these $\frac{1}{25}$ is the least and $\frac{1}{6}$ is the greatest.

$\therefore$ $\frac{24}{25}$ is the *greatest* and $\frac{5}{6}$ is the *least*. **Ans.**

Example 5. Arrange in order of magnitude $\frac{3}{8}$, $\frac{6}{25}$, $\frac{7}{22}$.

Sol. $\frac{3}{8}=\frac{3\div3}{8\div3}=\frac{1}{2\frac{2}{3}}$; $\frac{6}{25}=\frac{6\div6}{25\div6}=\frac{1}{4\frac{1}{6}}$;

$\frac{7}{22}=\frac{7\div7}{22\div7}=\frac{1}{3\frac{1}{7}}$

Now of these $\frac{1}{2\frac{2}{3}}$ is the greatest and $\frac{1}{4\frac{1}{6}}$ is the least (Rule 2).

$\therefore$ the order of magnitude is $\frac{3}{8}$, $\frac{7}{22}$, $\frac{6}{25}$. **Ans.**

EXERCISE 33.

Find the greatest fractions by the first method:—

1. $\frac{5}{22}$, $\frac{3}{22}$, $\frac{9}{22}$. 2. $\frac{5}{12}$, $\frac{3}{16}$, $\frac{7}{24}$. 3. $\frac{7}{8}$, $\frac{11}{12}$, $\frac{17}{20}$.

Find the least fractions by the second method:—

4. $\frac{3}{11}$, $\frac{3}{16}$, $\frac{3}{22}$. 5. $\frac{3}{7}$, $\frac{5}{9}$, $\frac{4}{13}$. 6. $\frac{3}{7}$, $\frac{9}{11}$, $\frac{12}{17}$.

Find the greatest and the least of the following fractions by the third method:—

7. $\frac{7}{8}$, $\frac{8}{9}$, $\frac{9}{10}$. 8. $\frac{4}{9}$, $\frac{5}{11}$, $\frac{10}{16}$. 9. $\frac{3}{11}$, $\frac{7}{29}$, $\frac{5}{26}$.

Arrange the following fractions in descending order of magnitude;—

10. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$. 11. $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$. 12. $\frac{5}{12}$, $\frac{7}{15}$, $\frac{13}{30}$.

13. $\frac{6}{11}$, $\frac{10}{22}$, $\frac{16}{33}$. 14. $\frac{7}{12}$, $\frac{8}{9}$, $\frac{7}{16}$. 15. $\frac{5}{8}$, $\frac{9}{16}$, $\frac{11}{18}$.

Arrange the following fractions in ascending order of magnitude:—

16. $\frac{7}{18}, \frac{10}{27}, \frac{15}{36}, \frac{5}{72}$.

17. $\frac{17}{21}, \frac{9}{14}, \frac{1}{7}, \frac{5}{6}$.

18. $\frac{3}{7}, \frac{13}{21}, \frac{16}{35}, \frac{13}{28}$.

19. $\frac{7}{19}, \frac{9}{38}, \frac{15}{57}, \frac{15}{76}$.

Find a fraction lying between:—

20. $\frac{5}{6}$ and $\frac{6}{7}$ whose denominator is 84.

21. $\frac{2}{5}$ and $\frac{29}{72}$ whose denominator is 720.

ADDITION OF FRACTIONS.

§13. You know that $\frac{3}{16}$ of a rupee $=3a$.

$\frac{5}{16}$ of a rupee $=5a$.

$\frac{7}{16}$ of a rupee $=7a$.

$\therefore$ $\frac{3}{16}+\frac{5}{16}+\frac{7}{16}$ of a rupee $=3+5+7$ or $15a$.

$=\frac{15}{16}$ of a rupee.

Rule. *If the fractions have a common denominator, add their numerators for the new numerator and put the common denominator underneath.*

§14. But if the frections have no common denominator, first reduce them to equivalent fractions having the least common denominator (L. C. D.) and then add as shown above.

Example 1. Add together $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}$.

Sol. $\frac{2}{3}, \frac{5}{6}, \frac{7}{8}=\frac{16}{24}, \frac{20}{24}, \frac{21}{24}$ $\because$ L. C. M. of 3, 6, 8 $=24$

$$\therefore \text{ sum}=\frac{16+20+21}{24}=\frac{57}{24}=1\frac{9}{8}=2\frac{3}{8} \quad \textbf{Ans.}$$

Note 1. If the answer be an improper fraction, reduce it to a mixed number.

Note 2. If the fraction in the answer be not in its lowest terms, reduce it to its lowest terms. The fractions in the questions should also be reduced to their lowest terms.

Example 2. Add togethere $\frac{3}{5}, \frac{7}{8}, \frac{9}{10}$.

Sol. $\frac{3}{5}+\frac{7}{8}+\frac{9}{10}=\frac{24+35+36}{40}$ { The L. C. M. of 5, 8, 10 $=40$.

$=\frac{95}{40}=2\frac{15}{40}=2\frac{3}{8}$. **Ans.**

Example 3. Add together $5\frac{3}{4}, 7\frac{5}{8}, 12\frac{5}{6}$.

Note. In adding the mixed numbers, add all the whole numbers on one side and the fractions on the other, and then combine both the sums.

Sol. $5\frac{3}{4}+7\frac{5}{8}+12\frac{5}{6}=5+7+12+\frac{3}{4}+\frac{5}{8}+\frac{5}{6}$

$$=24+\frac{18+15+20}{24}=24+\frac{53}{24}$$

$$=24+2\frac{5}{24}=26\frac{5}{24}$$ **Ans.**

EXERCISE 34.

Find the sum of the following:—

1. $\frac{3}{8}+\frac{7}{8}+\frac{5}{8}$.
2. $\frac{5}{12}+\frac{11}{12}+\frac{7}{12}$.
3. $\frac{5}{6}+\frac{7}{8}+\frac{9}{12}$
4. $\frac{3}{7}+\frac{5}{14}+\frac{11}{21}$.
5. $\frac{8}{25}+\frac{9}{75}+\frac{7}{15}$.
6. $\frac{7}{12}+\frac{9}{16}+\frac{11}{24}$.
7. $\frac{7}{15}+\frac{11}{25}+\frac{16}{45}$.
8. $\frac{3}{14}+\frac{16}{21}+\frac{17}{28}$.
9. $\frac{5}{11}+\frac{7}{22}+\frac{14}{33}$.
10. $\frac{2}{9}+\frac{7}{18}+\frac{13}{24}$.
11. $2\frac{1}{3}+3\frac{1}{4}+5\frac{1}{6}$.
12. $3\frac{1}{3}+4\frac{1}{4}+9\frac{2}{3}$.
13. $5\frac{7}{8}+9\frac{8}{9}+10\frac{9}{10}$.
14. $2\frac{3}{14}+\frac{9}{28}+7\frac{3}{35}$.
15. $\frac{1}{9}+\frac{1}{7}+\frac{6}{63}$.
16. $\frac{5}{16}+\frac{7}{24}+\frac{13}{48}$.
17. $5\frac{1}{4}+\frac{2}{3}+\frac{9}{16}+18$.
18. $\frac{1}{3}+\frac{11}{9}\frac{6}{}+1\frac{1}{27}+12$.
19. $6\frac{3}{4}+7\frac{2}{8}+\frac{29}{9}+\frac{18}{24}$.
20. $7\frac{5}{12}+9\frac{3}{16}+8\frac{1}{24}+15\frac{5}{48}$.

SUBTRACTION OF FRACTIONS.

§15. The process in the subtraction is similar to the process in addition. The following examples will illustrate the method:—

(*i*) **To subtract one fraction from another when they have the same denominator.**

Example 1. Subtract $\frac{4}{15}$ from $\frac{7}{15}$.

Sol. $\frac{7}{15}-\frac{4}{15}=\frac{7-4}{15}$ } For seven-fifteenths—four-fifteenths = three-fifteenths.

$=\frac{3}{15}=\frac{1}{5}$. **Ans.**

Rule. *Find the difference of the numerator for a new numerator and write the denominator of the given fractions as its denominator.*

(*ii*) **When the given fractions have different denominators.**

Example 2. Subtract $\frac{15}{26}$ from $\frac{38}{39}$.

Sol. $\frac{38}{39}-\frac{15}{26}=\frac{76}{78}-\frac{45}{78}=\frac{76-45}{78}=\frac{31}{78}$ **Ans.**

Rule. *Reduce the fractions to the least common denominator and then proceed as in case (i).*

(iii) **To subtract one mixed number from another mixed number.**

Example 3. Subtract $12\frac{3}{5}$ from $19\frac{5}{6}$.

Sol. $19\frac{5}{6}-12\frac{3}{5}=19-12+\frac{5}{6}-\frac{3}{5}$.

$$=7+\frac{25-18}{30}$$

$$=7+\frac{7}{30}=7\frac{7}{30}. \quad \text{Ans.}$$

Note. In mixed numbers, first subtract the whole numbers and then the fractions as shown above. But if the fraction of the subtrahend is greater than the fraction of the minuend, then take unity from the difference of the whole numbers and add to the minuend. (This is done by adding the least common denominator to the new numerator of the minuend). The following example will show the method clearly:—

Example 4. Subtract $18\frac{7}{8}$ from $29\frac{3}{7}$.

Sol. $29\frac{3}{7}-18\frac{7}{8}=29-18+\frac{3}{7}-\frac{7}{8}$.

$$=11+\frac{3}{7}-\frac{7}{8}=11+\frac{24-49}{56}$$

Now we cannot subtract 49 from 24. Take one from 11 and add to $\frac{3}{7}$; $1+\frac{3}{7}=\frac{10}{7}$. There will be 80 in place of 24 now *i.e.*, we will add the L. C. D. to the minuend.

$$\text{Thus } 11+\frac{24-49}{56}=10+\frac{(24+56)-49}{56}$$

$$=10+\frac{31}{56}=10\frac{31}{56}. \quad \text{Ans.}$$

(iv) **To subtract a fraction from another when they have a common numerator.**

Example 5. Subtract $\frac{7}{13}$ from $\frac{7}{8}$.

Sol. $\frac{7}{8}-\frac{7}{13}=\frac{(13-8)\times 7}{8\times 13}=\frac{5\times 7}{104}=\frac{35}{104}$ **Ans.**

Rule. *Multiply the difference of the denominators by the common numerator for the new numerator and put the product of the denominators for the new denominator.*

(v) **To subtract a mixed number from an integer.**

Example 6. Subtract $2\frac{2}{3}$ from 10.

Sol. $10-2\frac{2}{3}=8-\frac{2}{3}=7+1+\frac{2}{3}$.

$$=7+\frac{1}{3}=7\frac{1}{3}. \quad \text{Ans.}$$

EXERCISE 35.

Find the value of :—

1. $\frac{7}{8}-\frac{3}{8}$.
2. $\frac{12}{25}-\frac{7}{25}$.
3. $\frac{12}{35}-\frac{1}{7}$.
4. $\frac{15}{36}-\frac{7}{24}$.
5. $\frac{7}{24}-\frac{17}{120}$.
6. $\frac{19}{58}-\frac{9}{87}$.
7. $16\frac{7}{8}-12\frac{5}{6}$.
8. $14\frac{3}{4}-12\frac{5}{8}$.
9. $12\frac{7}{15}-10\frac{8}{25}$.
10. $16\frac{9}{64}-12\frac{7}{32}$.
11. $45-36\frac{7}{8}$.
12. $37-25\frac{7}{12}$.
13. $21\frac{17}{18}-11\frac{19}{24}$.
14. $99\frac{23}{100}-97\frac{24}{25}$.
15. $\frac{5}{8}+\frac{7}{12}-\frac{23}{24}$.
16. $\frac{8}{15}+2\frac{3}{4}-1\frac{7}{10}$.
17. $15\frac{7}{8}+12\frac{3}{4}-11\frac{23}{24}$.
18. $\frac{18}{25}+\frac{20}{45}-\frac{7}{30}$
19. $18\frac{3}{4}-12\frac{7}{8}-4\frac{3}{21}$.
20. $13\frac{3}{5}-7\frac{16}{25}-3\frac{7}{15}$.
21. $\frac{15}{39}-\frac{5}{26}$.
22. Find the difference between $\frac{5}{12}+\frac{7}{16}+\frac{11}{24}$ and $\frac{3}{4}+\frac{15}{16}+\frac{7}{24}$.
23. What number added to $(\frac{5}{12}-\frac{5}{18})$ will give 4 ?

MULTIPLICATION OF FRACTIONS

§16. To multiply a fraction by a whole number.

Rule. *Multiply the numerator of the fraction by the whole number.*

Example 1. Multiply $\frac{7}{24}$ by 3.

Sol. $\frac{7}{24}\times 3=\frac{7\times 3}{24}=\frac{21}{24}=\frac{7}{8}$. **Ans.**

Example 2 Multiply $8\frac{5}{6}$ by 15.

Sol. $8\frac{5}{6}\times 15=(8+\frac{5}{6})\times 15=8\times 15+\frac{5}{6}\times 15$.
$=120+\frac{25}{2}$.
$=120+12\frac{1}{2}=132\frac{1}{2}$. **Ans.**

☞ The process in the following example should be carefully noted :—

Example 3. Multiply $555\frac{554}{555}$ by 100.

Sol. Since $555\frac{554}{555}=556-\frac{1}{555}$.
$\therefore 555\frac{554}{555}\times 100=(556-\frac{1}{555})\times 100$.
$=55600-\frac{100}{555}$
$=55599+1-\frac{20}{111}$
$=55599+\frac{91}{111}$
$=55599\frac{91}{111}$. **Ans.**

§17. **To multiply a fraction by a fraction.**

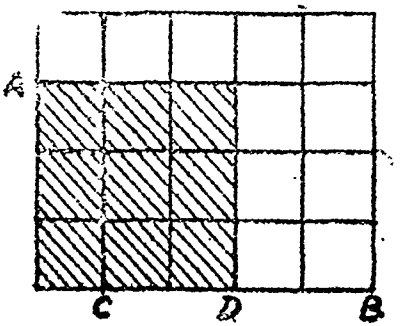
Example 4. Multiply $\frac{3}{4}$ by $\frac{2}{5}$.

Exp. The annexed figure represents a unity. $A\ B$ is $\frac{3}{4}$ of it and AC is one-fifth of $\frac{3}{4}$.

$\therefore$ AD is three-fifths of $\frac{3}{4}$.

But $A\ D$ also represents $\frac{3}{20}$ of the unity.

$\therefore$ $\frac{3}{4}\times\frac{2}{5}=\frac{3}{20}$.



Rule. *Multiply the numerators for a new numerator and denominators for a new denominator.*

Example 5. Simplify $1\frac{1}{2}\times\frac{4}{9}\times3\frac{5}{6}\times2\frac{1}{4}$.

Sol. Product $=\frac{3}{2}\times\frac{4}{9}\times\frac{23}{6}\times\frac{9}{4}$.

$$=\frac{3\times2\times2\times23\times3\times3}{2\times3\times3\times3\times2\times2\times2}=\frac{23}{4}=5\frac{3}{4}.$$ **Ans.**

Or, we may cancel the common factors of the numerators and the denominators without splitting them into prime factors. As,

$$\text{Product}=\frac{3\times4\times23\times9}{2\times9\times6\times4}=\frac{23}{4}=5\frac{3}{4}.$$ **Ans.**

2

Note. Before multiplying, mixed numbers must be turned into improper fractions, and if the product is an improper fraction it must be turned into mixed number.

EXERCISE 36.

Multiply :—

1. $5\frac{1}{3}$ by 9.
2. $7\frac{1}{8}$ by 56.
3. $6\frac{7}{8}$ by 28.
4. $9\frac{15}{16}$ by 24.
5. $17\frac{5}{21}$ by 35.
6. $13\frac{7}{19}$ by 57.
7. $111\frac{110}{111}$ by 10.
8. $999\frac{998}{999}$ by 100.
9. $\frac{5}{8}$ by $\frac{12}{25}$.
10. $\frac{35}{36}$ by $\frac{42}{75}$.
11. $7\frac{3}{25}$ by $1\frac{1}{79}$.
12. $16\frac{9}{16}$ by $2\frac{26}{27}$.
13. $5\frac{10}{13}$ by $4\frac{4}{75}$.
14. $37\frac{31}{37}$ by $2\frac{7}{13}$.

Simplify :—

15. $7\frac{1}{2}\times2\frac{1}{3}\times2\frac{1}{7}\times3\frac{1}{5}$.
16. $2\frac{41}{81}\times5\frac{2}{29}\times11\frac{5}{7}\times5\frac{1}{7}$.
17. $5\frac{3}{5}\times2\frac{1}{7}\times3\frac{1}{4}\times2\frac{1}{13}\times1\frac{1}{81}$.
18. $\frac{3}{4}\times\frac{5}{6}\times\frac{9}{10}\times1\frac{7}{9}$.

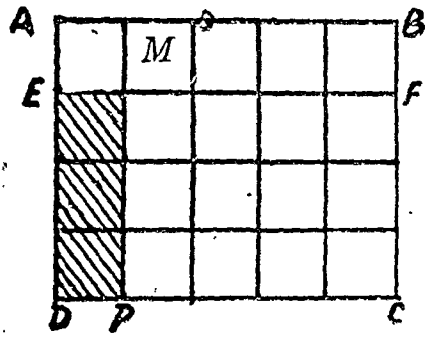
Find the continued product of :—

19. $\frac{3}{4}$, $1\frac{1}{7}$, $5\frac{1}{6}$, $1\frac{2}{13}$.
20. $5\frac{2}{7}$, $4\frac{2}{7}$, $1\frac{4}{17}$, $7\frac{1}{15}$, $1\frac{1}{6}$.

DIVISION OF FRACTIONS.

§18. To divide a fraction by a whole number.

Example 1. Divide $\frac{3}{4}$ by 5.



Graphical Proof. Let rectangle $ABCD$ represent a unity. Then $EFCD$ will represent $\frac{3}{4}$;

$EMPD$ is one-fifth of $EFCD$, *i.e.*, of $\frac{3}{4}$ which is three-twentieth of the whole figure.

$\therefore \frac{3}{4} \div 5 = \frac{3}{20}$.

Rule. *Multiply the denominator of the fraction by the whole number leaving the numerator as it was: or in other words multiply the fraction by the reciprocal of the whole number.*

Def. When a whole number or a fraction is inverted, *i.e.*, the numerator is turned into denominator, and the denominator into numerator, the new fraction is called the **Reciprocal** of the original; thus the reciprocal of $\frac{5}{9}$ is $\frac{9}{5}$; of 5 or $\frac{5}{1}$ is $\frac{1}{5}$. It is evident that the product of two reciprocal numbers is 1.

Example 2. Divide $\frac{7}{8}$ by 14.

Sol. $\frac{7}{8} \div 14 = \frac{\cancel{7} \times 1}{8 \times \cancel{14}_{2}} = \frac{1}{16}$. **Ans.**

Example 3. Divide $56\frac{7}{8}$ by 12.

Sol. $56\frac{7}{8} \div 12 = \frac{455}{8} \div 12.$

$= \frac{455}{8} \times \frac{1}{12} = \frac{455}{96} = 4\frac{71}{96}$. **Ans.**

Aliter. $56\frac{7}{8} \div 12 = (48 + 8\frac{7}{8}) \div 12$

$= 4 + \frac{71}{8} \times \frac{1}{12} = 4 + \frac{71}{96} = 4\frac{71}{96}$. **Ans.**

§ 12. To divide a fraction by a fraction.

As divison is the *reverse* of multiplication, hence the rule:

Rule. *Multiply the dividend by the reciprocal of the divisor.*

Example 4. Divide $\frac{7}{8}$ by $\frac{5}{12}$.

Sol. $\frac{7}{8} \div \frac{5}{12} = \frac{7}{8} \times \frac{12}{5}$.

$$= \frac{7 \times \cancel{4} \times 3}{\cancel{4} \times 2 \times 5} = \frac{21}{10} = 2\frac{1}{10}. \quad \textbf{Ans.}$$

EXERCISE 37.

Divide :—

1. $\frac{5}{12}$ by 10. 2. $\frac{15}{16}$ by 25. 3. $2\frac{3}{4}$ by 11.
4. $8\frac{8}{11}$ by 16. 5. $18\frac{1}{3}$ by 45. 6. $14\frac{3}{7}$ by 37.
7. $7\frac{4}{5}$ by $2\frac{8}{9}$. 9. $30\frac{1}{4}$ by $6\frac{3}{5}$. 9. $26\frac{1}{4}$ by $1\frac{7}{8}$.
10. $24\frac{5}{24}$ by $16\frac{5}{36}$. 11. $527\frac{7}{8}$ by $24\frac{15}{17}$. 12. $\frac{375}{8316}$ by $\frac{875}{3364}$
13. $\frac{475}{7524}$ by $\frac{625}{4356}$. 14. $\frac{2505}{3969}$ by $\frac{3006}{2646}$. 15. $\frac{3744}{4500}$ by $\frac{1664}{2625}$
16. $\frac{6720}{3168}$ by $\frac{8640}{3744}$. 17. $\frac{1028}{3035}$ by $\frac{1285}{5463}$. 18. $24\frac{287}{396}$ by $20\frac{1}{4}$
19. Divide 130 by the sum of $7\frac{1}{2}$ and $8\frac{3}{4}$.
20. A man's annual income is Rs. $2737\frac{1}{2}$; find his income per day.
21. The product of two numbers is $29\frac{1}{256}$; one of them is $22\frac{11}{112}$; find the other.
22. What number multiplied by $9 - 3\frac{4}{25}$ will give 1 as product ?

COMPOUND FRACTIONS.

§20. **A compound fraction** as defined in Art. 4, is a fraction of a fraction or several other fractions. Thus $\frac{1}{2}$ of $\frac{3}{4}$; $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{6}$ &c, &c, are compound fractions. *'Of'* indicates the sign of multiplication.

For instance, $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$. So also $\frac{2}{3}$ of $\frac{1}{2} = \frac{1}{3}$

Numbers connected by *of* are to be considered as a *single number, i.e.*, the word 'of' has the force of a pair of brackets. The student should, therefore, very carefully note the difference in meaning between $\times$ and *of*. Thus in $2\frac{1}{3} \div 1\frac{1}{2} \times \frac{3}{4}$, the sign $\div$ applies only to the next number $1\frac{1}{2}$, but in $2\frac{1}{3} \div 1\frac{1}{2}$ of $\frac{3}{4}$, the sign $\div$ applies to ($1\frac{1}{2}$ of $\frac{3}{4}$), as shown below :—

(1) $2\frac{1}{3} \div 1\frac{1}{2} \times \frac{3}{4} = \frac{7}{3} \div \frac{3}{2} \times \frac{3}{4}$

$$= \frac{7}{3} \times \frac{2}{3} \times \frac{3}{4} = \frac{7}{6} = 1\frac{1}{6}. \quad \textbf{Ans.}$$

(2) $2\frac{1}{3} \div 1\frac{1}{2}$ of $\frac{3}{4} = \frac{7}{3} \div (\frac{3}{2} \times \frac{3}{4})$

$= \frac{7}{3} \div \frac{9}{8}$

$= \frac{7}{3} \times \frac{8}{9} = \frac{56}{27} = 2\frac{2}{27}$. **Ans.**

This shows that if the sign $\div$ is immediately followed by numbers connected together by the word *'of'* then the product of all such numbers following the sign $\div$ is to be considered as a single number.

§21 Combined Operations.

☞ In simplifying fractions involving various signs and brackets, we should proceed in the following order:—

(*i*) all operations within brackets must be done first ;

(*ii*) then the quantities, connected together by *'of'* should be simplified ;

(*iii*) operations of division should then be done ;

(*iv*) next the operations of multiplication should be performed.

(*v*) last of all, the operations of addition and subtraction should be performed.

In doing steps (*iii*) and (*iv*) shown above, we should remember that the sign $\div$ and $\times$ apply only to the numbers following them.

§22. Illustrations.

1. $\frac{1}{7}$ of $3\frac{1}{2} + 2\frac{1}{2} = \frac{1}{7} \times \frac{7}{2} + \frac{5}{2} = \frac{1}{2} + \frac{5}{2} = 3$. **Ans.**

2. $8 - 2\frac{1}{3}$ of $2\frac{1}{7} = 8 - \frac{7}{3} \times \frac{15}{7} = 8 - 5 = 3$. **Ans.**

3. $2\frac{1}{2}$ of $\frac{4}{5} \times \frac{7}{8}$ of $2\frac{2}{3} = \frac{5}{2} \times \frac{4}{5} \times \frac{7}{8} \times \frac{8}{3} = \frac{14}{3} = 4\frac{1}{3}$. **Ans.**

4. $\frac{5}{6}$ of $\frac{7}{10} \div \frac{5}{8}$ of $\frac{7}{10} = (\frac{5}{6} \times \frac{7}{10}) \div (\frac{5}{8} \times \frac{7}{10})$

$= \frac{7}{12} \div \frac{7}{16} = \frac{7}{12} \times \frac{16}{7} = \frac{4}{3} = 1\frac{1}{3}$. **Ans.**

5. $\frac{5}{9} \times 3\frac{1}{3} \div 2\frac{2}{5} = \frac{5}{9} \times \frac{10}{3} \times \frac{5}{12} = \frac{125}{162}$. **Ans.**

6. $\frac{5}{9} \div 3\frac{1}{3} \times 2\frac{2}{5} = \frac{5}{9} \times \frac{3}{10} \times \frac{12}{5} = \frac{2}{5}$. **Ans.**

7. $\frac{5}{9} \div 3\frac{1}{3}$ of $2\frac{2}{5} = \frac{5}{9} \div (\frac{10}{3} \times \frac{12}{5})$

$= \frac{5}{9} \div 8 = \frac{5}{9} \times \frac{1}{8} = \frac{5}{72}$. **Ans.**

8. $\frac{5}{9} \div 3\frac{1}{3} \div 2\frac{2}{5} = \frac{5}{9} \times \frac{3}{10} \times \frac{5}{12} = \frac{5}{72}$. **Ans.**

Note. The result in 7 and 8 is the same, Observe carefully the difference in solution of Examples 6 and 7 above.

Example 1. Simplify:—

$$13\tfrac{1}{3} \div 6\tfrac{1}{2} \text{ of } \tfrac{10}{13} + 2\tfrac{2}{3} - \tfrac{2}{7} \text{ of } \tfrac{5}{8} \div 1\tfrac{11}{14} \times 3\tfrac{1}{3}$$

Sol. The exp. $= \frac{40}{3} \div (\frac{13}{2} \times \frac{10}{13}) + \frac{8}{3} - (\frac{2}{7} \times \frac{5}{8}) \div \frac{25}{14} \times \frac{10}{3}$.

$$= \tfrac{40}{3} \div 5 + \tfrac{8}{3} - \tfrac{5}{28} \div \tfrac{25}{14} \times \tfrac{10}{3}$$

$$= \tfrac{40}{3} \times \tfrac{1}{5} + \tfrac{8}{3} - \tfrac{5}{28} \times \tfrac{14}{25} \times \tfrac{10}{3}$$

$$= \tfrac{8}{3} + \tfrac{8}{3} - \tfrac{1}{3}$$

$$= \tfrac{15}{3} = 5. \quad \textbf{Ans.}$$

Example 2. Simplify:—

$$8\tfrac{5}{6} - \left\{ \tfrac{1}{3} + (1 + \tfrac{1}{12}) \right\} \div 3(1 - \tfrac{3}{8} \text{ of } 2\tfrac{1}{2}).$$

Sol. The exp. $= 8\tfrac{5}{6} - \left\{ \tfrac{1}{3} + \tfrac{13}{12} \right\} \div 3(1 - \tfrac{15}{16})$

$$= 8\tfrac{5}{6} - \left\{ \frac{4+13}{12} \right\} \div 3(1 - \tfrac{15}{16}).$$

$$= 8\tfrac{5}{6} - \tfrac{17}{12} \div \tfrac{3}{16}$$

$$= 8\tfrac{5}{6} - \tfrac{17}{12} \times \tfrac{16}{3}$$

$$= 8\tfrac{5}{6} - 7\tfrac{5}{9} = 1\tfrac{5}{18}. \quad \textbf{Ans.}$$

REMOVAL OF BRACKETS.

§23. A pair of brackets may be removed if it is preceded by the sign "+" without changing the signs of the terms inside them ; thus $8+(2+3-2)=8+2+3-2=11$. **Ans.**

But if the pair is preceded by "−," the sign of every term within the brackets is changed, *i.e.*, + to − and − to + ; thus $8-(2+3-2)=8-2-3+2=5$. **Ans.**

In a long expression in which different kinds of brackets are used, less comprehensive brackets are removed first and then more comprehensive ones. Thus in the expression $9-\{8-[2+4-(3-2)]\}$, we remove first the least comprehensive or the innermost brackets '( ).' After removing the innermost brackets the expression stands thus:—

$9-\{8-[2+4-3+2]\}$.

Now remove again the innermost brackets '[ ]' as

$9-\{8-2-4+3-2\}$.

Now we have only a pair of brackets. Remove this pair also and the expression $=9-8+2+4-3+2=6$. **Ans.**

Example 3. Remove the brackets and then simplify expression $2+\{3-[5-(2+4)]\}$.

Sol. The expression $=2+\{3-[5-2-4]\}$

$=2+\{3-5+2+4\}$

$=2+3-5+2+4=6$. **Ans.**

Example 4. Remove the brackets and then simplify the expression $1\frac{1}{2}+\{2\frac{1}{3}-[\frac{3}{4}+(\frac{5}{6}-\frac{3}{8}+\frac{1}{8})]\}$

Sol. The exp. $=1\frac{1}{2}+\{2\frac{1}{3}-[\frac{3}{4}+(\frac{5}{6}-\frac{3}{8}-\frac{1}{8})]\}$

$=1\frac{1}{2}+\{2\frac{1}{3}-[\frac{3}{4}+\frac{5}{6}-\frac{3}{8}-\frac{1}{8}]\}$

$=1\frac{1}{2}+\{2\frac{1}{3}-\frac{3}{4}-\frac{5}{6}+\frac{3}{8}+\frac{1}{8}\}$

$=1\frac{1}{2}+2\frac{1}{3}-\frac{3}{4}-\frac{5}{6}+\frac{3}{8}+\frac{1}{8}$

$=3+\frac{1}{2}+\frac{1}{3}-\frac{3}{4}-\frac{5}{6}+\frac{3}{8}+\frac{1}{8}$

$=3+\frac{12+8-18-20+9+3}{24}$

$=3+\frac{32-38}{24}=2+\frac{32+24-38}{24}$

$=2\frac{18}{24}=2\frac{3}{4}$ **Ans.**

EXERCISE 38.

Simplify:—

1. $\frac{1}{5}$ of $2\frac{1}{2}+4\frac{1}{2}$.
2. $19-5\frac{1}{4}$ of $2\frac{2}{7}$.
3. $5\frac{1}{3}$ of $2\frac{1}{4}\times\frac{8}{9}$ of $1\frac{1}{16}$.
4. $\frac{5}{8}$ of $\frac{9}{10}\div\frac{11}{12}$ of $\frac{3}{8}$.
5. $\frac{7}{8}\times2\frac{1}{4}\div\frac{5}{6}$ of $\frac{9}{10}$.
6. $1\frac{2}{3}\div2\frac{2}{9}\times5\frac{1}{3}$ of $4\frac{1}{5}$.
7. $\frac{5}{6}\div2\frac{1}{3}$ of $2\frac{1}{2}\times1\frac{3}{4}$.
8. $2\frac{1}{4}\div5\frac{5}{6}\div3\frac{1}{7}\times1\frac{2}{9}$.
9. $11\frac{1}{2}\div5\frac{1}{9}\times\frac{2}{3}-\frac{3}{2}+5\frac{1}{2}$.
10. $9\frac{1}{4}+\frac{1}{3}\div\frac{1}{2}$ of $\frac{1}{3}-5\frac{1}{2}+1\frac{7}{8}$.
11. $2\frac{1}{4}\times1\frac{1}{2}\div1\frac{1}{8}$ of $2\frac{2}{5}$.
12. $3\frac{7}{10}-2\frac{1}{3}\times\frac{1}{2}+2\frac{4}{5}$ of $\frac{9}{14}$.
13. $2\frac{1}{4}$ of $1\frac{1}{2}\div1\frac{1}{8}\times2\frac{2}{5}$.
14. $2\frac{1}{4}$ of $1\frac{1}{2}\div1\frac{1}{8}$ of $2\frac{2}{5}+\frac{5}{2}$.
15. $(\frac{1}{3}+\frac{2}{11}+2\frac{7}{4}-\frac{1}{3}\times\frac{2}{11}$ of $2\frac{7}{4})\div(1-\frac{1}{3}$ of $\frac{2}{11}-\frac{2}{11}$ of $\frac{7}{24}-\frac{7}{24}$ of $\frac{1}{3}$
16. $(\frac{5}{3}+\frac{11}{6}+1\frac{7}{8}+1\frac{11}{12}-5)\div(\frac{3}{2}+\frac{1}{3})$.
17. $\{1\frac{3}{7}$ of $26\frac{1}{2}$ of $(1-\frac{2}{3})\}\times\{2\frac{5}{8}$ of $(4\frac{1}{5}-3\frac{2}{3})$ of $\frac{45}{106}\}$.
18. $(1\frac{2}{3}$ of $2\frac{7}{9}-3\frac{17}{18})\times(5\frac{1}{3}$ of $4\frac{1}{2}-3\frac{1}{4}$ of $3\frac{1}{5})\times\frac{27}{37}$ of $\frac{3}{17}\times\frac{1}{2}$.
19. $\frac{22}{10}\frac{47}{17}\div\frac{903}{1107}\times\frac{774}{615}\div\frac{1926}{5665}$.
20. $(2\frac{1}{2}-1\frac{3}{4})\div(4$ of $1\frac{1}{3}+6\times\frac{1}{2})\times(5-\frac{5}{9})\div(\frac{1}{2}+\frac{2}{5})+\frac{5}{9}$.
21. $(2+\frac{1}{3}+\frac{1}{8}+\frac{1}{12}+\frac{1}{15}+\frac{1}{18})\div(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})$.

Remove the brackets and then simplify:—

22. $1\frac{1}{3}+\{2\frac{1}{9}-[\frac{7}{9}-(\frac{2}{3}-1\frac{1}{2})]\}$

23. $7-\{\frac{3}{4}+[2\frac{1}{2}-(1\frac{1}{2}-\frac{1}{3})]\}$

24. $8-\{2\frac{1}{2}+[3\frac{1}{2}-(5\frac{1}{6}-2\frac{1}{2}+1\frac{1}{4})]\}$

25. $7\frac{1}{8}+\{\frac{5}{12}-[\frac{3}{8}+(16\frac{1}{2}-4\frac{1}{3}+5\frac{1}{6})]\}$

COMPLEX FRACTIONS.

§24. Complex fraction as defined in Art 4 is one in which the numerator or the denominator or both are fractions.

Thus $\frac{\frac{5}{6}}{8}$, $\frac{4}{\frac{5}{6}}$, $\frac{\frac{3}{2}}{\frac{5}{6}}$, $\frac{5\frac{1}{2}}{8\frac{1}{2}}$, $\frac{\frac{1}{3}+\frac{3}{4}}{\frac{5}{6}+\frac{7}{8}}$ are all complex fractions.

The numerator is separated from the denominator by a thick line.

Note. A complex fraction is read by using the word by for divided by between the readings of numerator and denominator.

Thus, $\frac{\frac{3}{2}}{\frac{5}{6}}$ is read '$\frac{3}{2}$ by $\frac{5}{6}$'.

§25. In the sum of a whole number and a fraction, when the fraction is either complex or simple, the sign of addition is sometimes omitted, as in $7\frac{\frac{2}{3}}{4}$ which really means $7+\frac{\frac{2}{3}}{4}$; and in a product when one of the factors is put within brackets, the sign of multiplication is often omitted as in $\frac{3}{4}(\frac{2}{3}-\frac{1}{7})$, which really means $\frac{3}{4}\times(\frac{2}{3}-\frac{1}{7})$.

§26. Complex fractions are simplified by the application of the same rules as simple fractions and are reduced by regarding them as the quotient of the numerator divided by the denominator.

Example 1. Simplify the complex fractions given in §24.

Sol. 1. $\frac{\frac{5}{6}}{8}=\frac{5}{6}\div 8=\frac{5}{6}\times\frac{1}{8}=\frac{5}{48}$ **Ans.**

2. $\frac{8}{\frac{5}{6}}=8\div\frac{5}{6}=8\times\frac{6}{5}=\frac{48}{5}=9\frac{3}{5}$ **Ans.**

Note the difference between the two solutions.

3. $\frac{\frac{3}{2}}{\frac{5}{6}} = \frac{3}{2} \div \frac{5}{6} = \frac{3}{2} \times \frac{6}{5} = \frac{9}{5} = 1\frac{4}{5}$. Ans.

4. $\frac{5\frac{1}{2}}{8\frac{1}{2}} = 5\frac{1}{2} \div 8\frac{1}{2} = \frac{11}{2} \times \frac{2}{17} = \frac{11}{17}$. Ans.

5. $\frac{\frac{1}{3}+\frac{3}{4}}{\frac{5}{6}+\frac{7}{8}} = \frac{\frac{13}{12}}{\frac{41}{24}} = \frac{13}{12} \div \frac{41}{24} = \frac{13}{12} \times \frac{24}{41} = \frac{26}{41}$. Ans.

Or thus :—Multiply the numerator and denominator by the L. C. of the denominators 3, 4, 6, 8, *i.e.*, 24.

$$\therefore \frac{\frac{1}{3}+\frac{3}{4}}{\frac{5}{6}+\frac{7}{8}} = \frac{8+18}{20+21} = \frac{26}{41}. \quad \textbf{Ans.}$$

☞ This step is, in many cases, very short and convenient.

Example 2. Simplify $\frac{\frac{1}{7}+\frac{3}{4}\text{ of }2\frac{2}{3}}{\frac{5}{12}-\frac{2}{7}} \times \frac{5\frac{1}{6} \div 4\frac{3}{7} - \frac{1}{6}}{14\frac{4}{11}+5\frac{3}{4} \div 2\frac{7}{8}} - 1$

Sol. The exp. $= \frac{\frac{1}{7}+\frac{3}{4} \times \frac{8}{3}}{\frac{5}{12}-\frac{2}{7}} \times \frac{\frac{31}{6} \times \frac{7}{31} - \frac{1}{6}}{\frac{158}{11}+\frac{23}{4} \times \frac{8}{23}} - 1.$

$$= \frac{\frac{1}{7}+2}{\frac{5}{12}-\frac{2}{7}} \times \frac{\frac{7}{6}-\frac{1}{6}}{\frac{158}{11}+2} - 1$$

$$= \frac{\frac{15}{7}}{\frac{11}{84}} \times \frac{1}{\frac{180}{11}} - 1$$

$$= \frac{15}{7} + \frac{84}{11} \times 1 \times \frac{11}{180} - 1.$$

$$= 1 - 1 = 0. \quad \textbf{Ans.}$$

EXERCISE 39.

Simplify :—

1. $\frac{2\frac{1}{3}}{4\frac{2}{3}}$. 2. $\frac{3\frac{1}{3}}{4\frac{2}{3}}$ 3. $\frac{5\frac{1}{5}}{2\frac{3}{5}}$ 4. $\frac{5\frac{1}{6}}{3\frac{4}{9}}$.

5. $\frac{5\frac{2}{5}}{3\frac{1}{15}+7\frac{11}{15}}$ 6. $\frac{3\frac{5}{6}}{2\frac{4}{15}+2\frac{1}{3}}$ 7. $\frac{4\frac{1}{5}-2\frac{4}{5}}{1\frac{2}{5}}$ 8. $\frac{5\frac{4}{7}-3\frac{6}{7}}{3\frac{1}{7}}$.

9. $\frac{\frac{7}{10}-\frac{2}{3}}{\frac{9}{12}+\frac{1}{5}} \div \frac{\frac{1}{3}}{9\frac{1}{2}}$. 10. $\frac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{6}}$ 11. $\frac{15\frac{2}{3}+6-\frac{3}{4}}{7\frac{4}{9}+\frac{3}{12}}$.

12. $8-8\times\frac{2\frac{1}{5}-1\frac{2}{7}}{2-\frac{6}{35}}$. 13. $\frac{1\frac{1}{2}\text{ of }2\frac{1}{4}\text{ of }3\frac{1}{5}\text{ of }4\frac{1}{6}}{3\frac{2}{3}\text{ of }2\frac{5}{11}}$

14. $\frac{1+\frac{1}{2}+\frac{1}{3}-\frac{1}{4}-\frac{1}{12}}{\frac{1}{3}+\frac{1}{6}} \div \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}$.

15. $\dfrac{8\frac{1}{7}-1\frac{1}{6}}{\frac{2}{3}+1\frac{1}{2}}-\dfrac{1}{5\frac{3}{4}-1\frac{1}{5}}+\dfrac{\frac{4}{3}}{\frac{5}{6}-\frac{1}{6}}$.

16. $(\frac{1}{3}+\dfrac{\frac{3}{7}}{1\frac{3}{5}}+\frac{11}{14}+\frac{5}{9}-1)\div\frac{5}{9}$ of $\frac{5}{8}$ of $2\frac{5}{7}$.

17. $(4\frac{2}{3}-1\frac{1}{3})\times(3\frac{1}{2}-\frac{3}{8})\div(13\frac{1}{3}+7\frac{1}{2})$ of $\dfrac{3\frac{1}{3}}{1\frac{11}{9}}$

18. $\dfrac{2\frac{1}{2}-1\frac{3}{4}}{4 \text{ of } 1\frac{1}{3}+6\times\frac{1}{2}}\times\dfrac{5-\frac{5}{9}}{\frac{1}{2}+\frac{2}{5}}+\frac{5}{9}$. 19. $\dfrac{\frac{1}{4}+\frac{2}{9}}{4-\frac{5}{9} \text{ of } 5\frac{1}{2}}\div\dfrac{\frac{2}{5}+\frac{1}{4}}{\frac{7}{10} \text{ of } 4\frac{1}{2}-2\frac{1}{2}}$

20. $\dfrac{1\frac{1}{4}-\frac{5}{12}}{1\frac{1}{4}+\frac{5}{12}}+\frac{1}{6}$ of $\dfrac{9\times5}{14\times3}-\dfrac{14\frac{1}{4}}{15}$.

21. $\dfrac{1+2\frac{1}{2}+3\frac{1}{3}}{\frac{2}{3}+\frac{6}{7}+\frac{12}{13}}\times\dfrac{55\frac{2}{3}\div11}{1\frac{2}{11} \text{ of } 13\frac{2}{3}}$.

22. $\dfrac{7\frac{1}{2}}{6\frac{1}{3}}+\dfrac{11\frac{1}{2}-2\frac{2}{5}}{11\frac{1}{2}+2\frac{2}{5}}\times10\frac{9}{13}-6\frac{42}{273}$.

23. $\dfrac{17}{7+2\frac{2}{5}}\times\frac{20}{21}\frac{21}{93}\div(1\frac{37}{48}-\frac{15}{16})$.

24. $\dfrac{\frac{3}{7}-\frac{2}{9}}{\frac{3}{7}+\frac{2}{9}}$ of $2\frac{11}{26}\div\dfrac{4}{13-3\frac{8}{9}}+3\frac{11}{16}\times\dfrac{3}{3-2\frac{10}{13}}$.

25. $\dfrac{2\frac{1}{4}}{2\frac{2}{3}}+\dfrac{2\frac{1}{2}+5\frac{1}{5}}{3\frac{1}{3}+9\frac{1}{2}}+\frac{5}{9}$ of $\frac{9}{10}+\frac{3}{8}$ of $\frac{3}{20}$.

26. $\dfrac{\frac{5}{14}-\frac{3}{7} \text{ of } \frac{1}{2}}{\frac{5}{16}+\frac{7}{12} \text{ of } 3\frac{1}{4}-(\frac{7}{8} \text{ of } \frac{37}{21}-\frac{1}{3})}\div\dfrac{\frac{1}{3} \text{ of } \frac{1}{2}+\frac{3}{2} \text{ of } 5}{9\frac{1}{3}-1\frac{2}{3}}$

27. $\dfrac{(4\frac{2}{3}-1\frac{1}{3})\times(3\frac{1}{2}-\frac{3}{8})}{13\frac{1}{3}+7\frac{1}{2}}$ of $\dfrac{3\frac{1}{3}}{1\frac{11}{9}}$ 28. $5-5\times\dfrac{2+1\frac{1}{5}(2+1\frac{1}{5})}{1\frac{1}{5}+2(2+1\frac{1}{5})}-\frac{3}{19}$

29. $\frac{2}{9}\div\left\{\dfrac{2\frac{8}{81} \text{ of } \frac{1}{9\frac{1}{3}}}{\frac{4}{5}-\frac{3}{4}}\right\}-\frac{1}{3}$ of $\left\{\frac{1}{2}-\frac{2}{5} \text{ of } \dfrac{1}{3-1\frac{2}{7}}\right\}$

30. $\dfrac{7}{5-\frac{8}{3}}\div\dfrac{3-\frac{4}{3}}{4-\frac{3}{2}}-\frac{5}{7}$ of $\left\{\dfrac{1}{1\frac{3}{7}}+\frac{6}{5} \text{ of } \dfrac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2}\right\}$

31. $\dfrac{5\frac{3}{8}+4\frac{3}{5}}{8\frac{3}{5}-5\frac{3}{4}}\div\left\{\dfrac{\frac{7}{8}-\frac{6}{7}+\frac{5}{6}}{\frac{8}{7}-\frac{7}{6}+\frac{6}{5}} \text{ of } (1\frac{3}{11}\times3\frac{4}{5})\right\}$

32. $\left(\dfrac{\frac{4}{11} \text{ of } 1\frac{4}{7}-\frac{5}{6}+\frac{6}{7}}{\frac{3}{7} \text{ of } 1\frac{8}{3}+1\frac{3}{4}}\right)\times4\frac{3}{10}-\frac{5}{6} \text{ of } \frac{4}{5}$.

33. $\dfrac{\frac{2}{9}(1\frac{2}{3}-\frac{3}{8}\text{ of }1\frac{4}{9})+\frac{4}{17}}{\frac{3}{4}\times1\frac{4}{7}\div1\frac{1}{2}-\frac{11}{28}}\times\dfrac{\frac{3}{4}+\frac{2}{3}}{\frac{3}{4}-\frac{2}{3}}-20.$

34. $\dfrac{6\frac{3}{5}-4\frac{1}{8}}{5\frac{1}{2}-4\frac{2}{5}}-\dfrac{2\frac{2}{35}\div1\frac{11}{25}+1\frac{1}{4}-\frac{9}{13}\text{ of }3\frac{1}{4}}{\frac{3}{2}\times3\frac{5}{7}\times5\frac{3}{5}\div3\frac{4}{15}}\times33\frac{3}{7}.$

35. $\dfrac{(\frac{1}{3}+\frac{1}{7})\div(\frac{1}{7}-\frac{1}{9})}{(\frac{1}{3}+\frac{1}{7})\div\frac{1}{7}-\frac{1}{9}}\cdot\dfrac{1\frac{7}{8}+\frac{3}{28}}{\frac{7}{12}-\frac{1}{7}}-\dfrac{\frac{1}{2}\text{ of }\frac{1}{3}}{\frac{1}{2}\div\frac{1}{3}}.$

36. $\dfrac{\frac{3}{3}-\frac{1}{5}\text{ of }\frac{7}{8}+\frac{8}{9}}{\frac{4}{9}\div\frac{8}{7}-\frac{1}{5}+\frac{2}{3}}-\dfrac{\frac{6}{11}-\frac{4}{9}}{1-\frac{5}{9}\div\frac{3}{5}}.$

37. $\dfrac{\frac{5}{7}\text{ of }1\frac{3}{8}-\frac{5}{8}\text{ of }\frac{6}{7}}{1-\frac{1}{7}\times(\frac{5}{12}+\frac{1}{3})}\times\dfrac{\frac{1}{3}+\frac{1}{7}\div(\frac{1}{7}-\frac{1}{9})}{(\frac{1}{3}+\frac{1}{7})\div\frac{1}{7}-\frac{1}{9}}$

38. $\dfrac{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}}\times3\frac{1}{2}\div\dfrac{\frac{1}{15}+\frac{1}{20}-\frac{1}{24}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}}.$

39. $2\frac{3}{7}\text{ of}\dfrac{13\frac{1}{2}-9\frac{2}{3}}{15\frac{1}{5}-11\frac{7}{30}}\div3\frac{2}{7}+\dfrac{1\frac{7}{9}}{9\frac{2}{9}-8\frac{4}{9}}.$

40. $\dfrac{\frac{3}{11}(\frac{2}{19}\text{ of }2\frac{5}{7}+\frac{1}{3}\text{ of }1\frac{2}{7})}{\frac{2}{13}\times1\frac{5}{8}\times1\frac{1}{7}-\frac{1}{11}}\cdot\dfrac{\frac{1}{3}}{\frac{2}{9}\text{ of }3-\frac{1}{17}\text{ of }5\frac{2}{3}}$

41. $\left\{3+\dfrac{1\frac{3}{8}+\frac{11}{12}}{2\frac{11}{15}-1\frac{9}{20}}\text{of }\dfrac{11\frac{7}{8}6}{8\frac{4}{4}}\right\}\div\left\{(3\frac{1}{2}\times5\frac{1}{7})\div\frac{16}{5}+3\frac{3}{4})+\frac{1}{2}\right\}$

CONTINUED FRACTIONS.

§27. The fraction of the form of $\cfrac{1}{1+\cfrac{1}{1+\frac{2}{3}}}$ is called **a continued fraction.** The method of solving such fractions will be best illustrated by the following solved example:—

Example 1. Simplify the continued fraction

$$\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{1-\frac{5}{12}}}}$$

Sol. The fraction $=\cfrac{1}{4+\cfrac{1}{2-\cfrac{1}{\frac{7}{12}}}}$

$$= \cfrac{1}{4+\cfrac{1}{2-\frac{1}{\frac{7}{2}}}}$$

$$= \cfrac{1}{4+\cfrac{1}{\frac{2}{7}}}$$

$$= \frac{1}{4+\frac{7}{2}} = \frac{1}{\frac{15}{2}} = \frac{2}{15} \text{ Ans.}$$

Rule. *Begin at the bottom and work upwards.*

§28. **To convert a given fraction to a continued fraction.**

The following example shows how we convert a fraction to a continued fraction with unit numerators and with all signs positive.

Example 2. Convert $\frac{13}{59}$ to a continued fraction.

Sol. The fraction $= \frac{1}{\frac{59}{13}} = \frac{1}{4+\frac{7}{13}}$

$$= \cfrac{1}{4+\cfrac{1}{\frac{13}{7}}}$$

$$= \cfrac{1}{4+\cfrac{1}{1+\frac{6}{7}}}$$

$$= \cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{\frac{7}{6}}}}$$

$$= \cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{6}}}}$$

which is often written in the form

$$\frac{1}{4+}\ \frac{1}{1+}\ \frac{1}{1+}\ \frac{1}{6}$$

EXERCISE 40.

Simplify (1 to 12):—

1. $2+\cfrac{1}{5+\cfrac{1}{1+\frac{1}{2}}}$

2. $\cfrac{1}{10+\cfrac{1}{2+\frac{1}{30}}}$

3. $\cfrac{1}{4-\cfrac{1}{2-\cfrac{1}{1-\frac{5}{13}}}}$

4. $\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}}$

5. $1+\cfrac{2}{3+\cfrac{1}{4+\cfrac{1}{5\frac{2}{3}}}}$

6. $9+\cfrac{1}{1+\cfrac{1}{7+\frac{1}{5}}}$

7. $\cfrac{2}{2+\cfrac{2}{2-\cfrac{2}{2-\frac{2}{3}}}}$

8. $1-\cfrac{3}{7-\cfrac{3}{6-\cfrac{3}{5-\frac{3}{4}}}}$

9. $\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{2}}}}$

10. $9+\cfrac{1}{1-\cfrac{3}{2+\cfrac{4}{2+\frac{5}{6}}}}$

11. $\cfrac{5}{4+\cfrac{3}{3+\cfrac{2}{2+\cfrac{1}{1+\frac{1}{5}}}}}$

12. $\cfrac{1}{2+\cfrac{3}{4+\cfrac{1}{5-\cfrac{1}{7-\frac{2}{3}}}}}$

13. **Divide:—**

$2\frac{1}{3}+8\frac{6}{11}-\frac{1}{5}$ of $(7\frac{1}{2}-3\frac{1}{3})$ by $11+\cfrac{1}{1-\cfrac{1}{14+\cfrac{1}{8\frac{1}{11}}}}$

Simplify:—

14. $\dfrac{10\frac{2}{3}-(5\frac{2}{5}+4\frac{9}{20})}{10\frac{1}{3}-(2\frac{4}{5}-\frac{4}{15})-7} \div \dfrac{1}{6+\dfrac{1}{1+\frac{1}{6}}} + \dfrac{1}{4-\dfrac{1}{3-\dfrac{1}{2-\dfrac{1}{1-\frac{5}{13}}}}}$

15. $\dfrac{6\frac{41}{88}}{6-4\frac{4}{11}} + \frac{7}{9}\times 1\frac{22}{27} \text{ of } \frac{3}{56} - \dfrac{6+\dfrac{1}{6+\dfrac{1}{6-\frac{1}{6}}}}{6}$

16. (*i*) $4+\dfrac{1}{1+}\ \dfrac{1}{1+}\ \frac{1}{2}$. (*ii*) $1+\dfrac{1}{1+}\ \dfrac{1}{1+}\ \dfrac{1}{5+}\ \frac{1}{6}$.

17. Convert $\frac{10}{73}$ to a continued fraction.

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§29. **Application of fractions to compound quantities.**

☞ Study the following examples very carefully. They are very useful in solving questions of practice.

Example 1. Add together Rs. 15, 8*a*. $7\frac{1}{5}p$,, Rs. 18, 9*a*. $5\frac{1}{3}p$., Rs. 2, 3*a*. $6\frac{7}{9}p$.

Sol.

Rs.	*a.*	*p.*	
15	8	$7\frac{1}{5}$	$\frac{1}{5}+\frac{1}{3}+\frac{7}{9}=\frac{59}{45}=1\frac{14}{45}$.
18	9	$5\frac{1}{3}$	Put down $\frac{14}{45}p$. and carry on 1*p*.
2	3	$6\frac{7}{9}$	Now proceed further.
36	5	$7\frac{14}{45}$	**Ans.**

Example 2. Subtract £17, 8*s*. $5\frac{3}{4}d$. from £19, 7*s*. $3\frac{5}{8}d$.

Sol.

£.	*s.*	*d.*	
19	7	$3\frac{5}{8}$	Here $\frac{5}{8}$ is less than $\frac{3}{4}$, therefore add
17	8	$5\frac{3}{4}$	1 to $\frac{5}{8}$ making it $\frac{13}{8}$.
			Now $\frac{13}{8}-\frac{3}{4}=\frac{7}{8}d$. Put down $\frac{7}{8}d$.
1	18	$9\frac{7}{8}$.	Add. 1*d*. to 5*d*. and proceed further.

Example 3. Multiply Rs. 5, 7*a*. $6\frac{5}{6}p$. by 29.

Sol.

Rs.	*a.*	*p.*	
5	7	$6\frac{5}{6}$	$\because\ 29=7\times4+1$
		7	$\frac{5}{6}\times7=\frac{35}{6}=5\frac{5}{6}$, put down $\frac{5}{6}$ and carry
38	4	$11\frac{5}{6}$	on 5. Now proceed further.
		4	$\frac{5}{6}\times4=\frac{10}{3}=3\frac{1}{3}$, put down $\frac{1}{3}$ and carry on
153	3	$11\frac{1}{3}$	3 and proceed further.
5	7	$6\frac{5}{6}$	$\frac{1}{3}+\frac{5}{6}=\frac{7}{6}=1\frac{1}{6}$, put down $\frac{1}{6}$, carry on 1
158	11	$6\frac{1}{6}$	and proceed further.

Example 4. Find the value of $\frac{7}{8}$ of Rs. 3, 5*a*. 4*p*.

Sol.

Rs.	*a.*	*p.*
3	5	4
		7
8)23	5	4
2	14	8

Ans.

Or

Rs.	*a.*	*p.*
8)3	5	4
0	6	8
		7
2	14	8

Ans.

Example 5. Simplify Rs. 7, 8*a*. 9*p*. $\times 5\frac{3}{4}$.

Sol. Rs. *a.* *p.* Rs. *a.* *p.*

$7\ 8\ 9 \times 5\frac{3}{4} = 7\ 8\ 9$

Rs.	*a.*	*p.*
7	8	9
		3
4)22	10	3
5	10	$6\frac{3}{4}$

			5
37	11	9	} Add
5	10	$6\frac{3}{4}$	} Add
43	6	$3\frac{3}{4}$	**Ans.**

$\therefore$ Rs. 5 10 $6\frac{3}{4}$ = $\frac{3}{4}$ of Rs. 7, 8*a*. 9*p*.

Example 6. Divide Rs. 13, 10*a*. 8*p*. by $2\frac{1}{3}$ of $3\frac{3}{5}$.

Sol. $\because$ $2\frac{1}{3}$ of $3\frac{3}{5} = \frac{7}{3} \times \frac{18}{5} = \frac{42}{5}$.

$\therefore$ Rs. 13, 10*a*. 8*p*. $\div \frac{42}{5}$ = Rs. 13, 10*a*. 8*p*. $\times \frac{5}{42}$.

Rs.	*a.*	*p.*
= 13	10	8
		5
6)68	5	4
7)11	6	$2\frac{2}{3}$
1	10	$\frac{8}{21}$

Ans.

$42 = 6 \times 7$

Example 7. Divide £56, 7*s*. $5\frac{5}{8}$*d*. by 24.

Sol.

£.	*s.*	*d.*	
4)56	7	$5\frac{5}{8}$	Rem. $= 1\frac{5}{8} = \frac{13}{8}$.
6)14	1	$10\frac{13}{32}$	$\frac{13}{8} \div 4 = \frac{13}{8} \times \frac{1}{4} = \frac{13}{32}$.
2	6	$11\frac{141}{192}$	Rem. $= 4\frac{13}{32} = \frac{141}{32}$.

24

Ans. $\frac{141}{32} \div 6 = \frac{141}{192}$.

EXERCISE 41.

Add together :—

1.

£.	*s.*	*d.*
15	7	$3\frac{5}{6}$
16	3	$4\frac{3}{4}$
18	2	$3\frac{7}{8}$

2.

£.	*s.*	*d.*
21	9	$3\frac{7}{8}$
15	4	$6\frac{5}{24}$
15	3	$3\frac{1}{32}$

3.

Rs.	*a.*	*p.*
8	7	$4\frac{1}{2}$
12	13	$3\frac{5}{16}$
16	4	$7\frac{3}{32}$

	mds.	sr.	ch.
4.	21	15	$9\frac{1}{12}$
	56	18	$3\frac{2}{9}$
	18	25	$7\frac{5}{18}$
	16	21	$6\frac{5}{21}$

	ton	cwt.	qr.
5.	78	16	$3\frac{7}{8}$
	15	12	$2\frac{1}{12}$
	16	13	$1\frac{3}{4}$
	12	7	$1\frac{5}{16}$

	mi.	fur.	po.
6.	17	2	$35\frac{1}{9}$
	18	3	$16\frac{2}{3}$
	16	4	$15\frac{5}{18}$
	23	3	$18\frac{13}{21}$

Perform the following subtractions :—

	£.	*s.*	*d.*
7.	48	5	$7\frac{1}{8}$
	19	6	$8\frac{2}{8}$

	Rs.	*a.*	*p.*
8.	21	7	$4\frac{1}{3}$
	15	9	$8\frac{7}{18}$

	yd.	ft.	in.
9.	36	3	$7\frac{2}{9}$
	18	1	$9\frac{3}{4}$

10. Subtract 18 mds. 24 srs. $12\frac{3}{4}$ ch. from 29 mds. 36 srs. $7\frac{9}{16}$ chk.

Simplify :

11. Rs. 5, 12*a*. 9*p*. $\times \frac{2}{3}$.
12. Rs. 9, 13*a*. 4*p*. $\times \frac{1}{8}$.
13. Rs. 21, 11*a*. 9*p*. $\times 1\frac{3}{13}$.
14. Rs. 24, 13*a*. 6*p*. $\times 1\frac{4}{15}$.
15. 20 mds. 15 srs. 8 chk. $\times \frac{7}{8}$.
16. 16 mds. 21 srs. 8 chk. $\times 1\frac{5}{16}$.
17. 5 tons 10 cwt. 3 qr. $\times 1\frac{4}{9}$.
18. 12 tons 10 cwt. 2 qr. $\times 1\frac{7}{8}$.
19. Rs. 9, 10*a*. $8\frac{1}{2}$*p*. $\times 1\frac{5}{6}$.
20. Rs. 16, 11*a*. $7\frac{2}{3}$*p*. $\times 5\frac{1}{4}$.
21. 28 mds. 21 sr. $7\frac{1}{2}$ chk. $\times 5\frac{1}{3}$.
22. 16 tons 12 cwt. $3\frac{1}{2}$ qr. $\times 9\frac{1}{3}$.
23. 21 tons 13 cwt. $1\frac{5}{6}$ qr. $\times 7\frac{1}{4}$.
24. 15 yards 2 ft. $3\frac{1}{2}$ in. $\times 3\frac{1}{3}$.
25. 24 yards 1 ft. $7\frac{1}{3}$ in. $\times 2\frac{1}{4}$.
26. Rs. 307, 4*a*. 4*p*. $\div \frac{4}{23}$.
27. Rs. 45, 13*a*. 4*p*. $\div 1\frac{7}{9}$.
28. Rs. 173, 5*a*. 4*p*. $\div 20\frac{4}{5}$.
29. 12 mds. 27 srs. 8 chk. $\div 1\frac{7}{9}$.
30. 13 mds. 21 srs. 7 chk. $\div 5\frac{5}{6}$.
31. 18 tons 11 cwt. $3\frac{1}{2}$ qr. $\div 5\frac{1}{4}$.
32. 11 tons 18 cwt. 2 qr. 21 lb. $\div 9\frac{1}{3}$.
33. $\frac{5}{6}$ of Rs. 16, 12*a*. 8*p*. $\times 1\frac{3}{5}$ of Rs. 15, 9*a*. 7*p*. $\times 1\frac{5}{16}$ of Rs. 21, 13*a*. 4*p*.

34. $\frac{3}{5}$ of Rs. 3, 5*a*. 4*p*.$+\frac{5}{7}$ of Rs. 21, 14*a*.$+\frac{2}{11}$ of Rs. 47, 3*a*. 4*p*.

35. $\frac{15\frac{3}{5}}{7\frac{4}{5}}$ of £1$+\frac{1}{3}$ of £140, 10*s*. 6*d*.$+2\frac{13}{63}$ of half a guinea.

36. $\frac{3}{4}$ of 3 mds. 34 srs. $+\frac{3}{7}$ of 8 mds. 9 srs.$+\frac{2}{3}$ of 3 srs. 12 chk.

37. $\frac{43\frac{6}{69}}{}$ of Rs. 2. 9*a*. $+\frac{117}{480}$ of Rs. $7\frac{1}{2}+\frac{29}{333}$ of Rs. 9, 4*a*.

38. $£\frac{11}{12}+\frac{3}{4}$ of 2 guineas$-\frac{5}{9}$ of 3*s*. 9*d*.$+\frac{7}{8}$ of 1*s*.

39. $(2\frac{1}{2}\div 3\frac{1}{2})$ of £ 3, 6*s*.$+(\frac{2}{3})^2$ of 27*s*.$-3\frac{2\frac{1}{2}}{6}$ of 5*s*.

40. $\frac{1\frac{1}{8}\div 1\frac{1}{5}}{\frac{2}{3}\text{ of }\frac{5}{9}\div 10\frac{1}{8}}$ of $\frac{1\frac{1}{2}\text{ of }4\frac{1}{9}}{6\frac{15}{16}\times 5\frac{1}{3}}$ of Rs. 50

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§ 30. **To express one concrete quantity as a fraction of another quantity of the same kind.**

Rule. *By dividing one concrete number by another concrete number of the same kind we get an abstract number which may be an integer or a fraction. To find this number reduce both the quantities to the same denomination and divide the first by the second.*

Example 1. Express Rs. 2, 4*a*. as a fraction of Rs. 9.

Sol. The fraction $=\frac{\text{Rs. 2, 4}a}{\text{Rs. 9}}=\frac{36a.}{144a.}$

$=\frac{1}{4}$. **Ans.**

Note. A beginner makes a mistake by dividing Rs. 9 by Rs. 2, 4a.

Example 2. Express 2 mds. 27 srs. 8 chk. as a fraction of 8 mds. 2 srs. 8 chk.

Sol. The fraction $=\frac{\text{2 mds. 27 sr. 8 chk.}}{\text{8 mds. 2 sr. 8 chk.}}$

$=\frac{1720\text{ chk.}}{5160\text{ chk.}}=\frac{1}{3}$. **Ans.**

Note. We can solve the above example by reducing each compound quantity to the fraction of the highest denomination thus :–

2 mds. 27 srs. 8 chks.$=$2 mds. $27\frac{1}{2}$ srs.

$$=2\ \frac{55}{2\times40}\text{ or }2\tfrac{11}{16}\text{ mds.}=\tfrac{43}{16}\text{ mds.}$$

8 mds. 2 srs. 8 chks.$=$8 mds. $2\frac{1}{2}$ srs.

$$=8\ \frac{5}{2\times40}\text{ or }8\tfrac{1}{16}\text{ mds.}=\tfrac{129}{16}\text{ mds.}$$

The required fraction$=\frac{43}{16}\div\frac{129}{16}=\frac{43}{16}\times\frac{16}{129}=\frac{1}{3}$. **Ans.**

Note. We can make the statement of such questions in a number of different ways. For instance we can state the 1st example in the following ways also :–

1. Express Rs. 2, 4*a*. as a fraction of Rs. 9.
2. Reduce Rs. 2, 4*a*. to the fraction of Rs. 9.
3. What fraction is Rs. 2, 4*a*. of Rs. 9 ?
4. What fraction of Rs. 9, is Rs. 2, 4*a*. ?
5. What part is Rs. 2, 4*a*. in terms of Rs. 9 ?
6. Express Rs. 2, 4*a*. in terms of Rs. 9.
7. Find the measure of Rs. 2, 4*a*. in terms of Rs. 9.
8. Find the quotient when Rs. 2, 4*a*. is divided by Rs. 9.
9. Find the ratio of Rs. 2, 4*a*. to Rs. 9.
10. How many times is Rs. 9 contained in Rs. 2, 4*a*. ?
11. How often is Rs. 9 contained in Rs. 2. 4*a*. ?

Example 3. Simplify $\dfrac{\text{Rs. 2, 5}a\text{. 6}p\text{.}}{\text{Rs. 3, 12}a\text{.}}-\dfrac{\text{1 hr. 16 m. 46 sec.}}{\text{2 hr. 7 m. 55 sec.}}$

Sol. The exp. $=\dfrac{450p\text{.}}{720p\text{.}}-\dfrac{4605\text{ sec.}}{7675\text{ sec.}}=\frac{5}{8}-\frac{3}{5}=\frac{1}{10}$. **Ans.**

Example 4. Simplify $\dfrac{\text{Rs. 9, 1}a\text{.}}{\text{Rs. 17, 13}a\text{.}}-\dfrac{\text{1 sr. 9 chk}}{\text{5 sr. 15 chk.}}$

Sol. The quantity $=\dfrac{145a\text{.}}{285a\text{.}}-\dfrac{25\text{ chk.}}{95\text{ chk.}}$

$$=\frac{145}{285}-\frac{25}{95}$$
$$=\frac{29}{57}-\frac{5}{19}=\frac{14}{57}.$$

EXERCISE 42.

1. Express 5*a*. 4*p*. as a fraction of Re. 1.
2. Express 13*a*. 4*p*. as a fraction of Rs. 5.
3. Express Rs. 3, 10*a*. 8*p*. as a fraction of Rs. 11.
4. Reduce Rs. 5, 1*a*. 4*p*. to the fraction of Rs. 7, 10*a*.

5. What fraction of Rs. 21 is Rs. 5, 4*a*. ?

6. Find the quotient when Rs. 5, 8*a*. 9*p*. is divided by Rs. 15, 8*a*. 6*p*.

7. What fraction is £6, 13*s*. 4*d*. of £10 ?

8. Find the measure of £25, 16*s*. 8*d*. in terms of £41, 6*s*. 8*d*.

9. Express 15 mds. 27 srs. 8 chks. in terms of 25 mds. 4 srs.

10. How many times is 8 hours 40 minutes 20 seconds contained in 2 hours 10 minutes 5 seconds ?

11. What part of 8 miles is 1 mile 586 yards 2 feet ?

12. Reduce 3 miles 110 yards to the fraction of 8 miles 176 yards.

13. Express 4 cwt. 2 qrs. 7 lbs. in terms of 14 cwt. 7 lbs.

14. What is the ratio of Rs. 12, 13*a*. 4*p*. to Rs. 19, 10*a*. 8*p*. ?

15. What is the ratio of £15, 16*s*. 8*d*. to £213, 15*s*. ?

16. Reduce $\frac{1}{8}$ of Rs. 2, 10*a*. 8*p*. + $\frac{2}{5}$ of Rs. 3, 5*a*. 4*p*. to the fraction of Rs. 10, 13*a*. 4*p*.

17. Express $\frac{1}{9}$ of a guinea + $\frac{3}{8}$ of £1 + $\frac{5}{24}$ of 1*s*. + $\frac{1}{4}$ of 1*d*. as the fraction of £ 24, 3*s*.

18. Express $\frac{2}{5}$ of Rs. 3, 8*a*. + $\frac{3}{7}$ of Rs. 5, 4*a*. − $\frac{2}{9}$ of Rs. 10, 8*a*. as the fraction of Rs. 39, 8*a*.

19. Express $\frac{2}{5}$ of $8\frac{1}{3}$ mds. + $\frac{1}{11}$ of $5\frac{1}{2}$ mds. − $\frac{2}{9}$ of $4\frac{1}{2}$ mds. as the fraction of $\frac{1}{5}$ of $23\frac{1}{3}$ mds.

20. Express $8\frac{3}{4}$ days of $\frac{2}{7}$ − $5\frac{1}{4}$ days of $\frac{2}{3}$ + $6\frac{3}{4}$ days of $\frac{2}{9}$ as the fraction of $8\frac{1}{4}$ days of $\frac{1}{11}$.

In the following examples express the former of the two quantities as a fraction of the latter :—

21. $(8\frac{4}{7} - 3\frac{3}{5})$ of £5, 9*s*. $11\frac{1}{2}d$. ; $\frac{7\frac{4}{5}}{4\frac{2}{3}}$ of £30, 13*s*. $2\frac{3}{4}d$.

22. $\frac{4}{15}$ of £$1\frac{1}{2}$ + $\frac{5}{8}$ of 5*s*. 4*d*. − $8\frac{1}{2}$ of $\frac{1}{4\frac{1}{4}}$ of 5*s*. $3\frac{3}{4}d$. ; 2*s*. $1\frac{1}{2}d$.

23. $\frac{3\frac{1}{2}}{4\frac{1}{2}}$ of $\left\{ \text{£}\frac{3}{4} - \frac{3}{7} \text{ of } 5s. \right\}$; $\frac{2}{3}$ of 27*s*.

24. **Simplify :—**

$$\left(5\tfrac{5}{6}-\tfrac{5}{42}\right)\text{of}\left(\frac{\frac{5}{6}}{3\frac{1}{3}}\div\tfrac{7}{8}\text{ of }\tfrac{3}{4}\right)\div\tfrac{5}{7}\text{ of}\frac{3\text{ tons }3\text{ cwt.}}{9\text{ cwt.}}$$

25. **Find the value of :—**

$$12\times\left(\tfrac{2}{29}-\tfrac{1}{24}-\tfrac{1}{58}-\tfrac{1}{174}-\tfrac{1}{232}\right)+3\tfrac{23}{361}\div\frac{2\frac{5}{8}}{2\frac{5}{7}}\text{of}\frac{11s.\ 4d.}{12s.\ 3d.}$$

26. **Simplify :—**

$$\frac{\frac{3}{5}+\frac{7}{8}}{\frac{8}{9}+\frac{9}{10}}\text{of}\frac{13s.\ 5d.}{9s.\ 10d.}\div\tfrac{2}{3}\left(\tfrac{3}{7}+\tfrac{8}{9}\right)\text{ of }\frac{3\text{ tons }3\text{ cwt.}}{4\text{ tons }3\text{ cwt.}}$$

27. What fraction of a ton added to $\frac{11}{28}$ of 2 cwt. will make it equal to 1 cwt. 2 qr. 11 lbs. ?

28. What fraction of Rs. 29, 12*a*. must be added to $\frac{3\frac{1}{2}}{4\frac{1}{3}}$ of $(3\frac{3}{7}+1\frac{2}{5})$ of Rs. 6, 9*a*. to make the sum equal to Rs. 32, 8*a*. ?

29. Find the weight which is the same fraction of 15 cwt. 2 qrs. 13 lbs. as £1, 11*s*. $10\frac{1}{2}d$. is of £3, 10*s*. $1\frac{1}{2}d$.

30. What fraction of Re. 1, 13*a*. 7*p*. must be added to $\frac{3\frac{1}{2}}{2\frac{1}{3}}$ of $(\frac{3}{4}+\frac{4}{3})$ of 1*a*. 4*p*. to make the sum equal to Re. 1

31. Express $\frac{3}{14}$ of 3*s*. 6*d*.$+\frac{2}{7}$ of £ 11, 7*s*. 6*d*.$-\frac{5}{8}$ of £4, 17*s*. 4*d*.$+1d$. to the fraction of Rs. 12 when one rupee$=$1*s*. 6*d*.

32. Reduce $\frac{1}{3}$ of Rs. 30, 6*a*. 6*p*.$+\frac{2}{5}$ of 13*a*. 9*p*.$-\frac{1}{2}$ of Rs. 2. 9*a*. 4*p*. to the fraction of £5 when one rupee$=$1*s*. 4*d*.

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§24. **G. C. M. and L. C. M. of Fractions.**

The theory of the G. C. M. and L. C. M. of whole numbers is also applicable when the given numbers are fractions.

Rule. *If the fractions are of the same denominator, find the G. C. M. or L. C. M. of the numerators and put it above the denominator. But if the fractions are not of the same denominator reduce them to their least common denominator and then find the G. C. M. or L. C. M. of the new numerators and write it above the common denominator.* Thus—

Example 1. Find the G. C. M. and L. C. M. of $\frac{5}{23},\frac{10}{23},\frac{15}{23}$.

Sol. The fractions have the same denominator. The G. C. M of the numerators $=5$,
and L. C. M. of the numerators $=30$.
$\therefore$ the required G. C. M. $=\frac{5}{23}$. **Ans.**
and ,, L. C. M. $=\frac{30}{23}$ or $1\frac{7}{23}$ **Ans.**

Example 2. Find the G. C. M. and L. C. M. of $\frac{5}{12}$, $\frac{5}{18}$, and $\frac{5}{36}$.

Sol. The fractions when reduced to their L. C. D. $=\frac{15}{36}$, $\frac{10}{36}$ and $\frac{5}{36}$.
$\therefore$ G. C. M. $=\frac{5}{36}$ and L. C. M. $=\frac{30}{36}$ or $\frac{5}{6}$. **Ans.**

From the examples solved above we can deduce the following rules also:—

(*i*) the G. C. M. of any number of fractions

$$= \frac{\text{G. C. M. of numerators}}{\text{L. C. M. of denominators}}$$

(*ii*) the L. C. M. of any number of fractions

$$= \frac{\text{L. C. M. of numerators}}{\text{G. C. M. of denominators}}$$

Note. When the given fractions are not in their lowest terms they must be reduced before applying the rule.

EXERCISE 43.

Find the G. C. M. and L. C. M. of:—

1. $\frac{5}{12}$, $\frac{7}{12}$, $\frac{15}{12}$.
2. $\frac{7}{40}$, $\frac{14}{40}$, $\frac{21}{40}$.
3. $\frac{9}{38}$, $\frac{15}{38}$, $\frac{21}{38}$.
4. $\frac{5}{12}$, $\frac{3}{21}$, $\frac{5}{18}$.
5. $\frac{7}{15}$, $\frac{10}{25}$, $\frac{21}{40}$.
6. $2\frac{1}{2}$, $\frac{3}{7}$, $\frac{5}{9}$, $\frac{5}{12}$.
7. $3\frac{1}{4}$, $\frac{5}{8}$, $\frac{13}{24}$, $\frac{11}{36}$.
8. $\frac{13}{17}$, $\frac{21}{34}$, $\frac{35}{51}$, $\frac{3}{68}$.
9. 3, $\frac{5}{11}$, $\frac{4}{11}$, $\frac{5}{33}$.

10. One pendulum ticks 57 times in 58 seconds, another 608 times in 609 seconds. If they are started together, how often will they have ticked together in the first hour?

11. What English sum of money can buy an exact number of rupees or dollars? Re. 1=1s. 4*d*. and 1 dollar=3s. 8*d*.

12. One sixth of the number of books in a library consist of Mathematics, $\frac{3}{5}$ of the remainder of Fiction, $\frac{8}{9}$ of what still remains of History and the remaining books are on Science. What should be the least number of books in the library to satisfy these conditions?

13. 5 competitors start together and continue running round a circular course $1\frac{1}{2}$ miles in circumference. They run at the rate of 2, 3, 4, 5 and 6 miles an hour respectively. How soon will they all be at the starting place?

14. One Imperial maund $=82\frac{2}{7}$ lbs. avoirdupois, one Madras maund $=26\frac{14}{35}$ lbs. avoirdupois. Find the least number of pounds that can be expressed as an exact number of Imperial and Madras maund.

☞ Some Important Typical Examples in Fractions.

Study the following examples very carefully:—

Example 1. A man owns $\frac{3}{5}$ of a house and sold $\frac{1}{3}$ of it for Rs. 250 : find the value of the house.

Sol. The portion he sold $=\frac{1}{3}$ of $\frac{3}{5}=\frac{1}{5}$.

$\because$ he sold $\frac{1}{5}$ for Rs. 250,

$\therefore$ the value of the whole $=$ Rs. $250\times 5=$ Rs. 1250. **Ans.**

Example 2. A person bequeathed his property to his three sons thus : to the first $\frac{2}{5}$ and to the second $\frac{1}{3}$ of the remainder and the remaining property worth Rs. 1500 to the third ; find the value of the property.

Sol. First boy's share $=\frac{2}{5}$

Remainder $=1-\frac{2}{5}=\frac{3}{5}$

$\therefore$ second boy's share $=\frac{1}{3}$ of $\frac{3}{5}=\frac{1}{5}$

Now remaining property $=1-(\frac{2}{5}+\frac{1}{5})=\frac{2}{5}$

$\because$ $\frac{2}{5}$ of the property is worth Rs. 1500

$\therefore$ whole property is worth Rs. $1500\times\frac{5}{2}$

$=$ Rs. 3750. **Ans.**

Example 3. One-seventh of a sum exceeds its one-ninth by Rs. 192 ; find the sum.

Sol. $\frac{1}{7}-\frac{1}{9}=\frac{2}{63}$.

$\because$ $\frac{2}{63}$ of the sum $=$ Rs. 192

$\therefore$ the sum $=$ Rs. $192\times\frac{63}{2}=$ Rs. 6048. **Ans.**

Example 4. A man was given $\frac{11}{12}$ of a sum and was asked what the sum was. He multiplied the given sum by $\frac{11}{12}$ and so got the answer too little by Rs. 115 ; find the sum.

Sol. Suppose the sum $=$ Re. 1

$\therefore$ the sum given $=$ Re. $\frac{11}{12}$

$\therefore$ the answer found $=\frac{11}{12}\times\frac{11}{12}=$ Re. $\frac{121}{144}$

$\therefore$ the diff. $=1-\frac{121}{144}=$ Re. $\frac{23}{144}$

$\therefore$ the actual sum $=$ Rs. $115\times\frac{144}{23}=$ Rs. 720. **Ans.**

EXERCISE. 44.

1. $\frac{3}{16}$ of an estate is worth Rs. 501 ; find the value of the whole estate.

2. $\frac{3}{8}$ of an estate is worth Rs. 125, 5*a*. 3*p*. ; find the value of $\frac{5}{12}$ of the estate.

3. A man owns $\frac{9}{16}$ of a building and sells $\frac{2}{3}$ of it for Rs. 948 ; find the value of the whole building.

4. *A* had some money, he spent $\frac{2}{5}$ of it and found that $\frac{2}{3}$ of the remainder is equal to Rs. 13, 7*a*. 4*p*. ; find the amount he had.

5. Peter spent $\frac{3}{5}$ of his money and found that $\frac{3}{5}$ of the remainder is equal to Re. 1, 8*a*. What amount of money had he ?

6. *A* has Rs. 14, 7*a*. 11*p*. with him which is $3\frac{3}{4}$ times of *B's* money ; find the amount of *B*.

7. A man divided some apples among three boys ; to the first he gave $\frac{1}{3}$ of them and to the second $\frac{2}{3}$ of the remainder and to the third, the remaining 10. How many apples had he ?

8. A person distributed some money among 3 beggars ; the first got $\frac{1}{5}$ of the money and the second $\frac{4}{5}$ of what was then left and the third got 4*a*. only. What did the first and second beggars get ?

9. If a person gets a bequest of $\frac{5}{7}$ of an estate of 2000 acres, and sells $\frac{2}{5}$ of his share, how many acres does he retain ?

10. A person willed that the elder son should get $\frac{1}{2}$ of his property, the younger $\frac{1}{4}$ and wife $\frac{1}{6}$. The remaining property worth Rs. 2000 is utilized in opening a public library ; find the value of the property.

11. A person divided a piece of land among his three sons thus : he gave $16\frac{2}{3}$ acres to the first, $\frac{1}{3}$ of the whole to the second and $\frac{1}{2}$ to the third. How many acres did he divide ?

12. A father divided a piece of land among his three sons thus : he gave 35 acres to the first, $\frac{3}{20}$ of the whole to the second and to the third as much as to the other two together; how many acres did the third get ?

13. A person bequeathed his property to his three sons thus : $\frac{3}{5}$ of the property to the first, $\frac{2}{5}$ of the remainder to the second and to the third Rs. 2000 more than to the second. What is the value of the property ?

14. A person sold $\frac{3}{5}$ of $\frac{7}{8}$ of $\frac{5}{6}$ of an estate for Rs. 501, 1*a*. 4*p*. ; find the value of $\frac{8}{9}$ of it.

15. What is the least number which when divided by $\frac{2}{5}$, $\frac{9}{10}$, $\frac{11}{15}$ gives a whole number as quotient in each case ?

16. What is the greatest amount which is contained an exact number of times in both Rs. $15\frac{3}{4}$ and Rs. $18\frac{3}{4}$?

17. Find the greatest length that is contained an exact number of times in $5\frac{1}{4}$ ft. and $9\frac{1}{3}$ ft.

18. Four bells begin to toll together and toll respectively at intervals of $1\frac{1}{2}$, $1\frac{3}{4}$, $3\frac{1}{3}$ and $4\frac{1}{2}$ seconds. What time will elapse before they all toll together again ?

19. A man owns $\frac{1}{3}$ of an estate he sold $\frac{11}{25}$ of $\frac{2\frac{7}{11}}{8\frac{7}{10}}$ of it for Rs. 960, find the value of $\frac{5}{12}$ of $\frac{2\frac{1}{2}}{3\frac{1}{3}}$ of the estate.

20. There is a stick, $\frac{1}{10}$ of which is red, $\frac{1}{20}$ black, $\frac{1}{30}$ yellow, $\frac{1}{40}$ orange, $\frac{1}{50}$ blue, $\frac{1}{60}$ white and the remaining 302 inches of violet colour. Find the length of the stick.

21. What sum is that, $\frac{2}{3}$ of $\frac{1}{5}$ of which is $\frac{1}{7}$ of $\frac{2}{9}$ of Rs. 5. 10*a*. ?

22. Find the length, $\frac{27}{80}$ of which is $\frac{3}{5}$ of $7\frac{1}{2}$ of $16\frac{1}{2}$ yards.

23. Find the sum $\frac{13}{13\frac{3}{50}}$ of which is $(4\frac{14}{21}-10\frac{44}{45}+9\frac{3}{45}-\frac{52}{117})$ of 8*p*. Find also what fraction it is of $\frac{8}{75}$ of Rs. 6. 8*a*.

24. In a book on Arithmetic an example was printed thus : "add together $\frac{1}{14\frac{2}{3}}$, $\frac{1}{19\frac{1}{4}}$, $\frac{1}{\quad}$, $\frac{1}{13\frac{3}{4}}$," the denominator of one fraction being accidentally omitted. The answer given at the end of the book was $\frac{11}{28}$. Find the missing denominator.

25. For two parts of an examination there are 572 candidates, $\frac{8}{11}$ of the number entered for part I, and $\frac{9}{13}$ for part II, how many entered for both parts ?

26. One-fifth of a number exceeds its one-seventh by 54 ; find the number.

27. A boy was given $\frac{12}{13}$ of a sum and was asked what the sum was. He multiplied the given sum by $\frac{12}{13}$ and so got the answer too little by Rs. 83, 5*a*. 4*p*. ; find the sum.

CHAPTER VIII.

DECIMAL FRACTIONS

§1, In a number like 3333, you know that the local value of 3 decreases ten fold at each step from left to right.

First 3 on the left indicates 3000,
Second 3 indicates 300,
Third 3 ,, 30,
Fourth 3 ,, 3.

If we place a dot (called the decimal point) to the right of the unit's figure and then write another 3 as 3333·3 the value of this last 3 will also decrease ten fold, *i.e.*, it will represent three tenths and another 3 will represent three-hundredths and so on. The following diagram will make the method of **notation** clear:—

etc.	thousands.	hundreds.	tens.	units.	tenths.	hundreths.	thousandths.	etc.
	2	3	5	4	7	3	5	
	2000	300	50	4	$\frac{7}{10}$	$\frac{3}{100}$	$\frac{5}{1000}$	

The number indicated above is "two thousand three hundred fifty four and seven-tenths, three-hudredths, five-thousandths" and is read as two thousand, three hundred, fifty-four **decimal** seven, three, five. Thus the above number is written as 2354·735.

Note. In reading the figures after the decimal point' they are read in order. Thus ·735 is read as "decimal seven three and five" and not as "decimal seven hundred and thirty-five."

The number expressed in the above notation is called a **decimal fraction**; because each figure to the right of the decimal point indicates a fraction having 10 or some power of ten for its denomintor. Hence the definition

Definition, A decimal fraction is a fraction which has *ten* or some power of *ten* for its denominator.

$$5{\cdot}24 = 5 + \tfrac{2}{10} + \tfrac{4}{100}.$$

Note 1. The decimal point is always placed towards the top of the figures and not in the middle. Thus 6·9 is six decimal nine and 6.9 is 6×9. The boys should note the difference very carefully.

Note 2. Number of figures after the decimal point is called the number of decimal places. In 5·24, there are two decimal places and in 5·024 there are three decimal places and so on.

Note 3. The part to the left of the decimal point is the integral part and to the right, decimal part. Thus in 5·24, 5 is the integral part and 24 is the decimal part.

EXERCISE. 45.

Read the following:—

1.	·1.	2.	·01.	3.	·001.	4.	5·2
5.	5·02.	6.	5·5502.	7.	16·789.	8.	16·0789.
9.	13·7089,	10.	3·0025.	11.	30·0025.	12.	30·2005.

Express into vulgar fractions as $15{\cdot}02 = 15 + \frac{2}{100}$**:—**

13.	7·125.	14.	8·034.	15.	12·003.	16.	16·203.
17.	20·0025.	18.	15·1354.	19.	7·0105.	20.	21·1205.

Express as decimals:—

21. Six tenths.
22. Six hundredths.
23. Six thousandths.
24. Six ten thousandths.
25. Five and three tenths.
26. Fifteen and nine tenths and six thousandths.

27. $\frac{3}{10} + \frac{5}{100}$.
28. $\frac{3}{10} + \frac{5}{100} + \frac{7}{1000}$.
29. $\frac{9}{100} + \frac{5}{1000} + \frac{9}{10000}$.
30. $\frac{8}{10} + \frac{7}{1000} + \frac{9}{10000}$.
31. $5 + \frac{3}{100} + \frac{8}{1000} + \frac{7}{10000}$.
32. $8 + \frac{9}{1000} + \frac{7}{10000}$.
33. $15 + \frac{3}{10} + \frac{9}{100} + \frac{5}{10000}$.
34. $21 + \frac{6}{1000} + \frac{8}{10000}$.
35. $15 + \frac{7}{100} + \frac{9}{10000}$.
36. $15 + \frac{8}{10} + \frac{7}{10000}$.

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§2. To convert a decimal into a vulgar fraction.

Observe the following examples:—

Example. Convert ·2053, 15·0253 into vulgar fractions.

Sol. *(i)* $\cdot 2053 = \frac{2}{10} + \frac{5}{1000} + \frac{3}{10000}$.

$$= \frac{2000 + 50 + 3}{10000} = \frac{2053}{10000}.$$ **Ans.**

(ii) $15{\cdot}0256 = 15 + \frac{2}{100} + \frac{5}{1000} + \frac{6}{10000}$

$= 15 + \frac{200+50+6}{10000}$

$= 15 + \frac{256}{10000} = \frac{150256}{10000}$. **Ans.**

Hence the following rule :—

Rule. *Write down the given number omitting the decimal point for the numerator and one followed by as many zeroes as there are decimal places in the given number for its denominator. Simplify the fraction thus obtained, if possible.*

Thus $5{\cdot}002 = \frac{5002}{1000} = \frac{2501}{500}$

Note. The student should commit to memory the following results

$\cdot 5 = \frac{1}{2}$, $\cdot 25 = \frac{1}{4}$, $\cdot 75 = \frac{3}{4}$, $\cdot 125 = \frac{1}{8}$, $\cdot 375 = \frac{3}{8}$, $\cdot 625 = \frac{5}{8}$, $\cdot 875 = \frac{7}{8}$.

§3. **Conversely** we can convert every vulgar fraction having 10 or any power of 10 as denominator into a decimal fraction. Observe the following examples :—

$\frac{3}{10}$ means $\cdot 3$

$\frac{3}{100}$ „ $\cdot 03$

$\frac{3}{1000}$ „ $\cdot 003$

Hence the following rule :

Rule. *Put the decimal point leaving as many figures in the numerator on the right hand as there are zeroes after 1 in the denominator. If the number of figures in the numerator be less than the number of zeroes in the denominator, prefix necessary zeroes in the numerator.*

Thus $\frac{1256}{100} = 12{\cdot}56$ and $\frac{1256}{1000} = 1{\cdot}256$

and $\frac{1256}{10000} = \cdot 1256$ and $\frac{1256}{100000} = \cdot 01256$. etc.

Note. This is really a division by 10 or any power of 10.

EXERCISE 46

Express as vulgar fractions in their lowest terms :—

1.	$\cdot 5$.	2.	25	3.	$\cdot 75$.	4.	$\cdot 125$.
5.	$\cdot 375$.	6.	$\cdot 625$.	7.	$\cdot 05$.	8.	$\cdot 025$.
9.	$\cdot 0125$.	10.	$\cdot 124$.	11.	$\cdot 576$.	12.	$\cdot 728$.
13.	$\cdot 504$.	14.	$\cdot 054$.	15.	$\cdot 0504$.	16.	$5{\cdot}1$
17.	$5{\cdot}029$.	18.	$5{\cdot}625$.	19.	$16{\cdot}25$.	20.	$18{\cdot}75$.
21.	$21{\cdot}375$.	22.	$90{\cdot}5$.	23.	$90{\cdot}05$.	24.	$900{\cdot}55$.

Express as decimals :—

25.	$\frac{7}{10}$.	26.	$\frac{17}{100}$.	27.	$\frac{177}{100}$.	28.	$\frac{75}{100}$.
29.	$\frac{185}{1000}$.	30.	$\frac{1825}{10000}$.	31.	$\frac{9}{10}$.	32.	$\frac{9}{100}$.
33.	$\frac{9}{1000}$.	34.	$\frac{375}{1000}$	35.	$\frac{75}{1000}$.	36.	$\frac{5}{1000}$
37.	$5\frac{7}{100}$.	38.	$8\frac{9}{1000}$.	39.	$15\frac{175}{10000}$	40.	$\frac{500}{10000}$.
41.	$\frac{40}{10000}$.	42.	$\frac{540}{10000}$.	43.	$\frac{78935}{100}$.	44.	$\frac{756356}{1000}$.
45.	$\frac{75001}{10000}$.	46.	$\frac{15}{1000}$.	47.	$\frac{7}{10000}$.	48.	$\frac{101}{100000}$.

§4. **The following facts should be noted very carefully :—**

(*i*) *The value of a decimal fraction is not changed by affixing zeroes to the right of the last figure.*

Thus $\cdot 175 = \cdot 1750 = \cdot 17500$ etc.

Since $\cdot 175 = \frac{175}{1000}$.

and $\cdot 1750 = \frac{1750}{10000} = \frac{175}{1000}$

also $\cdot 17500 = \frac{17500}{100000} = \frac{175}{1000}$.

$\therefore$ $\cdot 175 = \cdot 1750 = \cdot 17500$, etc., etc.

(*ii*) *Zeroes affixed just after the decimal point decrease its value ten fold.*

Thus $\cdot 1 = \frac{1}{10}$; $\cdot 01 = \frac{1}{100}$; $\cdot 001 = \frac{1}{1000}$ etc.

§5. **To multiply or divide a decimal by 10 or any power of 10.**

It has been explained that the local value of figures increases ten fold at each step from right to left and decreases from left to right. Hence to multiply a decimal by 10, 100, 1000, etc., etc., remove the decimal point 1, 2, 3, etc., etc., places respectively to the right, and to divide the decimal by 10, 100, 1000, etc., etc., remove the decimal point 1, 2, 3, etc., etc., places to the left. Thus

$\cdot 537 \times 10$	$= 5 \cdot 37$	$153 \cdot 7 \div 10$	$= 15 \cdot 37$
$\cdot 537 \times 100$	$= 53 \cdot 7$	$153 \cdot 7 \div 100$	$= 1 \cdot 537$
$\cdot 537 \times 1000$	$= 537$	$153 \cdot 7 \div 1000$	$= \cdot 1537$
$\cdot 537 \times 10000$	$= 5370$	$153 \cdot 7 \div 10000$	$= \cdot 01537$

Note. Deficiency of figures to the right or to the left may be made up by adding zeroes as shown above.

EXERCISE 47. (Oral)

Multiply the following by10 :—

1. ·1 2. ·01. 3. ·001. 4. ·0001.
5. 1·01. 6. 1·001. 7. 1·0001. 8. 2·5.
9. 2·55. 10. 25·25. 11. 25·002. 12. ·15.
13. ·115. 14. ·1015.

Divide the following by 10 :—

15. 6·1. 16. 8·25. 17. 10·123. 18. 15·3.
19. 15·03. 20. 12·156. 21. ·153, 22. ·1057.
23. ·1425. 24. ·01. 25. ·001. 26. ·0001.

Multiply the following by 100:—

27. 1·001. 28. ·1001. 29. ·01001. 30. 16·023.
31. 15·125. 32. 21·1307. 33. 154·25. 34. 123·1.
35. 158·02. 36. 103·002. 37. 103·0002.

Divide the following by 100:—

38. 728·13. 39. 1226·02. 40. 1202·09. 41. 12020·9.
42. ·1. 43. ·01. 44. ·001. 45. ·0001.
46. 2·001. 47. 20·001. 48. 200·01.

Multiply the following by 1000:—

49. ·02. 50. ·01. 51. ·001. 52. ·003.
53. ·004. 54. ·504. 55. 4·03. 56. 15·003.
57. 21·007. 58. ·1002. 59. ·0012. 60. ·00025.

Divide the following by 1000:—

61. ·003. 62. ·516. 63. 4·123. 64. 16·123.
65. 1830·116. 66. 56210·02. 67. 70231·05.
68. 212·31. 69. 723·45. 70. 5530·5.

§6. ADDITION OF DECIMALS

Rule. *Write the numbers under one another so that the decimal points may fall in a vertical straight line and the digits of the same local value exactly under each other; then proceed as in simple addition.*

Example. Add together 2·05, 15·912, ·002 and 21.

Sol.

(1)		(2)	
	2·050		2·05
	15·912		15·912
	·002		·002
	21·000		21
	38·964		38·964 Ans.

Proof:—

$$2{\cdot}05+15{\cdot}912+{\cdot}002+21=\tfrac{205}{100}+\tfrac{15912}{1000}+\tfrac{2}{1000}+21$$

$$=\frac{2050+15912+2+21000}{1000}$$

$$=\tfrac{38964}{1000}=38{\cdot}964 \text{ [Art. 3]}.$$

Note. The student may, in the beginning fill in with zeroes the missing power of 10 as shown in solution (1). It may also be noted that an integer is expressed as decimal by affixing zeroes in the decimal part. Thus 21=21·0=21·00=21·000 etc.

EXERCISE 48.

Add together:—

1. ·002, ·9002, 5·1, 12.
2. ·0005, ·8, 3·125, 15.
3. 15·1, 15·001, 150·01, 18.
4. ·1, ·001, 101·1, 10·1, 25.
5. ·02, 25·023, 100·2, 117.
6. 124·1235, 1·02, 18·075, 11.
7. ·052, ·5, ·351, 54·501, 31.
8. 501·5607,5·02,5003·24, ·8.
9. ·72801, ·01, ·612,556·1
10. 137·023, 124·135, ·00235,56.

§7. SUBTRACTION OF DECIMALS

Rule. *Write the smaller number underneath the greater as explained in Art. 6 and proceed as in simple subtraction.*

Example. Subract 5·00235 from 11·1.

Sol.

11·10000
5·00235
6·09765 Ans.

Proof. $11{\cdot}1-5{\cdot}00235=\frac{111}{10}-\frac{500235}{100000}$

$$=\frac{1110000-500235}{100000}$$

$$=\frac{609765}{100000}=6{\cdot}09765.$$

EXERCISE 49.

Subtract :—

1. ·25 from ·5.
2. ·45 from ·51.
3. ·025 from ·1.
4. ·078 from ·201.
5. 12·1 from 121.
6. ·8736 from ·985.

Find the difference of :—

7. 2·00035 and 15·87.
8. 7·12302 and 8·01.

Simplify :—

9. $56-45{\cdot}0235$.
10. $41-40{\cdot}9356$.
11. $43-42{\cdot}00235$.
12. $121-111{\cdot}980765$.
13. $105{\cdot}34-45{\cdot}43-15{\cdot}012-30{\cdot}012$.
14. $17{\cdot}143-14{\cdot}01235+21{\cdot}0123-2{\cdot}123$.
15. $162{\cdot}013+{\cdot}00235-156{\cdot}9135-2{\cdot}0023$.

§8. MULTIPLICATION OF DECIMALS.

(*i*) **To multiply a decimal by a whole number.**

Example 1. Multiply 2·35 by 21.

Sol. $2{\cdot}35\times21=\frac{235}{100}\times21$

$$=\frac{4935}{100}=49{\cdot}35 \quad \textbf{Ans}$$

There are two places of decimals in the multiplicand and so in the product too. Hence the following rule ;

Rule. *Multiply the numbers as in simple multiplication and point off in the product as many places of decimals as there are decimal places in the multiplicand. Prefix zeroes if necessary.*

Example 2. Multiply ·0026 by 15.

Sol.

$$\begin{array}{r} {\cdot}0026 \\ 15 \\ \hline 390 \end{array}$$

The number of decimal places should be 4, because there are 4 decimal places in the multiplicand.

$\therefore$ The product $={\cdot}0390={\cdot}039$. **Ans.**

(ii) To multiply a decimal by a decimal.

Example 3. Multiply 5·0023 by ·00025.

Sol. $5{\cdot}0023 \times {\cdot}00025 = \frac{50023}{10000} \times \frac{25}{100000}$

$= \frac{1250575}{1000000000} = {\cdot}001250575$ **Ans.**

There are 9 places of decimals in the multiplicand and multiplier and so in the product too. Hence the following rule ;

Rule. *Multiply the numbers as in simple multiplication and point off in the product as many places of decimals as there are decimal places in the multiplicand and the multiplier together. Prefix zeroes if necessary.*

Example 4. Multiply 121·50987 by 5·25 correct to 3 decimal places.

Sol. 121·50987 five decimal places.

5·25 two ,, ,,

60754935

24301974

60754935

637·9268175 seven ,, ,,

Since the answer is required correct to 3 decimal places

∴ the required product = 637·927. **Ans.**

EXERCISE 50.

Multiply:—

1. 4·2 by 5.
2. 15·25 by 16.
3. 18·026 by 25.
4. 21·023 by 20.
5. 51·908 by 35.
6. 31·0235 by 46.
7. 12·25 by 2·5.
8. 13·2102 by 3·25.
9. 18·023 by 16·124.
10. 21·0005 by 1·0025.
11. 8·001 by ·8001.
12. 75·005 by 5·0016.

Multiply the following correct to 4 decimal places :—

13. 16·5803 by 8·5302.
14. 31·8035 by 165·02.
15. 156·7835 by 9·1356.
16. 121·7802 by 35·125.

Find the continued product of :—

17. ·1 × ·01 × ·001 × ·0001.
18. ·002 × ·0005 × 16.
19. ·4 × ·04 × ·05 × ·005.
20. 1·6 × 16 × 1600 × ·016.

§ 10. DIVISION OF DECIMALS

(*i*) **To divide a decimal by a whole number.**

Example 1. Divide 5·025 by 25.

Sol. $5{\cdot}025 \div 25 = \frac{5025}{1000} \times \frac{1}{25}$.

$= \frac{201}{1000} = {\cdot}201$. **Ans.**

There are three places of decimals in the dividend and so in the quotient too. Hence the following rule :—

Rule. *Proceed as in simple division and then point off in the quotient as many places of decimals as there are decimal places in the dividend. Deficiency of figures, if any, may be supplied by zeroes.*

Example 2. Divide ·2565 by 15

Sol.
$$15)\overline{{\cdot}2565}$$
$$\overline{171}$$

There are four decimal places in the dividend and so there should be in the quotient too.

∴ the reqd. quotient = ·0171. **Ans.**

(*ii*) **To divide a decimal by a decimal.**

Example 3. Divide 5·125 by ·25.

Sol. $5{\cdot}125 \div {\cdot}25 = \frac{5{\cdot}125}{{\cdot}25}$

To make the divisor a whole number, multiply the dividend and the divisor by 100.

$$\therefore \frac{5{\cdot}125}{{\cdot}25} = \frac{5{\cdot}125}{{\cdot}25} \times \frac{100}{100} = \frac{512{\cdot}5}{25}$$

Now proceed as in example 2.

The required quotient = 20·5 **Ans.**

From the above example we deduce the following rule :

Rule. *Remove the decimal point from the divisor and move the decimal point as many places to the right in the dividend as there were decimal places in the divisor, and then proceed as in Art. 10 (i).*

§ 11. In dividing if there is any remainder left after the last digit from the dividend has been brought down, we should add zeroes to the right of the dividend and proceed till there is no remainder left.

Example 4. Divide 1·365 by 1·25

Sol. 1·25)1·365=

125)136·500(1·092 **Ans.**

125

1150

1125

250

250

EXERCISE 51.

Divide :—

1. 25·6 by 16.
2. 17·28 by 32.
3. 897·2 by 40.
4. 773·682 by 13.
5. 773·682 by 78.
6. 773·682 by 169.
7. 773·682 by 1014.
8. ·750116 by 677.
9. ·750116 by 1354.
10. ·750116 by 10832.
11. 35·9424 by 7·02.
12. 11·444495 by 4·735.
13. 89·725 by ·005.
14. 4·2225 by ·000015.
15. ·020872522 by ·08635.
16. ·39538 by 5300.
17. ·1 by ·01.
18. ·01001 by ·001.
19. 99 by ·0009.
20. 9864·1698175 by 35·0645.
21. 124·59993 by 3194·87.
22. 1·365 by 1·25.
23. 1·365 by 12·5.
24. 1·365 by ·00125.
25. 1·365 by 12500.
26. ·0064096 by 2·003.
27. 614·50824 by ·0010201.

Divide and find the quotient to 3 places of decimals :—

28. 210·8972 by 128·25.
29. 156·8935 by 115·89.
30. 5·1352 by 892·7893.
31. 117·003 by 21·00235
32. 59·78312 by 21·563.
33. 12·354 by 7256·04.

Simplify :—

34. $\dfrac{500\times\cdot00025}{\cdot625}$

35. $\dfrac{\cdot001\times\cdot1421}{2\cdot03}$

36. $\dfrac{4\cdot25\times5\cdot135}{10\cdot27\times\cdot0085}$

37. $\dfrac{15\cdot001\times\cdot004}{8\times300\cdot02}$

38. $\dfrac{\cdot1\times\cdot01\times\cdot0001\times16}{\cdot2\times\cdot02\times\cdot002\times4}$

39. $\dfrac{5\cdot24\times7\cdot28}{104\cdot8\times\cdot091.}$

§12. **To reduce a vulgar fraction to a decimal.**

Rule. *Reduce the vulgar fraction to its lowest terms and divide the numerator by the denominator as in division of decimal.*

Example 1. Convert $\frac{5}{32}$ into decimal.

$$
\begin{array}{r}
\cdot 15625 \quad \textbf{Ans.} \\
\hline
32)5\cdot 00000 \\
3\cdot 2 \\
\hline
1\cdot 80 \\
1\cdot 60 \\
\hline
200 \\
192 \\
\hline
80 \\
64 \\
\hline
160 \\
160 \\
\hline
\end{array}
$$

Example 2. Arrange $\frac{9}{155}$, $\frac{15}{186}$, $\frac{25}{372}$, $\frac{27}{62}$, $\frac{9}{124}$ in order of magnitude by reducing to decimals.

$\frac{9}{155} = \cdot 058$.....least or $\frac{9}{155} = \frac{1}{17\cdot 2}$ least .....(5)

$\frac{15}{186} = \cdot 080$..... or $\frac{15}{186} = \frac{1}{12\cdot 4}$ .....(2)

$\frac{25}{372} = \cdot 067$..... or $\frac{25}{372} = \frac{1}{14\cdot 8}$ .....(4)

$\frac{27}{62} = \cdot 435$.....greatest or $\frac{27}{62} = \frac{1}{2\cdot 2}$ greatest.....(1)

$\frac{9}{124} = \cdot 072$..... or $\frac{9}{124} = \frac{1}{13\cdot 7}$ .....(3)

∴ order of magnitude is $\frac{27}{62}$, $\frac{15}{186}$, $\frac{9}{124}$, $\frac{25}{372}$, $\frac{9}{155}$.

☞ Note this solution very carefully.

EXERCISE 52.

Convert into decimals :—

1. $\frac{1}{4}$. 2. $\frac{1}{8}$. 3. $\frac{1}{25}$. 4. $\frac{5}{8}$.
5. $\frac{7}{20}$. 6. $\frac{11}{40}$. 7. $\frac{4}{5}$. 8. $\frac{3}{8}$.

9. $\frac{7}{25}$. 10. $\frac{32}{125}$. 11. $\frac{25}{128}$. 12. $\frac{16}{125}$.

13. $\frac{32}{625}$. 14. $\frac{21}{250}$. 15. $\frac{41}{320}$. 16. $\frac{124}{256}$.

17. $\frac{132}{192}$. 18. $\frac{15}{128}$.

Reduce the following to five decimal places :—

19. $\frac{513}{130}$. 20. $\frac{729}{140}$. 21. $\frac{253}{156}$. 22. $56\frac{71}{75}$.

23. $59\frac{64}{75}$. 24. $56\frac{121}{526}$.

Arrange in the descending order of magnitude, by reducing to decimals :—

25. $\frac{5}{12}$, $\frac{7}{16}$, $\frac{3}{4}$. 26. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{7}{8}$.

Convert the following into decimals :—

27. $3\frac{1}{8}$ of $2\frac{4}{5}$. 28. $\frac{3}{5}+\frac{7}{8}+\frac{9}{10}+\frac{7}{32}$.

29. $\frac{6\frac{3}{4}}{11\frac{1}{4}}$. 30. $\frac{17\frac{1}{8}}{12\frac{1}{2}}$. 31. $\frac{5\frac{1}{4}}{4\frac{1}{5}}\times\frac{\frac{3}{8}}{\frac{5}{16}}$.

32. $4\frac{4}{7}$ of $\frac{7}{5\frac{1}{2}}$. 33. $3\frac{5}{8}$ of ·003—·0011 of $7\frac{1}{4}$.

Find the greatest and the least fractions by reducing to decimals : —

34. $\frac{13}{144}$, $\frac{15}{156}$, $\frac{17}{168}$, $\frac{5}{48}$. 35. $\frac{7}{16}$, $\frac{9}{24}$, $\frac{16}{56}$, $\frac{36}{79}$.

---

§18. **G. C. M. and L. C. M. of decimals.**

Example 1. Find the G. C. M. and L. C. M. of ·5, ·15 and ·25.

Sol. ·5, ·15, ·25 $=\frac{5}{10}, \frac{15}{100}, \frac{25}{100}$.

$=\frac{50}{100}, \frac{15}{100}, \frac{25}{100}$.

G. C. M. $=\frac{5}{100}$ or ·05.

L. C. M. $=\frac{150}{100}$ or 1·5. } **Ans.**

From the above we conclude the following rule :—

Rule. *Make the same number of decimal places in all the numbers by affixing zeroes and then find the G. C. M. or L.C.M. of the new numbers as if they were integers and then mark off in the result the said number of decimal places prefixing zeroes if necessary.*

We shall apply the rule in the first example thus :—

·5, ·15, ·25=·50, ·15, ·25.

G. C. M. of 50, 15, 25 =5 and L. C. M.=150.

∴ the required G. C. M. =·05

and the required L. C. M. =1·50 or 1·5 } **Ans.**

Example 2. Find the G. C. M. and L. C. M. of ·05, ·7 and ·002.

Sol. ·05, ·7 and ·002 $=$·050, ·700, ·002
G. C. M. of 50, 700 and $2=2$
L. C. M. of 50, 700 and $2=700$
∴ the required G. C. M. $=$·002
and ,, L. C. M. $=$·700 or ·7. } **Ans.**

EXERCISE 53.

Find the G. C. M. and L. C. M. of :—

1. ·14, ·21, ·28, ·56.
2. ·11, ·22, ·33, ·44.
3. ·15, ·25, ·45, ·75.
4. ·13, ·26, ·39, ·52.
5. ·14, ·42, ·56, ·84.
6. ·5, ·25, ·025, ·005.
7. ·7, 1·4, ·007, ·0014.
8. ·45, ·09, ·009, 8·1.
9. 1·2, ·24, 3·6, 10·8.
10. ·00121, 1·21, ·0011, ·11.

---

§14. Complex fractions involving decimals.

Example 1. Simplify $\dfrac{3\cdot5+2\frac{1}{3}\text{ of }\frac{9}{7}}{2\cdot5+5\frac{1}{6}\div1\frac{7}{24}}+\dfrac{1\cdot05+\cdot95}{5}$

Sol. The expression $=\dfrac{3\cdot5+\frac{7}{3}\times\frac{9}{7}}{2\cdot5+\frac{31}{6}\times\frac{24}{31}}+\dfrac{2}{\cdot5}=\dfrac{3\cdot5+3}{2\cdot5+4}+4$.

$$=\frac{6\cdot5}{6\cdot5}+4=1+4=5. \quad \textbf{Ans.}$$

Example 2. Simplify $\dfrac{1\cdot75}{\frac{5}{4}}+\dfrac{2\cdot35-1\cdot05}{4\cdot7-\frac{21}{10}}+\dfrac{57\cdot5}{92}$

Sol. The expression $=\dfrac{1\cdot75}{1\cdot25}+\dfrac{1\cdot3}{4\cdot7-2\cdot1}+\dfrac{575}{920}$

$$=\tfrac{175}{125}+\tfrac{13}{26}+\tfrac{575}{920}$$
$$=\tfrac{7}{5}+\tfrac{1}{2}+\tfrac{5}{8}$$
$$=1\cdot4+\cdot5+\cdot625=2\cdot525. \quad \textbf{Ans.}$$

Note 1. In such questions, sometimes vulgar fractions are converted to decimal fractions and sometimes decimal fractions are converted to vulgar. A careful study in decimals and constant practice will enable the student to understand which method is simple in an example.

Note 2. The result in such questions should be expressed in its decimal form.

EXERCISE 54.

Simplify the following :—

1. $\dfrac{1\frac{1}{9} \text{ of } 10{\cdot}26}{2\frac{2}{7} \text{ of } 1{\cdot}05}$

2. $\dfrac{1\frac{1}{4}\times{\cdot}6+{\cdot}25}{2\frac{1}{3} \text{ of } 8\frac{1}{4}-18{\cdot}25}$

3. $\dfrac{2{\cdot}37 \text{ of } (5{\cdot}21+6{\cdot}84)}{{\cdot}1185}$

4. ${\cdot}05 \text{ of } \dfrac{3{\cdot}4}{2\frac{1}{3} \text{ of } 1{\cdot}2} \div \frac{1}{14}$

5. ${\cdot}399\times{\cdot}007 \div {\cdot}000019 \div \dfrac{{\cdot}003125\times{\cdot}48}{{\cdot}0000125}$.

6. $\dfrac{3\frac{4}{11}\times(1\frac{2}{5} \text{ of } 1{\cdot}08)}{1\frac{11}{13}\times({\cdot}6+\frac{2}{9})}$

7. $\dfrac{4}{7{\cdot}5}\left\{\dfrac{\frac{1}{3}+\frac{{\cdot}75}{4{\cdot}5}}{\frac{1-{\cdot}25}{2-{\cdot}5}}+\frac{7}{8}\right\}$

8. $\dfrac{1{\cdot}25}{\frac{5}{4}}+\dfrac{4{\cdot}85-2{\cdot}25}{{\cdot}85-{\cdot}81}-55{\cdot}0025.$

9. $\dfrac{1}{1\frac{1}{20}}\times\dfrac{1+{\cdot}0025\times{\cdot}05}{1{\cdot}0025-{\cdot}05}+\dfrac{45\times\frac{16}{45}}{8}$

10. $\dfrac{{\cdot}67\times{\cdot}67\times{\cdot}67-{\cdot}001}{{\cdot}67\times{\cdot}67+{\cdot}067+{\cdot}01}+\dfrac{{\cdot}57}{1+\dfrac{1}{3\frac{1}{14}}}$

11. $\dfrac{1{\cdot}125}{\frac{9}{8}}+\dfrac{5{\cdot}6 \text{ of } (6\frac{1}{4}-1{\cdot}25)}{1{\cdot}4 \text{ of } {\cdot}2\div 10}$

12. $\dfrac{\frac{2}{13}\times{\cdot}39+\frac{7}{25}}{\frac{17}{21}+(1{\cdot}375 \text{ of } 2\frac{1}{2})}\times\dfrac{14{\cdot}27}{{\cdot}0102}$.

13. $\dfrac{3\frac{1}{5} \text{ of } 3\frac{3}{4}\div 8}{3{\cdot}2+2\frac{4}{5}}+\dfrac{{\cdot}527\times{\cdot}527-{\cdot}223\times{\cdot}223}{{\cdot}527-{\cdot}223}$

14. $\dfrac{7{\cdot}5}{6{\cdot}5}+\dfrac{11{\cdot}5-2{\cdot}4}{11{\cdot}5+2{\cdot}4}\times 10\frac{6}{13}-6\frac{42}{273}.$

15. $\dfrac{8\frac{11}{63}}{\frac{7}{8} \text{ of } 2{\cdot}25} \text{ of } \dfrac{{\cdot}1625}{\frac{1}{15} \text{ of } 5\frac{4}{7}}+(\frac{2}{21}+\frac{7}{81})+\frac{1}{567}$

16. $\dfrac{3\frac{3}{4}+4\frac{4}{5}}{5\frac{1}{10}-4\frac{3}{20}}\div\dfrac{4{\cdot}75+3{\cdot}8}{6{\cdot}1-5{\cdot}15}\times\dfrac{{\cdot}021\times{\cdot}0021\times 210}{{\cdot}007\times{\cdot}14}$

RECURRING DECIMALS.

§15. In the conversion of a vulgar fraction to a decimal the division in some cases does *not* terminate. For the sake of illustration we solve a few examples of this type here.

Examples. Express $\frac{1}{3}, \frac{5}{11}$ and $\frac{17}{90}$ into decimals.

(*i*) $\frac{1}{3}$ $= 3)1\cdot000$

$\cdot333\ldots$

(*ii*) $\frac{5}{11} = 11)5\cdot0000$

$\cdot45,45\ldots$

(*iii*) $\frac{17}{90} = 90)17\cdot000$

$\cdot188\ldots\ldots$

We see that the division in each of these cases does not terminate and can be extended to unlimited length. Such decimals are called **Repeating or Circulating or Recurring** decimals.

Definition. *A decimal in which a figure or a set of figures repeat continually is called a Recuring decimal and the whole set of figures which recurs constantly in the same order is called the period.*

The first two examples are examples of **Pure recurring** decimals and the third is an example of **mixed recurring** decimal.

Definition. *A decimal in which all the figures recur is called a pure recurring decimal; and in which some figures do not recur is called a mixed recurring decimal.*

Notation. The result is expressed by placing a dot (·) over the repeated figure or figures. Thus the result in the first example will be expressed as $\cdot\dot{3}$ where the dot indicates that the figure 3 is continually repeated. Similarly the results in the second and third examples will be written as $\cdot\dot{4}\dot{5}$ and $\cdot1\dot{8}$.

$\cdot\dot{3}$ will be read as decimal 3, three recurring and $,4\dot{5}$ will be read as decimal 4,5 four five recurring.

§ 16. Terminating or Non-terminating.

If we were to know that a certain vulgar fraction will produce a *treminating* decimal or not, we should reduce the

given vulgar fraction to its lowest terms. If its denominator be wholly made up of the factors 2 and 5 it will be expressed a terminating decimal, otherwise not.

Example. Find whether $\frac{3}{80}$ and $\frac{5}{36}$ are terminating or non-terminating.

Sol. (*i*) $\frac{3}{80} = \left(\frac{3}{2\times2\times2\times2\times5}\right)$; since its denominator is wholly made up of the factors 2 and 5, therefore it can be expressed as a terminating decimal. The fraction $= \cdot375$.

(*ii*) $\frac{5}{36} = \left(\frac{5}{2\times2\times9}\right)$; since its denominator is not wholly made up of the factors 2 and 5, therefore it cannot be expressed as a terminating decimal. The fraction in this case $= \cdot138$

EXERCISE 55.

State in each case, whether the equivalent decimal is terminating or non-terminating:

1. $\frac{1}{9}$. 2. $\frac{5}{21}$. 3. $\frac{9}{64}$. 4. $\frac{11}{84}$. 5. $\frac{96}{125}$.
6. $\frac{81}{128}$. 7. $\frac{91}{168}$. 8. $\frac{128}{925}$. 9. $\frac{101}{728}$. 10. $\frac{101}{875}$.

Express the following as a recurring decimal:

11. $\frac{5}{6}$. 12. $\frac{4}{9}$. 13. $\frac{10}{11}$. 14. $\frac{6}{7}$. 15. $\frac{5}{13}$.
16. $\frac{7}{90}$. 17. $\frac{7}{36}$. 18. $\frac{2}{27}$. 19. $\frac{9}{56}$. 20. $\frac{3}{37}$.
21. $5\frac{4}{7}$. 22. $3\frac{2}{13}$. 23. $9\frac{8}{11}$. 24. $5\frac{8}{27}$. 25. $14\frac{3}{22}$.
26. $\frac{5}{\cdot12}$ 27. $\frac{8}{\cdot27}$ 28. $\frac{11}{1\cdot3}$ 29. $\frac{12}{2\cdot1}$ 30. $\frac{5\frac{1}{2}}{8\frac{1}{4}}$

To convert a pure recurring decimal into its equivalent vulgar fraction.

Example 1. Convent $\dot{5}$ into its equivalent vulgar fraction.

Sol. Let $x = \dot{5} = \cdot5555\ldots\ldots$

Multiplying both sides by 10: $10x = 5\cdot555\ldots\ldots(1)$

But $x = \cdot555\ldots\ldots(2)$

$\therefore$ Subtracting (2) from (1), we have $9x = \cdot5$,

x, *i. e.*, $\dot{5} = \frac{5}{9}$ **Ans.**

Example 2. Convert $\cdot\dot{4}\dot{5}$ into its equivalent vulgar fraction.

Sol. Let $x = \cdot\dot{4}\dot{5} = \cdot 454545\ldots\ldots$

Multiplying by 100 we get $100x = 45\cdot 454545\ldots\ldots(1)$

but $x = \cdot 454545\ldots\ldots(2)$

Subtracting (2) from (1), we get $99x = 45$

$\therefore x$, *i.e.*, $\cdot\dot{4}\dot{5} = \frac{45}{99}$ **Ans.**

From these solutions we deduce the following rule:

Rule. *Make the period the numerator of the fraction and put a number of as many nines as there are figures in the period in the denominator; then reduce the fraction to lowest terms, if passible.*

§ 18. To convert a mixed recurring decimal into its equivalent vulgar fraction.

Example 3. Express $\cdot 4\dot{5}$ to an equivalent vulgar fraction.

Sol. Let $x = \cdot 4\dot{5} = \cdot 4555\ldots\ldots$

$\therefore \quad 10x = 4\cdot 555\ldots\ldots(1)$

and $\quad 100x = 45\cdot 555\ldots\ldots(2)$

Subtracting (1) from (2) $\quad 90x = 45 - 4.$

$\therefore x$, *i. e.* $\cdot 4\dot{5} = \frac{45-4^*}{90} = \frac{41}{90}$ **Ans.**

Example 4. Express $\cdot 456\dot{9}$ to its equivalent vulgar fraction.

Sol. Let $x = \cdot 456\dot{9} = \cdot 456969\ldots\ldots$

$\therefore \quad 100x = 45\cdot 6969\ldots\ldots(1)$

and $\quad 10000x = 4569\cdot 6969\ldots\ldots(2)$

Subtracting (1) from (2), $9900x = 4569 - 45$

$\therefore x$, *i.e.* $\cdot 456\dot{9} = \frac{4569-45^*}{9900} = \frac{4524}{9900} = \frac{377}{825}$ **Ans.**

From asterisked* figures in the above solutions we have the following rule:—

Rule. *Subtract the figures which do not recur from the given expression and make the difference the numerator of the fraction and underneath it write a number consisting of as many nines as there are figures that recur followed by as many zeroes as there are figures that do not recur.*

Note. 1. If a pure or mixed recurring decimal be preceded by a whole number, the whole number may be kept separate.

Note 2. From the above rules we conclude that

$\dot{9} = \frac{9}{9} = 1$, $\cdot0\dot{9} = \cdot1$ and $\cdot00\dot{9} = \cdot01$.

$\therefore 4\cdot\dot{9} = 5$, $4\cdot5\dot{9} = 4\cdot6$, $4\cdot02349 = 4\cdot0235$; etc.

and $\cdot\dot{9}\dot{9} = 1$, $\cdot9\dot{9}\dot{9} = 1$, $\cdot\dot{9}999\dot{9} = 1$, $5\cdot3\dot{9}\dot{9} = 5\cdot4$. etc.

That is to say, in simplifying recurring decimals containing the figure 9 only, the recurring part may be omitted and the preceding figure be increased by one. But you must not be tempted to apply the same rule if the recurring portion contain 9 along with other digits in any position.

EXERCISE 56.

Express as vulgar fraction in their lowest terms (Questions asterisked to be done mentally) :—

1. $\cdot\dot{4}$.	2. $\cdot2\dot{5}$.	3. $\cdot2\dot{7}$.	4. $\cdot4\dot{5}$
5. $\cdot0\dot{8}\dot{1}$.	6. $\cdot1\dot{1}\dot{7}$	7. $\cdot3\dot{3}\dot{3}$.	8. $\cdot5\dot{7}\dot{6}$.
9. $2\cdot0\dot{1}8$.	10. $5\cdot4\dot{4}4\dot{4}$.	11. $7\cdot1\dot{4}8$.	12. $10\cdot2\dot{9}6$
*13. $7\cdot9\dot{9}\dot{9}$	*14. $12\cdot9999\dot{9}$.	15. $18\cdot\dot{2}12\dot{7}$.	16. $15\cdot\dot{7}27\dot{2}$
17. $\cdot\dot{6}15384\dot{}$.	18. $\cdot\dot{8}57142\dot{}$.	19. $\cdot\dot{9}23076\dot{}$.	20. $\cdot\dot{7}14285$
21. $\cdot8\dot{1}$.	22. $\cdot1\dot{2}\dot{7}$.	23. $\cdot2\dot{5}\dot{6}$.	24. $\cdot2\dot{6}$.
*25. $\cdot4\dot{9}$.	*26. $5\cdot9\dot{9}$.	*27. $\cdot0\dot{9}\dot{9}$.	*28. $\cdot4\dot{1}\dot{9}$.
29. $25\cdot9\dot{0}4\dot{5}$.	30. $21\cdot5638\dot{}$.	31. $2\cdot257\dot{3}4\dot{5}$.	

32. Prove that

$$\frac{1}{9} = \frac{\cdot\dot{1}}{1} = \frac{\cdot\dot{2}}{2} = \frac{\cdot\dot{3}}{3} = \frac{\cdot\dot{4}}{4} = \frac{\cdot\dot{5}}{5} = \frac{\cdot\dot{6}}{6} = \frac{\cdot\dot{7}}{7} = \frac{\cdot\dot{8}}{8} = \frac{\cdot\dot{9}}{9}$$

33. Prove that $\frac{1}{11} = \frac{\cdot0\dot{9}}{1} = \frac{\cdot1\dot{8}}{2} = \frac{\cdot2\dot{7}}{3} = \frac{\cdot3\dot{6}}{4} = \frac{\cdot4\dot{5}}{5}$ etc.

34. Prove that $\frac{1}{7} = \frac{\cdot1428\dot{5}\dot{7}}{1} = \frac{\cdot2857\dot{1}\dot{4}}{2} = \frac{\cdot4285\dot{7}\dot{1}}{3}$ etc.

35. Prove that $25\cdot856\dot{2}35714\dot{} = 50\cdot856 + \frac{2}{7000}$.

36. Write at sight the following as recurring decimals:

$\frac{85}{999}$, $\frac{526}{990}$, $\frac{26}{990}$, $\frac{28}{9990}$, $\frac{141}{9990}$.

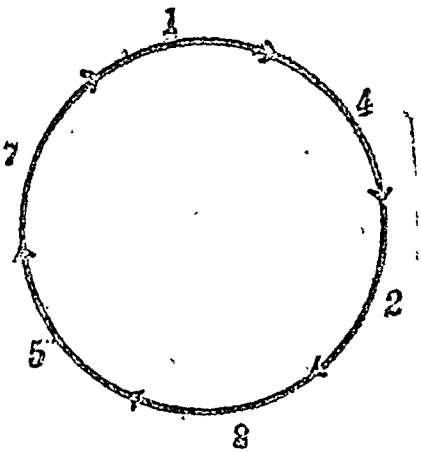
§19. **Cyclic order of digits.**

The following forms should be verified and committed to memory :—

$\frac{1}{7} = \cdot\dot{1}42857\dot{}$; $\frac{2}{7} = \cdot\dot{2}85714\dot{}$

$\frac{3}{7} = \cdot 428571\dot{}$; $\frac{4}{7} = \cdot\dot{5}71428\dot{}$

$\frac{5}{7} = \cdot\dot{7}14285\dot{}$; $\frac{6}{7} = \cdot\dot{8}57142\dot{}$



If the digits of the decimal of $\frac{1}{7}$ be placed round the circle we can obtain the results of $\frac{2}{7}$, $\frac{3}{7}$ etc., etc., by taking the digits in order in the direction of the arrow heads. It will also be noted that the decimals equivalent to fractions with denominator 7 are all pure recurring decimals.

The decimals equivalent to fractions with denominator 13 are pure recurring decimals and can be arranged in cyclic order in two sets. Verify and commit these results also to memory, if possible.

$\frac{1}{13} = \cdot\dot{0}76923\dot{}$	$\frac{2}{13} = \cdot\dot{1}53846\dot{}$
$\frac{3}{13} = \cdot\dot{2}30769\dot{}$	$\frac{5}{13} = \cdot\dot{3}84615\dot{}$
$\frac{4}{13} = \cdot\dot{3}07692\dot{}$	$\frac{6}{13} = \cdot\dot{4}61538\dot{}$
$\frac{9}{13} = \cdot\dot{6}92307\dot{}$	$\frac{7}{13} = \cdot\dot{5}38461\dot{}$
$\frac{10}{13} = \cdot\dot{7}69230\dot{}$	$\frac{8}{13} = \cdot\dot{6}15384\dot{}$
$\frac{12}{13} = \cdot\dot{9}23076\dot{}$	$\frac{11}{13} = \cdot\dot{8}46153\dot{}$

§20. **To begin the period in a given recurring decimal.**

In a given recurring decimal the period may be supposed to begin at any point we please after the first repeating figure. Thus

$\cdot 253636\ldots\ldots = \cdot 25\dot{3}6\dot{} = \cdot 2536\dot{3}6\dot{} =$ etc., etc.

Note. Number of figures in the period of a recurring decimal may be any multiple of that number without changing the value of the decimal. Thus

$\cdot 1\dot{2}5\dot{} = \cdot 12\dot{5}25\dot{} = \cdot 1252\dot{5}25\dot{} =$ etc.

§21. **Similar recurring decimals.**

When recurring decimals have the same number of non-recurring figures, and also the same number of recurring figures, they are said to be **Similar.** Thus

$\cdot 42\dot{0}\dot{2}\dot{3}$, $\cdot 1\dot{5}\dot{8}9\dot{7}$ and $8\cdot 2\dot{8}\dot{0}\dot{5}\dot{5}$ are similar recurring decimals.

§22. **To make the given recurring decimals similar.**

Example. Make $4\cdot 0\dot{2}\dot{3}$, $\cdot 2\dot{5}\dot{6}\dot{3}$ and $\cdot 5\dot{6}\dot{4}$ similar.

Exp. To make similar recurring decimals, there should be the same number of non-recurring figures and also the same number of recurring figures in each case. As the highest number of non-recurring figures in the given number is 2, extend each decimal to two non-recurring figures. Also the recurring numbers of figures in the periods are 1, 3 and 2 respectively; so to make the same number of recurring figures in each case, extend each decimal to 6 recurring figures which is the L. C. M. of 1, 3 and 2.

$$\begin{array}{rcl|l} \text{Thus } 4\cdot 0\dot{2}\dot{3} & = & 4\cdot 02 & \dot{3}3333\dot{3} \\ \cdot 2\dot{5}\dot{6}\dot{3} & = & \cdot 25 & \dot{6}35635\dot{6} \\ \cdot 5\dot{6}\dot{4} & = & \cdot 56 & \dot{4}64646\dot{4} \end{array}$$

EXERCISE 57.

In each of the following recurring decimals begin the period at the fourth decimal place :—

1. $\cdot 5\dot{2}3\dot{4}$. 2. $\cdot 5468$. 3. $\cdot 56\dot{7}$. 4. $\cdot 21\dot{5}\dot{6}$.
5. $\cdot 235\dot{6}\dot{3}$. 6. $\cdot 25438$. 7. $\cdot \dot{2}564$. 8. $\cdot 123\dot{5}6\dot{7}$.

9. Extend $\cdot 2\dot{4}$, $\cdot 5\dot{7}\dot{6}$ and $\cdot 5\dot{7}$ so that they may have the same number of figures in the period.

Make the following sets of recurring decimals similar :—

10. $\cdot 25$, $\cdot 5\dot{6}$. 11. $3\dot{5}\dot{6}$, $\cdot 7\dot{8}$.
12. $\cdot 3\dot{4}5\dot{6}$, $\cdot 3\dot{7}\dot{8}$. 13. $\cdot 0\dot{2}\dot{3}$, $\cdot 5\dot{6}7\dot{8}$.
14. $\cdot 5$, $\cdot 0\dot{0}\dot{5}$, $\cdot 0\dot{0}5\dot{5}$. 15. $\cdot 0\dot{6}\dot{5}$, $\cdot 0\dot{8}\dot{5}$, $\cdot 0\dot{3}\dot{5}$.
16. $\cdot 2\dot{2}5$, $\cdot 02\dot{2}\dot{5}$, $\cdot 00\dot{2}2\dot{5}$. 17. $\cdot 5\dot{7}\dot{6}$, $\cdot 0\dot{0}\dot{8}56$, $\cdot 0004\dot{3}\dot{2}\dot{5}$
18. $3\cdot 1256$, $5\,0123\,\dot{6}\,\dot{2}\,\dot{3}\dot{2}\,\dot{5}$.
19. Extend $\cdot 2\dot{3}\dot{4}$ $2\dot{3}\cdot\dot{5}$ and $\cdot 2\dot{3}\cdot\dot{5}$ so that they may have

§23. **Addition of Recurring decimals.**

Rule. *Make the decimals similar and then to be accurate in answer retain two more figures of each than the required number and then add them as usual.*

Example 1. Add together $\cdot 23\dot{5}$, $5\cdot 00\dot{2}\dot{4}$, and $6\cdot 7\dot{1}\dot{4}$.

Sol.

$\cdot 23\dot{5}$	=	$\cdot 23$	$5\dot{5}5555\dot{5}$	55
$5\cdot 00\dot{2}\dot{4}$	=	$5\cdot 00$	$\dot{2}4242\dot{4}$	24
$6\cdot 7\dot{1}\dot{4}$	=	$6\cdot 71$	$\dot{4}7147\dot{1}$	47

sum = $11\cdot 95\dot{2}6945\dot{1}$ **Ans.**

§24. **Subtraction of Recurring decimals.**

Rule. *Proceed as in the case of addition.*

Example 2. Subtract $15\cdot 00\dot{3}\dot{4}\dot{5}$ from $18\cdot 00\dot{2}\dot{3}\dot{5}$.

Sol.

$18\cdot 00\dot{2}\dot{3}\dot{5} = 18\cdot 002$	$\dot{3}5235\dot{2}$	35
$15\ 00\dot{3}\dot{4}\dot{5} = 15\cdot 003$	$\dot{4}5454\dot{5}$	45

Difference $= 2\cdot 998\dot{8}97806$ **Ans.**

Note. If the answer in addition or subtraction is required correct to certain decimal places, the extra figures are neglected and the retained portion is increased by unity if the first figure of the neglected portion is either 5 or greater than 5.

EXERCISE 58.

Add together accurately:—

1. $\cdot 3\dot{2}$, $\cdot 48$, $\cdot 5\dot{8}$, $\cdot 4\dot{5}$.
2. $2\cdot 0\dot{2}$, $5\cdot 4\dot{3}$, $8\cdot 5642\dot{5}$, $9\cdot 89\dot{3}\dot{5}$.
3. $4\cdot 2\dot{5}$, $\cdot 00\dot{2}\dot{3}$, $3\cdot 7\dot{8}\dot{4}$, $\cdot 526\dot{3}\dot{2}$.
4. $15\ 2\dot{3}\dot{5}$, $\cdot 0\dot{2}35\dot{6}$, $9\cdot 8\dot{0}2\dot{3}$, $15\cdot \dot{1}\dot{2}$.
5. $2\cdot 57\dot{6}\dot{9}$, $7\cdot 20\dot{1}\dot{2}\dot{3}$, $8\cdot 235\dot{6}$, $5\cdot 7\dot{1}$.
6. $21\cdot 2\dot{3}5\dot{6}$, $5\cdot 4\dot{3}\dot{2}$, $8\cdot 0\dot{2}\dot{3}$, $6\cdot 235\dot{4}$.
7. $12\cdot 532\dot{6}$, $18\cdot 2\dot{5}\dot{6}\dot{3}$, $7\cdot 5\dot{6}3402$, $2\cdot \dot{1}$,
8. $13\cdot 2\dot{3}5\dot{6}$, $21\cdot 0\dot{2}35\dot{6}$, $12\cdot 0\dot{2}\dot{3}$, $8\cdot 24\dot{6}$.
9. $7\cdot 12\dot{3}5\dot{6}$, $11\cdot 2\dot{3}56$, $2\cdot 02\dot{3}\dot{5}$, $8\cdot \dot{9}356$.
10. $16\cdot \dot{2}\dot{3}$, $21\cdot 5\dot{6}$, $45\cdot 2\dot{3}5\dot{6}$, $9\cdot 82\dot{3}\dot{5}$

Subtract:—

11. $15\cdot \dot{2}3\dot{6}$ from $21\cdot 562\dot{3}$.
12. $19\cdot 503\dot{4}$ from $35\cdot 9\dot{8}\dot{3}\dot{5}$.
13. $12\cdot 3\dot{7}5\dot{6}$ from $18\cdot 9835\dot{6}$.
14. $32\cdot 743\dot{2}$ from $48\cdot 7\dot{4}9\dot{5}$.
15. $28\cdot 2356\dot{7}\dot{8}$ from $35\cdot 2356\dot{7}\dot{8}$.

§25 To multiply a recurring decimal by a whole number or by a terminating decimal.

Rule. *Extend the decimal 2 or 3 places beyond the end of the period in order to ensure the correctness of the last digit retained and proceed in the usual way. In the product, point off as many decimal places as there are decimal places in both the multiplicand and multiplier together. The product will also be a recurring decimal of the same kind as the multiplicand. i.e., with a period containing the same number of digits.*

Example 1. Multiply $5{\cdot}24\dot{6}\dot{5}$ by 215.

Sol. (1)

$5{\cdot}24\dot{6}\dot{5}$	65
215	
$26{\cdot}2328$	
$52{\cdot}465$	
$1049{\cdot}31$	

(2)

$5{\cdot}24\dot{6}\dot{5}$	65
215	
$26{\cdot}2328$	28
$52{\cdot}4656$	56
$1049{\cdot}3131$	31
$1128{\cdot}01\dot{1}\dot{6}$	Ans.

Extend each line by repeating the digits of the period and add as shown in (2).

Example 2. Multiply $3{\cdot}54\dot{2}6\dot{8}$ by $\cdot144$.

Sol.

$3{\cdot}54\dot{2}6\dot{8}$	268
$\cdot144$	
1417073	
1417073	
354268	
$=\cdot5141463\dot{0}$	Ans.

1417073	073
14170730	730
35426826	826
$\cdot5141463\dot{0}$	629

§To divide a recurring decimal by a whole number.

Example 3. Divide $15{\cdot}\dot{1}23\dot{5}$ by 6.

Sol. $6)15{\cdot}\dot{1}23\dot{5} = 6)\ 15{\cdot}1235123512351235.$

$2{\cdot}5205853918725205$

$2{\cdot}52058539187\dot{2}$ **Ans.**

Example 4. Divide $15{\cdot}9\dot{3}5\dot{6}$ by 124.

Sol, 124)15·9356356356356356356(·1285131906099648035

124

353
248

1055
992

636.....
620

163
124

395
372

230
124

1123
1116

756
744

1235
1116

1196
1116

803
744

595
496

996
52

435
372

636 . . . .
620

16

·1285132906099648035. Ans.

§27. **To multiply or divide a recurring decimal by a recurring decimal.**

Rule. *Reduce the recurring decimals to vulgar fractions, then multiply or divide as the case may be and reduce the resulting fraction back to the equivalent decimal.*

Example 5. Multiply $\cdot\dot{1}0\dot{5}$ by $\cdot\dot{3}$.

Sol. $\cdot\dot{1}0\dot{5}=\frac{105}{999}=\frac{35}{333}$. and $\cdot\dot{3}=\frac{3}{9}=\frac{1}{3}$.

$\therefore$ the product $=\frac{35}{333}\times\frac{1}{3}=\frac{35}{999}=\cdot\dot{0}3\dot{5}$. **Ans.**

Example 6. Divide $1\cdot2\dot{8}$ by $\cdot05\dot{4}$.

Sol. $1\cdot2\dot{8}=\frac{128-12}{90}=\frac{116}{90}=\frac{58}{45}$.

$\cdot06\dot{4}=\frac{64-6}{900}=\frac{58}{900}$.

$\therefore$ The quotient $=\frac{58}{45}\div\frac{58}{900}=\frac{58}{45}\times\frac{900}{58}=20$. **Ans.**

EXERCISE. 59.

Simplify:—

1. $\cdot5\dot{1}\dot{2}\times6$.
2. $\cdot\dot{3}3\dot{5}\times15$.
3. $\cdot02\dot{5}\times36$.
4. $1\cdot5\dot{1}\dot{6}\times24$
5. $2\cdot0\dot{3}\dot{1}\dot{5}\times45$.
6. $6\cdot\dot{3}0\dot{2}\dot{5}\times216$.
7. $4\cdot12\dot{5}\div7$.
8. $3\cdot15\dot{6}\div12$.
9. $6\cdot\dot{5}6\dot{1}\div16$.
10. $124\cdot01\dot{2}\dot{5}\div124$
11. $15\cdot02\dot{8}\dot{6}\div136$.
12. $8\cdot7\dot{2}\div288$.
13. $\cdot\dot{8}\dot{1}\times\cdot0\dot{1}\dot{1}$.
14. $\cdot\dot{3}\dot{7}\times\cdot\dot{2}9\dot{7}$.
15. $\cdot03\dot{3}\times\cdot\dot{6}\dot{6}$.
16. $\cdot2\dot{2}85714\dot{4}\times\cdot4\dot{6}$.
17. $\cdot6\dot{4}2857\dot{1}\times2\cdot0\dot{7}\dot{4}$.
18. $\cdot2\dot{1}42857\dot{1}\times1\cdot\dot{3}$.
19. $49\cdot\dot{3}\times\cdot2\dot{9}5\dot{4}$.
20. $2\cdot3\dot{5}71428\dot{8}\div10\cdot2\dot{1}42857\dot{1}$.
21. $1\cdot9\dot{2}\dot{7}\div\cdot58\dot{1}$.
22. $11\cdot8\dot{3}\div\cdot24\dot{9}$.
23. $\cdot\dot{1}42857\dot{1}\div1\cdot\dot{8}57142\dot{1}$.
24. $\cdot89\dot{1}\div1\cdot2\dot{9}$.
25. $\cdot12\dot{5}\div\cdot25\dot{1}$.
26. $\cdot\dot{1}53846\dot{1}\div1\,0\dot{7}6923\dot{1}$.
27. $\cdot\dot{4}2857\dot{1}\div\cdot\dot{2}85714\dot{1}$.
28. $\cdot\dot{3}07692\dot{1}\div\cdot\dot{5}38461\dot{1}$.
29. $7\cdot39\div\cdot079$.
30. $\cdot857142\dot{1}\times\cdot\dot{5}38461\dot{1}\div1\cdot\dot{1}53846\dot{1}$.
31. $2\cdot1\dot{4}28571\dot{1}\div\cdot07\dot{6}92307\dot{1}\times2\cdot\dot{3}$.

DECIMAL MEASURES

§28 Reduction of simple decimal quantity from one unit to another.

Example 1. Reduce Rs. 4·16625 to pies.

Sol. 4·16625 rupees.
16
66·66000 annas.
12
799·92 p.es. **Ans.**

Example 2. Reduce 95445 lbs. to tons.

Sol. $28 = 4 \times 7$

4)95445 lbs.
7)23861·25
4)3408·7500 qr.
20) 852·187500 cwt.
42·609375 tons.
Ans.

EXERCISE 60.

Reduce (*based on Ex. 1.*) :—

1. Rs. ·375 to pies.
2. Rs. 15·1875 to pies.
3. ·01125 of Rs. 120 to pies.
4. £·15625 to pence.
5. ·035375 of £60 to pence.
6. ·15875 days to seconds.
7. 8·125 of 20 days to min.
8. 2·1625 maunds to chks.
9. 21·1325 of 20 mds. to srs.
10. 4·1225 tons to lbs.
11. 2·625 of ·125 tons to qrs.
12. 1·1625 of 20 tolas to ratties.

Reduce (*based on Ex. 2.*) :—

13. 5 chks. to maunds.
14. 117 pies to rupees.
15. 1645 lbs. to tons.
16. 12321 pence to £.
17. 7875 seconds to hours.
18. 121 yards to miles.
19. 15620·4 chataks to maunds.
20. ·825 of 1600 pies to rupees.

---

§29. Reduction of compound quantities to simple quantities and *vice versa*.

The method will be best illustrated by the following solved examples :—

Example 1. Reduce Rs. 12·109375 to Rs. *a*. *p*.

Sol. Rs. 12·109375

16

1·750000 *a.*

12

9·00 *p.*

The Rs. 12 is not reduced to annas.

The anna 1 is not reduced to pies.

∴ Rs. 12·109375=Rs. 12. 1*a*. 9*p*. Ans.

Example 2. Reduce Rs. 15, 7*a*. 9*p*. to rupees.

Sol.

12)9·00*p*.

·75*a*.

7·

16)7·750000*a*.

·484375

15

Rs. 15·484375 Ans.

Example 3. Find the value of 3·725 of 12 tons 12 cwt. 2qr.

Sol.

4)2·0 qr.

·5 cwt.

12·

20)12·500 cwt.

·625 tons.

12·

12·625 tons.

∴ 5·725 of 12 tons 12 cwt. 2 qr.=12·625×3·725 tons.

=12·625

3·725

63125

25250

88375

37875

47·028125 Tons.

20

0·562500 cwt.

4

2·2500

∴ 47 tons 2·25 qr. Ans.

Note. Sometimes the compound quantity is not reducible to decimals. In such cases we proceed thus :–

12 tons 12 cwt. 2 qr. $=1010$ qr.
3·725 of 12 tons 12 cwt. 2 qr. $=1010\times3{\cdot}725$ qr.
$=3762{\cdot}25$ qr.

4)3762·25 qr.
20) 940 cwt. 2·25 qr.
47 tons 0 cwt. 2·25 qr. Ans.

Aliter. $3{\cdot}725=\frac{3725}{1000}=\frac{149}{40}$.

∴ 3·725 of 12 tons 12 cwt. 2 qr.

$=\frac{(12 \text{ tons } 12 \text{ cwt. } 2 \text{ qr.}) \times 149}{40}$ and so on.

Example 4. Find the value of double of £ 31·258$\dot{3}$.

Sol.

31·258$\dot{3}$ 33
2
£62·516 /63
20
s. 10·333$\dot{3}$/20
12
d. 3·9998/40.

∴ the required value=£62. 10*s.* 4*d.* Ans

EXERCISE 61.

Reduce to compound quantity:

1. Rs. 12·129375.
2. Rs. 16·22875.
3. 16·29625 maunds.
4. 24·9125 maunds.
5. £16·1625.
6. £ 21·4875.
7. 12·18725 yards.
8. 12·16925 tons.

Express the following as a decimal of its highest denomination:—

9. Rs. 15, 8*a.* 6*p.*
10. Rs. 21, 13*a.* 9*p.*
11. £18, 12*s.* 9*a.*
12. £12, 17*s.* 6*d.*
13. 5 tons 15 cwt. 2 qr. 7 lb.
14. 16 tons 16 cwt. 1 qr. 14 lb.
15. 12 yards 2 ft. 3 in.
16. 12 mds. 28 sr. 4 chk.

Find the value of (by use of 1st method Ex. 3 if possible)

17. ·25 of Rs. 12, 13*a*. 9*p*.
18. ·5 of Rs. 2, 6*a*. 4·8*p*.
19. ·125 of Rs. 21, 8*a*. 6*p*.
20. $3\cdot\dot{0}\dot{6}$ of Rs. 2, 1*a*.
21. ·625 of 2 yards 1 ft. 6 in.
22. ·375 of £ 12; 13*s*. 9*d*.
23. $\cdot4\dot{2}\dot{8}$ of £3, 8*s*.
24. $\cdot41\dot{3}\dot{5}$ of £1.
25. $\cdot458\dot{3}$ of 1*s*.
26. $\cdot358\dot{1}$ of £4, 18*s*. 6*d*.
27. $\cdot0\dot{1}\times\cdot0\dot{1}$ of Rs. 749, 4*a*.
28. 8·71875 of 5*a*. 4*p*.+1·146875 of Rs. 3, 5*a*. 4*p*.—·0625 of Rs. 10, 8*a*.
29. ·325 of £1, 1*s*.+$\cdot\dot{5}\dot{4}$ of 8*s*. 3*d*.+$\cdot0\dot{2}\dot{7}$ of £ 14, 6*s*.
30. ·025 of 7 mds.+2·12 of 15 mds.—3 15 of 4 mds.
31. $\cdot\dot{7}$ of 7*s*. 6*d*.—$\cdot8\dot{4}$ of 16*s*. 6*d*.+$\cdot9\dot{2}\dot{7}$ of £2, 10*s*. 5*d*.
32. ·5 of Rs. 7, 8*a*.—·05 of Rs. 4. 6*a*.+·02 of Rs. 21, 9*a*.
33. ·7365 of £3. 6*s*. 8*d*.+ 504 of £15, 12*s*. 6*d*.+2·102083 of £2.
34. $\cdot3\dot{8}\dot{7}$ of £8, 16*s*. 3*d*.+$6\frac{1}{2}$ of $\frac{16}{65}$ of 7*s*. $8\frac{1}{2}d$.+$\frac{7}{11}$ of 1*d*.

---

§30. **To express one concrete quantity as the decimal of another concrete quantity of the same kind.**

Example 1. Reduce Rs. 2, 1*a*. 6*p*, to the decimal of Rs. 8, 6*a*.

Sol. The reqd. decimal $=\frac{\text{Rs. 2, 1}a\text{. 6}p\text{.}}{\text{Rs. 8, 6}a\text{.}}=\frac{2\cdot09375}{8\cdot375}$

$=8375)2093\cdot75(\cdot25$ **Ans.**

16750

41875

41875

Aliter. The reqd. decimal $=\frac{\text{Rs. 2, 1}a\text{. 6}p\text{.}}{\text{Rs. 8, 6}a\text{.}}=\frac{\frac{67}{32}}{\frac{67}{8}}$

$=\frac{67}{32}\times\frac{8}{67}=\frac{1}{4}=\cdot25$. **Ans.**

Rule. *Express both quantities in fraction of the same denomination and then proceed as in Art. 30 Chapter VII.*

EXERCISE 62.

Reduce the first of the two given quantities to the decimal of the second :—

1. Rs. 2, 8*a.* ; Rs. 5.
2. Rs. 5, 4*a.* ; Rs 21.
3. Rs. 5, 8*a.* ; Rs. 27, 8*a.*
4. Rs. 7, 8*a.* 9*p.*; Rs.30,3*a.*
5. Rs. 21, 2*a.* 6*p.* ; Rs. 169, 4*a.*
6. £2, 8*s.* 9*d.* ; £9, 7*s.* 6*d.*
7. £25, 1*s.* 6*d.* ; £50, 3*s.*
8. 15 mds. 4 sr. 8 chk. ; 75 mds. 22 sr. 8 chk.
9. 17 mds. 8 sr. 6 chk. ; 68 mds. 33 sr. 8 chk.
10. 12 tons 16 cwt, 2 qr. ; 51 tons 6 cwt.
11. Rs. 21. 2*a.* 4*p.* : Rs. 169, 2*a.* 8*p.*
12. Rs. 16, 8*a.* 8*p*, ; Rs. 66, 2*a.* 8*p.*
13. £5, 8*s.* 4*d.* ; £43, 6*s.* 8*d.*
14. £12, 13*s.* 8*d.* ; £50, 14*s.* 8*d.*
15. 7 *ft.* 4 *in.* ; 9 yards 2 ft. 4 in.
16. Reduce $5\frac{1}{2}$ sq. yds. to the decimal of an acre.
17. What decimal of £ 2, 13*s.* 4*d.* is ·0625 of $2\cdot\dot{6}$ of £1, 6*s.* 8*d.* ?
18. **Simplify.**

$$1-\frac{2}{3+\frac{4}{5-\frac{6}{7+\frac{8}{9}}}} \div 2\cdot08\dot{3} \text{ of } \frac{3 \text{ tons } 2 \text{ cwt. } 2 \text{ qr. } 20 \text{ lb.}}{10 \text{ cwt. } 2 \text{ qr. } 11 \text{ lb.}}$$

and reduce the result to the decimal of $1\cdot\dot{1}$.

19. Find the value of $1\frac{3}{17}$ of $\cdot0\dot{1}2\dot{3}\dot{6}$ of Rs. 5, 11*a.* 8*p.* and taking the rupee as worth 1*s.* $4\frac{1}{2}d.$ express the result as the decimal of one shilling.

20. Express $\frac{3}{8}$ of 7*s.* 6*d.* + 1·25 of 5*s.* − $\cdot5\dot{4}\dot{5}$ of 9*s.* 2*d.* as a decimal of £10.

§21. We now close this chapter with the following miscellaneous exercise :—

EXERCISE 63.

Simplify and give answer in decimals if possible :—

1. $\dfrac{\dot{8}\dot{1}\times\cdot005}{\cdot45}$ 2. $\dfrac{\cdot00\dot{3}\times\cdot15}{\cdot0035}$

3. $\dfrac{2\cdot4\dot{6}-2\cdot\dot{3}0}{\dot{3}+\cdot12\dot{7}}+\dfrac{4\frac{1}{5}}{19}$. 4. $\dfrac{\cdot125\times(\cdot175 \text{ of } \cdot\dot{2}85714)}{\cdot00025}$.

5. $\dfrac{\cdot0025+1\cdot\dot{6}}{3\dot{6}-25}\div\dfrac{\cdot7\dot{1}\dot{3}+2\cdot625}{12\cdot7-10\cdot2}$.

6. $\dfrac{\cdot12 \text{ of } (\cdot0104-\cdot002)+\cdot36\times\cdot002}{\cdot12\times\cdot12}$.

7. $\left\{37+\dfrac{3\cdot70\dot{2}\dot{7}}{100}\right\}\times\cdot54$.

8. $\left\{41+\dfrac{4\cdot10\dot{1}}{100}\right\}\div\cdot8\dot{2}-\frac{61}{111}$.

9. $\dfrac{4\cdot\dot{2}+3\cdot\dot{1}\dot{4}}{1\cdot\dot{3}+2\cdot10\dot{2}}$ of $\dfrac{1\cdot3 \text{ of } 4}{\cdot37 \text{ of } 8\cdot\dot{8}\dot{1}}$

10. $\dfrac{\cdot2857\dot{1}\dot{4}\times4\frac{2}{3}}{1\cdot\dot{3}}+\dfrac{\cdot\dot{8}\dot{1}}{1\frac{2}{5}}+\cdot\dot{3}$.

11. $\dfrac{2\frac{1}{5}-1\frac{1}{3}}{2\frac{1}{4}-\frac{1}{2} \text{ of } 3\frac{1}{3}}$ of $\dfrac{4\cdot3}{\cdot25}$ of Rs. 15.

12. $\dfrac{\frac{1}{2}+\frac{3}{4}}{\frac{5}{6}-\frac{7}{8}+\frac{9}{10}}\div\cdot\dot{1}42857$ of $10\frac{20}{103}$.

13. $\dfrac{3\cdot125}{2\cdot16}$ of $\dfrac{\cdot\dot{2}\dot{4}}{\cdot125}-\dfrac{2\cdot\dot{2}}{1\cdot\dot{5}}$ of $\dfrac{187\cdot5}{3\cdot42}$

14. $\left(\dfrac{\cdot0019}{3\cdot16} \text{ of } \dfrac{4\cdot4}{\cdot0095}\right)\div\left(\dfrac{3\cdot\dot{8}}{7} \text{ of } \dfrac{4}{5\cdot625}\right)$

15. $\dfrac{2\cdot8 \text{ of } 2\cdot\dot{2}\dot{7}}{1\cdot1\dot{3}\dot{6}}+\dfrac{4\cdot4-2\cdot8\dot{3}}{1\cdot6+2\cdot\dot{6}29}$ of $\dfrac{6\cdot8 \text{ of } 3}{2\cdot25}$

16. $\dfrac{\cdot\dot{1}42857\times\cdot0\dot{7}3923}{\cdot010989}+\dfrac{2\cdot75\times11\cdot25}{6\cdot2}+\frac{1}{3}$.

17. $\dfrac{1\cdot8\dot{3}+2\cdot041\dot{6}+\cdot\dot{3}-3\frac{1}{3}}{1\cdot0025+\cdot0625-1\frac{1}{16}}+\dfrac{66\frac{2}{3}}{100}$.

18. $\cdot42\dot{6}$ of $\dfrac{3\cdot\dot{3}}{\cdot08}$ of $\dfrac{\cdot3}{\cdot\dot{7}3\dot{5}}$ of $\dfrac{\cdot147\times4\cdot\dot{4}}{11\cdot1}$ of £1, 17*s.* 6*d.*

19. $\dfrac{2\frac{1}{4}+\frac{3}{4}\text{ of }2\frac{2}{3}-1\frac{1}{4}}{3\cdot6+\cdot15\times4\div\cdot24-2\cdot1}$ of $\dfrac{20\cdot1\text{ of }\frac{1}{2}\text{ md. }30\frac{1}{4}\text{ sr.}}{17\cdot5625\text{ of }2\text{ md. }20\frac{1}{2}\text{ sr.}}$ of Rs. 2, 8*a.*

20. $\dfrac{5\frac{1}{2}\text{ of }\cdot\dot{2}\text{ of }2\cdot\dot{5}71428\dot{5}-1\div(\frac{1}{6}+\cdot5)}{1-\frac{3}{14}\text{ of }\left\{\cdot5+\frac{1}{2}\text{ of }\dfrac{\cdot05}{\cdot\dot{1}42857\text{ of }1\frac{1}{20}}\right\}}$.

21. $(2\cdot36\dot{4}-1\cdot69\dot{7})+1\cdot\dot{3}\times(2\cdot\dot{4}+7\cdot\dot{5})+\dfrac{2}{2+\dfrac{1}{2+\frac{1}{3}}}$

22. $\dfrac{\cdot0016\times\cdot025}{\cdot325\times\cdot05}\div\dfrac{\cdot1216\times\cdot105\times\cdot002}{\cdot08512\times\cdot625\times\cdot039}\times\dfrac{1\cdot\dot{6}}{2\cdot\dot{3}}$

23. $\left[\dfrac{(3\frac{2}{3}\times5\frac{1}{4})-(3\frac{1}{2}\times4\frac{5}{6})}{3\frac{2}{3}-(5\frac{1}{4}\div2\frac{1}{2})+4\frac{5}{6}}\text{of Rs. }10\right]+\left[\dfrac{1\cdot405}{2\cdot1}\times\dfrac{6\cdot3}{28\cdot1}\text{ of Rs. }7\frac{1}{2}\right]-\cdot0\dot{2}\dot{7}\text{ of Rs. }125,\ 7a.\ 6p.$

24. $\dfrac{2\cdot5-6\cdot08+4\cdot7}{(\cdot2\times8\cdot3)-(2\times\cdot55)}+\dfrac{(3\frac{2}{3}\text{ of }\frac{1}{8})-\frac{5}{12}}{(\cdot\dot{3}\text{ of }\frac{1}{16})+(\frac{3}{16}-\frac{1}{6})}$.

25. $\dfrac{2\frac{1}{5}-1\frac{2}{3}}{3\cdot\dot{3}\times1\frac{2}{5}}\div\dfrac{\frac{2}{7}-\frac{1}{8}}{\frac{2}{7}+\frac{1}{8}}+\dfrac{\cdot05\times\cdot7}{\cdot071}\text{of }\dfrac{\text{Rs. }2,\ 7a.}{\text{Rs. }1,\ 11a.}$

26. $\dfrac{5\cdot75-\cdot\dot{4}2857\dot{1}\text{ of }15\frac{3}{4}+2\frac{2}{3\frac{1}{5}}\div1\cdot44}{2\frac{11}{35}\times5\cdot6\div3\cdot36}\times\dfrac{2\cdot8\times1\cdot13\dot{6}}{31\cdot8\dot{1}}$

27. How many times can ·34 be subtracted from 27·67 and what will be the magnitude of the remainder?

CHAPTER IX

APPROXIMATION

§1. The need for approximation.

Suppose we have to divide a distance of 10 yards 2 ft. 3·6 inches into 100 equal parts, then each part is, by calculation 3·876 inches. If we are to mark these divisions, we should draw a straight line and mark off along it a length of 3·87 in. To do this, we first take 3 inches ; then along the next inch, which is divided into 10 equal parts, we take 8 tenths or 8 small sub-divisions. To get the remaining 7 hundredths, we must sub-divide the next tenth of an inch into 10 equal parts and take 7 of these. This is very difficult to do, for a tenth of an inch is too small to admit of further equal sub-divisions. If, moreover, we wish to get a straight line 3·876 inches long, a hundredth of an inch is to be sub-divided into 10 equal parts and 6 of these are to be taken. This is a very small line which cannot be judged by the eye or measured with any ordinary instrument. Thus owing to either imperfection of our senses or want of very delicate instruments, we are obliged to impose certain limitations on the extent of our measurements. In the calculation of lengths expressed in inches, we should be satisfied with two decimal places and if there is a third decimal place as in 3·876 in the figure in the third place, *viz.*, six thousandths may be roughly taken to be one hundreth of an inch. Thus 3·876 in. is *approximately* equal to 3·88 inches.

§2. Error.

Approximation always involves some error, *i.e.*, there must be some difference between the actual value and that roughly taken for our purpose. In the above example, when 3·876 in. is changed to 3·88 in., the error is 3·88—3·876 or ·004 in., but if 3·876 in. is changed to 3·87 in., the error s ·006 inch. Since the error ·004 is less than the error ·006 in., the former approximation, *viz.*, 3·88 is nearer the truth, *i.e.*, the actual value of 3·876, 3·88 is said to be correct to the nearest hundredth or *correct to two decimal places.*

Again $\frac{5}{17}$=·29411764.....If we wish to *approximate* to the result by terminating the operation at the 5th place, we write $\frac{5}{17}$=·29412, but if at the fourth place, then we write $\frac{5}{17}$=·2941, and so on. From this we evidently have the following rule:—

Rule. *Increase the last figure in the result by 1, if the next figure is 5 or greater than 5.*

The reason for this is evident from the illustration given above.

§3. Significant Figures

The following examples explain clearly the meaning of the expression 'significant figures:—

(*a*) (*i*) The distance between two places is 1700 miles correct to the *nearest hundred.* Here the unit of measurement is one *hundred miles* and the distance is stated to be 17 such units *correct to the nearest units.* The figures 17, which give the number of units, are said to be **significant**; while the two zeroes, which express the *magnitude* of the unit, are said to be **non-significant.**

(*ii*) The distance between two places is 1700 miles correct to the *nearest mile*; here the unit is one mile and hence all the figures are significant.

(*b*) The length of a line is ·07 inch, correct to the 2nd decimal place.

This means that the length is 7 hundredths of an inch correct to the nearest *hundredth*; here the unit is *hundredth* of *an inch.*

∴ the *significant* figure is 7 and the zero at the beginning is non-significant.

Note Thus zeroes at the very beginning of a decimal are always non-significant. From the examples given above it is clear that significant figures are those which in any approximate result express the number of units, correct to the nearest such unit

§4. Absolute error, relative error and percentage error.

Errors are of two kinds: (1) Absolute error and (2) Relative error.

Absolute error = Actual value − approximate value.

For example, if 1 metre = 39·3707 inches and if we take 40 inches to be approximate equivalent to 1 metre, then the absolute error = 40 − 39·3707 = ·6293 inch.

$$\text{Relative error} = \frac{\text{Absolute error}}{\text{True value}}$$

In the above example, the relative error $= \frac{\cdot 6293 \text{ in.}}{39\cdot 3707 \text{ in.}}$

$$= \frac{\cdot 6293}{39\cdot 3707} = \cdot 0159$$

The percentage error = the relative error × 100.

∴ the percentage error in the above example
$= \cdot 0159 \times 100 = 1\cdot 59.$

The relative error is more important than the absolute error. *e.g.*, if 1008 is taken as 1000 and 63 is taken as 60, the absolute error 8 is the same in both cases ; but the relative error in the first case is $\frac{8}{1000} = \frac{1}{125}$ and in the last case it is $\frac{8}{60} = \frac{2}{15}$. Hence the relative error is much greater in the latter than in the former case.

§5. Contracted Addition and Subtraction.

In all approximate additions and subtractions where a certain degree of accuracy is required it is enough to retain in each quantity two more places of decimals than the number specified.

Example 1. Find correct to three decimal places, the sum of 16·81984 ; ·01$\dot{6}$; ·$\dot{1}$42357 and $\frac{5}{6}$.

Sol.

		Explanation.
16·819	84	A vertical line is drawn after
·016	66	the third decimal figure to show that
·142	85	the figures following are unnecessary
$\frac{5}{6}$ = ·833	33	except for determining the figure to
17·812	68	be carried to the third decimal place.

∴ the sum is 17·813 correct to the third decimal place. **Ans**

Example 2. Subtract $361\frac{5}{7}$ from 1082·8$\dot{3}\dot{5}$, showing the remainder correct to four decimal places.

Sol. $1082\cdot\dot{8}3\dot{5} = 1082\cdot8358 \mid 35$
$361\frac{5}{7} = 361\cdot7142 \mid 85$
$721\cdot1215 \mid 50$

∴ the remainder correct up to four decimal places is 721·1216. **Ans.**

§6. **Contracted Multiplication.**

In multiplying one long decimal fraction by another it is generally required to get the product *approximately* correct, *i. e.*, as far as a certain number of decimal places. The following Rule enable us to shorten the work :

Rule. *Mark off in the decimal parts of the multiplicand as many figures as is one more than the number of decimal places we are required to retain in the product ; under the last of these figures place the unit's figure of the multiplier writing the figures in the reverse order. Omit decimal points of both the multiplicand and multiplier and add zeroes (if necessary) in the multiplicand, so that every figure of the multiplier shall have a a figure above it. Begin the multiplication with the last right hand figure of the multiplier and multiply in succession by each of the other figures, in each case begining the multiplication from the figure above the one we are multiplying by taking care to carry to it the nearest ten from its product with the next figure on the right in the multiplicand. Place the unit's figure of all these partial products in the same vertical line ; add as usual and mark off the required number of decimal places in the result striking out the last figure.*

Note. In carrying nearest ten, if the product is a number from 5 to 14 carry 1 ; if from 15 to 24 carry 2 and so on. If the product is 4 or less than 4, carry 0, i. e., reject it (Art. 2).

Example 3. Multiply 459·63524 by 25·4637 giving the product correct to 3 decimal places.

Sol.

4596352	4
73645	2
9192704	8
2298176	2
183854	1
27578	1
1378	9
321	7
11704·013	8

∴ 11704·014. **Ans.**

Example 4. Multiply ·00040635 by 241·6358 retaining 6 places of decimals.

Sol.

4063	50	
85361	42	
8127	00	
1625	40	
40	64	
24	38	
1	22	
	20	
	3	
·09818	87	∴ ·098189 **Ans.**

Example 5. Multiply ·453 by ·01694 correct to 4 decimal places.

Sol.

4530	0	
49610	0	
453		
272		
41		
2		
·00768		∴ ·0076. **Ans.**

Note. ·01694 is written as 0·01694.

§7. Contracted Division.

In dividing one decimal by another where the quotient is required correct up to certain number of decimal places we observe the following rule :

Rule. *Make the divisor a whole number ; try to find out—by mere inspection by taking one single step in the ordinary way, —the nature of the required quotient, i. e., how many integral figures are e xpected in the quotient and also the whole number of figures in the quotient ; from the left of the divisor cut off this number of figures and one more for approximation and strike out he rest. Proceed one step with this new divisor but in multiplying its first figure by the quotient figure, carry the nearest ten (see Note Art. 4) from its product with the next figure on the right. Instead of bringing down a figure to the remainder, strike off another figure from the divisor and proceed as before, till all the*

figures in the divisor are exhausted. If the number of figures in the divisor is less than the number of figures to be cut off in the very beginning, then proceed in the ordinary way until the number of figures still to be found in the quotient is one less than the number of figures in the divisor and then apply the Rule.

Example 6. Divide 2508·928065051 by 92·410357 correct to four decimal places.

Sol. Evidently the integral part of the quotient will contain 2 figures and since 4 places of decimals are to be retained therefore 2+4=6 figures are retained in the divisor and 1 more for *approximation* makes the divisor have 7 figures, *viz.* it is written as 924103,5

```
× × × × × ×
9 2 4 1 0 3,5)2508928...(271498
              1848207
               660721   here the divisor is 924103 and carry
               646872   2 from 7×3.
                13849   . . . . . .
                 9241
                 4608   here the divisor is 924 and carry
                 3696   nothing from 4×1.
                  912   . . . . . .
                  832
                   80   here the divisor is 9 and carry 2 from 8×2
                   74
```

With each new figure in the quotient a × mark is put upon that figure in the divisor which is not to be taken in the next step and so on until all the figures in the divisor are exhausted.

∴ the quotient is 27·1498. **Ans.**

§8. Multiplication and division combined.

Example 7. Find the value of $\frac{346{\cdot}25 \times 32{\cdot}164}{732{\cdot}96}$ correct to 3 places of decimals.

Sol. The exp. $= \frac{34{\cdot}623 \times 3{\cdot}2164}{7{\cdot}3296}$, whence we estimate the nature of the answer roughly thus: $\left(\frac{35 \times 3}{7} = 15\right)$. *i. e.*, the integral part in the result shall contain 2 figures. But as the result is required to be correct to 3 decimal places, we require (2+3) or 5 *significant* figures in the result, so that 5+1=6 figures must be retained in the divisor, but as there are only 5, we shall proceed in the division one step by the ordinary method: (or we could put one zero after 6 in the divisor, making it 7·32960).

Again the rough estimate of the numerator shows that to make the first step of the division, 6 figures will be required in the numerator, but as the numerator (34×3) has 3 figures in its integral part, the product is to be correct to 3 decimal places.

The work, therefore, is as follows :—

						×	×	×	×	×			
34625	0	.	.	.	.	7	3	2	9	6	111367	(	15194
4612	3										73296		
103875	0										38071		
6925	0										36648		
346	3										1423		
207	7										733		
13	8										690		
111367											659		
											31		
											29		

i. e., the product is 111·367 up to 3 decimal places.

∴ the required answer is 15·194. **Ans.**

Example 8. Divide ·257917 by 2·03458 approximately correct to 7 places of decimals.

Sol. In this case, evidently the quotient will have no integral part. Since 7 places of decimals are to be retained, the divisor must consist of 8 figures; but as there are only 6 figures in the divisor, proceed in the ordinary way for 8−6=2 figures in the quotient, when the number of figures still to be obtained will be one less than the number of figures in the divisor. Then apply the rule given above.

× × × ×
2 0 3 4 5, 8)25791·70(·1267667
20345·8
5445·90
4069·16
1376·74 here begin to apply the Rule
1220·75
155·99
142·42
13·57
12·20
1·37
1·22
15
14

∴ the quotient is ·1267667. **Ans.**

Example 9. Divide ·549532676 by 931·2167 correct up to 7 places of decimals.

Sol. By inspection, we find that there will be 3 zeroes after the decimal point in the quotient, hence 7−3=4 figures are wanted in the quotient, ∴ we retain 4+1=5 figures in the divisor and proceed thus—

× × ×
9 3 1 2, 1)54953...(5901
46561
8392
8381
11
9 ∴ the quotient is ·0005901. **Ans.**

§9. SERIES.

The value of a *series* is often required to be correct to a certain number of decimal places. In such cases, we observe the following rule :

Rule. *Work out each term in decimals, taking help in each step, of the preceding term and continue this method till a term becomes so small that it does not affect the required result.*

Example 10. Find the value of

$1+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\ldots$ correct to 6 decimal places.

Sol. $1=1$		$=1{\cdot}000000$	0
$\frac{1}{1.2}$	$=\frac{1}{2}$ of previous result	$=\ {\cdot}500000$	0
$\frac{1}{1.2.3}$	$=\frac{1}{3}$ of ,, ,,	$=\ {\cdot}166666$	6
$\frac{1}{1.2.3.4}$	$=\frac{1}{4}$ of ,, ,,	$=\ {\cdot}041666$	6
$\frac{1}{1.2.3.4.5}$	$=\frac{1}{5}$ of ,, ,,	$=\ {\cdot}008333$	3
$\frac{1}{1.2.3.4.5.6}$	$=\frac{1}{6}$ of ,, ,,	$=\ {\cdot}001388$	8
Similarly next term		$=\ {\cdot}000198$	4
,, ,, ,,		$=\ {\cdot}000024$	8
,, ,, ,,		$=\ {\cdot}000002$	7
,, ,, ,,		$=\ {\cdot}000000$	2 .
		$=1{\cdot}718281$	4

.. the required value is 1·718281. **Ans.**

Note. We stopped at the term, which being converted to decimal produced six zeroes in the first six places and such this term as well as the term following it would not evidently affect the result.

☞ See the solution of the following two examples very carefully.

Example 11. Find the value of $1+\frac{1}{25}+\frac{1}{25^2}+\frac{1}{25^3}+\ldots$ to four places of decimals.

Sol. Suppose the sum $=x$.

$$\therefore \quad x=1+\frac{1}{25}+\frac{1}{25^2}+\frac{1}{25^3}+\ldots\ldots$$

$$\text{and}\ \frac{1}{25}x=\quad \frac{1}{25}+\frac{1}{25^2}+\frac{1}{25^3}+\ldots\ldots$$

$\therefore \frac{24}{25}x=1$ [*By Subtraction.*

or $x=\frac{25}{24}=1{\cdot}0417$. **Ans.**

Example 12. Find the value of $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots$ to three places of decimals.

Sol. $x=1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots\ldots$

and $-\frac{1}{2}x=\ -\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+\ldots\ldots$

$\therefore \quad \frac{3}{2}x=1$

EXERCISE 64.

Write down the following numbers correct to *three significant* figures :—

1. ·640834. 2. ·487634. 3. 9·06784.
4. ·0008346 5. $8\cdot48\dot{2}$ 6. ·008092.
7. Given that 1 cwt.=2465 grammes, find the equivalent of 1 cwt. in grammes correct to two *significant* figures.

Find the result correct to *two* significant figures :—

8. 150·3806+64·0098+·1009+0·8345.
9. 1008·053−678·349.
10. $\frac{3}{7}+\frac{5}{6}-\frac{2}{9}$. 11. 818·6025+41·3892−·008765.
12. Find the value of $51\cdot\dot{1}42857-5\cdot3\dot{2}\dot{3}$ correct to 7 places of decimals.
13. Find the sum of 5·23, 162·4302, 12·0789 and 14·3096 to within one thousandth.
14. Multiply ·0008347 by 1·0856 correct to 3 significant figures.

Give the following products correct to *two* decimal places:—

15. 0·6208×3·453 16. 8·4376×153·2986.
17. 708·00083×·0004684. 18. ·005683×101278·3

Find the products in the following correct to the third decimal place:—

19. 0·0007895×108·394 20. $(1\cdot3825)^2$
21. 62·843×·008345. 22. 6·0008375×·000483.
23. Multiply ·86858896 by 1·0986123 retaining 5 places of decimals.

Divide :—

24. 865·345 by 12·438 to 3 significant figures.
25. ·056789 by 139·28 „ „ „
26. ·834952 by 156·24 „ „ „
27. Divide 12·384698 by 168·0876 correct to the second decimal place.
28. Divide 1·0896543 by ·689763 correct to the second decimal place.
29. Divide 0·38465 by 0·48796 correct to the second decimal place.

30. Divide 1 by 3·1415926 to four places of decimals.

31. Find the reciprocal of 6·8438 to 3 places of decimals.

32. Simplify $\frac{1461}{88479}$ correct to 3 places of decimals.

33. Divide $2\cdot3472\dot{1}$ by $2\cdot2792\dot{4}$ retaining 7 places of decimals.

Find the value of:—

34. $1+\frac{1}{1.3}+\frac{1}{1.3.5}+$.....to 7 places of decimals.

35. $\frac{1}{5}+\frac{1}{3}\times\frac{1}{5^3}+\frac{1}{5}\times\frac{1}{5^5}+\frac{1}{7}\times\frac{1}{5^7}+$.....to 6 places of decimals.

36. $\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}+$..... to 3 places of decimals.

37. $1+\frac{1}{4}+\frac{1}{4.6}+\frac{1}{4.6.8}+$.....to 3 ,, ,, ,,

38. $\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+$..... to 4 ,, ,, ,,

39. $\frac{3}{16}+\left(\frac{3}{16}\right)^2+\left(\frac{3}{16}\right)^3+$...to 5 ,, ,, ,,

40. $\frac{1}{3.5}+\frac{2}{3^2.5^3}+\frac{2}{3^3,5^5}+$.....to 7 ,, ,, ,,

Evaluate the following correct to two decimal places :—

41. $\frac{8\cdot785\times16\cdot403}{48\cdot349}$ 42. $\frac{\cdot000567\times26\cdot893}{\cdot4183}$

43. Find the value of $(1\cdot07)^{16}$ correct to four decimal places.

44. Find correct to 5 places of decimals the value of $\frac{1}{9}+\frac{1}{3.9^3}+\frac{1}{5.9^5}+\frac{1}{7.9^7}+\cdots$

45. Find the value of $\frac{1}{1.4}+\frac{1}{3.4^3}+\frac{1}{5\cdot4^5}+$ .....correct to four decimals.

CHAPTER X

§1. **Aliquot part.** An *Aliquot part* of a quantity is a fraction of it, having unity for its numerator *i. e.*, when one quantity is an exact part of another quantity it is called an *aliquot part*. Thus $8a. = \frac{1}{2}$ of a rupee and is therefore an aliquot part of a rupee. For the sake of convenience we give below a table of aliquot parts which are generally used in practice ;—

Table of Aliquot Parts.

Of a Rupee		
8*a*.	$=\frac{1}{2}$	Rupee
5*a*. 4*p*.	$=\frac{1}{3}$	,,
4*a*.	$=\frac{1}{4}$	,,
2*a*. 8*p*.	$=\frac{1}{6}$	,,
2*a*.	$=\frac{1}{8}$	,,
1*a*. 4*p*.	$=\frac{1}{12}$	,,
1*a*.	$=\frac{1}{16}$	,,
Of an anna		
6*p*.	$=\frac{1}{2}$	Anna
4*p*.	$=\frac{1}{3}$	,,
3*p*.	$=\frac{1}{4}$	,,
2*p*.	$=\frac{1}{6}$	,,
$1\frac{1}{2}p$.	$=\frac{1}{8}$	,,
1*p*.	$=\frac{1}{12}$	,,
Of a Maund		
20. sr.	$=\frac{1}{2}$	Maund
10 ,,	$=\frac{1}{4}$	,,
8 ,,	$=\frac{1}{5}$	,,
5 ,,	$=\frac{1}{8}$	,,
2 sr. 8 chk.	$=\frac{1}{16}$	,,
1 ,, 4 ,,	$=\frac{1}{32}$	,,
1 sr.	$=\frac{1}{40}$	,,
Of a Seer		
8 chk.	$=\frac{1}{2}$	Seer
4 ,,	$=\frac{1}{4}$	,,
2 ,,	$=\frac{1}{8}$	,,
1 ,,	$=\frac{1}{16}$	,,
Of a Pound		
10*s*.	$=\frac{1}{2}$	Pound
6*s*. 8*d*.	$=\frac{1}{3}$	,,
5*s*.	$=\frac{1}{4}$	,,
4*s*.	$=\frac{1}{5}$	,,
3*s*. 4*d*.	$=\frac{1}{6}$	,,
2*s*. 6*d*.	$=\frac{1}{8}$	,,
2*s*.	$=\frac{1}{10}$	,,
1*s*. 8*d*.	$=\frac{1}{12}$	,,
1*s*. 4*d*.	$=\frac{1}{15}$	,,
1*s*. 3*d*.	$=\frac{1}{16}$	,,
1*s*.	$=\frac{1}{20}$	,,
Of a Shilling		
6*d*.	$=\frac{1}{2}$	Shilling
4*d*.	$=\frac{1}{3}$	,,
3*d*.	$=\frac{1}{4}$	,,
2*d*.	$=\frac{1}{6}$	,,
$1\frac{1}{2}d$.	$=\frac{1}{8}$	,,
1*d*.	$=\frac{1}{12}$	,,

Of a Ton			Of a quarter		
10 cwt.	$= \frac{1}{2}$	Ton.	14 lbs.	$= \frac{1}{2}$	quarter.
5 ,,	$= \frac{1}{4}$	,,	7 ,,	$= \frac{1}{4}$	,,
4 ,,	$= \frac{1}{5}$	,,	4 ,,	$= \frac{1}{7}$	,,
2 ,, 2 qr.	$= \frac{1}{8}$	,,	$3\frac{1}{2}$,,	$= \frac{1}{8}$	,,
2 ,,	$= \frac{1}{10}$	,,	2 ,,	$= \frac{1}{14}$	,,
1 ,, 1 qr.	$= \frac{1}{16}$	,,	$1\frac{3}{4}$,,	$= \frac{1}{16}$	,,
1 ,,	$= \frac{1}{20}$	,,	1 lb.	$= \frac{1}{28}$	,,

Practice is a short method of calculating the cost of a given quantity by means of aliquot parts when the cost of a unit is given.

There are two kinds of *Practice*—Simple and Compound.

1. SIMPLE PRACTICE.

§2. The following examples will explain the rule for Simple Practice:—

Example 1. Find the price of 25 mds. of rice at Rs. 5 13*a*. 4*p*, per md.

Sol.

	Rs.	*a*.	*p*.	
	25	0	0	= price at Re. 1 per md.
			5	
	125	0	0	= ,, ,, Rs. 5 ,,
8*a*. $= \frac{1}{2}$ of Re. 1.	12	8	0	= ,, ,, 8*a*. ,,
5*a*. 4*p*. $= \frac{1}{3}$ of Re. 1.	8	5	4	= ,, ,, 5*a*. 4*p*. ,,
	146	13	4	**Ans.**

Note. It is some times very convenient to subtract an aliquot part (see Ex. 2.)

Example 2. Find the value of $121\frac{3}{4}$ articles at Rs. 5. 14*a*. 8*p*. per article.

Sol. Re. $\frac{3}{4}$ = 12*a*. and Rs. 5, 14*a*. 8*p*. = Rs. 6—1*a*. 4*p*.

The price of $121\frac{3}{4}$ articles at Re. 1 = Rs. 121, 12*a*.

	Rs.	*a*.	*p*.	
	121	12	0	= Price at Re. 1 per article.
			6	
	730	8	0	= ,, ,, Rs. 6 ,, ,,
1*a*. 4*p*. $= \frac{1}{12}$ of Re. 1 by subtraction	10	2	4	= ,, ,, 1*a*. 4*p*. ,, ,,
	720	5	8	= ,, ,, Rs. 5, 14*a*. 8*p*.

Example 3. Find the price of 2155 tables at £22, 13*s*. $4\frac{1}{2}$*d*. per 100.

Sol.

	£		
	21·55	= Price at £1 per 100 tables	
	22		
	4310		
	4310		
	474·10	= „ „ £22 „ „	„
10s. = $\frac{1}{2}$ of a £	10·775	= „ „ 10s. „ „	„
2s. = $\frac{1}{5}$ of 10s.	2·155	= „ „ 2s. „ „	„
1s. = $\frac{1}{2}$ of 2s.	1·0775	= „ „ 1s. „ „	„
3d. = $\frac{1}{4}$ of 1s.	·269375	= „ „ 3d. „ „	„
$1\frac{1}{2}d.$ = $\frac{1}{2}$ of 3d.	·1346875	= „ „ $1\frac{1}{2}d.$ „	„
	488·5115625		
	20		
	10·2312500		
	12		
	2·77500		

∴ the required price is £488 10*s.* 2·775*d.* **Ans.**

Example 4. Find the price of $124\frac{1}{3}$ articles at Rs. 21, 7*a.* 7*p.* per article.

Sol.

	Rs.	*a.*	*p.*			
	124	5	4	= price at Re. 1 per article.		
			7			
	870	5	4	= „	Rs. 7 „	„
			3			
	2611	0	0	= „	Rs. 21 „	„
7*a.* = $\frac{1}{16}$ of Rs. 7	54	6	4	= „	7*a.* „	„
7*p.* = $\frac{1}{16}$ of 7*a.*	4	8	$6\frac{1}{3}$	= „	7*p.* „	„
	2669	14	$10\frac{1}{3}$.	**Ans.**		

Note. The student should very carefully note the aliquot parts in such examples.

EXERCISE 65.

Find by practice, the price of the following articles :—

1. 32 at 2*a.* each.
2. 40 at 5*a.* 4*p.* each.
3. 50 at 6*a.* each.
4. 65 at 10*a.* each.
5. 75 at 13*a.* 4*p.* each.
6. 85 at 12*a.* each.
7. 121 at Rs. 2, 6*a.* each.
8. 124 at Rs. 4, 10*a.* each,

Use one aliquot part only (9—22) :—

9. 125 at Rs. 3, 3*a.* each.
10. 127 at Rs. 5, 5*a.* each.
11. 145 at Rs. 7, 7*a.* ,,
12. 123 at Rs. 9, 9*a.* ,,
13. 127 at Rs. 11, 11*a.* ,,
14. 128 at Rs. 13, 13*a.* ,,
15. 125 at Rs. 21, 7*a.* ,,
16. 137 at Rs. 27, 9*a.* ,,
17. 150 at Rs. 26, 13*a.* ,,
18. 145 at Rs. 45, 15*a.* ,,
19. 156 at Rs. 12, 14*a.* 8*p.* ,,
20. 147 at Rs. 7, 5*a.* 4*p.* ,,
21. 147 at Rs. 9, 13*a.* 4*p.* ,,
22. 121 at Rs. 3, 10*a.* 8*p.*,,
23. 165 at Rs. 8, 9*a.* 6*p.* ,,
24. 172 at Rs. 15, 11*a.*4*p.*,,

25. 160 at Rs. 9, 9*a.* 9*p.* each
26. 180 at Rs. 11, 11*a.* 11*p.* ,,
27. 192 at Rs. 28, 7*a.* 7*p.* ,,
28. 156 at Rs. 45, 9*a.* 9*p.* ,,

Use two aliquot parts only.

29. $232\frac{1}{2}$ at Rs. 25, 7*a.* 8*p.* ,,
30. $221\frac{1}{2}$ at Rs. 45, 12*a.* 8*p.* ,,
31. $321\frac{1}{3}$ at Rs. 16, 14*a.* 8*p.* ,,
32. $521\frac{1}{4}$ at Rs. 32, 11*a.* 8*p.* ,,
33. $416\frac{1}{2}$ at Rs. 11, 10*a.* 8*p.* ,,
34. $512\frac{1}{3}$ at Rs. 24, 13*a.* 8*p.* ,,
35. $320\frac{1}{2}$ at Rs. 27, 9*a.* 8*p.* ,,
36. $728\frac{1}{8}$ at Rs. 15, 7*a.* $8\frac{2}{5}$*p.* ,,
37. $728\frac{1}{4}$ at £53, 3*s.* 4*d.* ,, [53 = 10 × 5 + 3]
38. $325\frac{1}{3}$ at £59, 7*s.* 6*d.* ,, [59 = 10 × 6 − 1]
39. $751\frac{2}{3}$ at £87, 16*s.* 8*d.* ,, [87 = 17 × 5 + 2]
40. $975\frac{5}{6}$ at £107, 17*s.* 6*d.* ,, [107 = 12 × 9 − 1]
41. 1625 at £25, 16*s.* 6*d.* per hundred.
42. 4524 at £40, 12*s.* $7\frac{1}{2}$*d.* per hundred.
43. 175 at Rs. 62, 8*a.* per score.
44. Find the price of 127 pen-holders at $10\frac{1}{2}$*p.* each.
45. Find the price of 155 pencils at $4\frac{1}{2}$*p.* each.
46. A bankrupt's debts are Rs. 15926, 10*a.* 8*p.* and he can pay the creditors 7*a.* 4*p.* in a rupee. Find his assets.
47. A bankrupt's debts are Rs. 96728, 5*a.* 4*p.* and he can pay the creditors 13*a.* 4*p.* in a rupee. Find his assets.

II. COMPOUND PRACTICE.

§3. The following examples will illustrate the rule of Compound Practice :—

Example 1. Find the cost of 12 maunds 27 sr. 8 ch. at Rs. 5 10*a*. 6*p*. per md.

Sol.

	Rs.	*a*.	*p*.	
	5	10	6	=price of 1 md.
			12	
	67	14	0	= ,, ,, 12 ,,
20 sr. = $\frac{1}{2}$ of 1 md.	2	13	3	= ,, ,, 20 sr.
5 sr. = $\frac{1}{4}$ of 20 sr.	0	11	$3\frac{3}{4}$	= ,, ,, 5 ,,
$2\frac{1}{2}$ sr. = $\frac{1}{2}$ of 5 sr.	0	5	$7\frac{7}{8}$	= ,, ,, $2\frac{1}{2}$,,
	71	12	$2\frac{5}{8}$.	**Ans.**

Example 2. Find the cost of 12 tons 15 cwt. 2 qr. 21 lb. at £5 16*s*. 8*d*. per ton.

Sol.

	£.	*s*.	*d*.	
	5	16	8	=price of 1 ton.
			12	
	70	0	0	=price of 12 tons.
10 cwt. = $\frac{1}{2}$ of 1 ton.	2	18	4	= ,, ,, 10 cwt.
5 cwt. = $\frac{1}{2}$ of 10 cwt.	1	9	2	= ,, ,, 5 cwt.
2 qr. = $\frac{1}{10}$ of 5 cwt.	0	2	11	= ,, ,, 2 qr.
14 lb. = $\frac{1}{4}$ of 2 qr.	0	0	$8\frac{3}{4}$	= ,, ,, 14 lb.
7 lb. = $\frac{1}{2}$ of 14 lb.	0	0	$4\frac{3}{8}$	= ,, ,, 7 lb.
	74	11	$6\frac{1}{8}$.	**Ans.**

Example 3. Find the value of 41 bales of cotton each weighing 4 mds. 37 srs. 8 chks. at Rs. 13. 7*a*. 6*p*. per maund.

Sol. 4 mds. 37 sr. 8 chk. = 5 md. $-2\frac{1}{2}$ sr. ; $41 = 8 \times 5 + 1$.

	Rs.	*a*.	*p*.	
	13	7	6	=value of 1 md.
			5	
	67	5	6	= ,, ,, 5 mds.
$2\frac{1}{2}$ sr. = $\frac{1}{16}$ of a md.	0	13	$5\frac{5}{8}$	= ,, ,, $2\frac{1}{2}$ srs.
by subtraction =	66	8	$\frac{3}{8}$	= ,, ,, 1 bale.
			8	
	533	0	3	= ,, ,, 8 bales.
			5	
	2660	1	3	= ,, ,, 40 ,,
	66	8	$\frac{3}{8}$	= ,, ,, 1 bale.
	2726	9	$3\frac{3}{8}$	= ,, ,, 41 bales.

Example 4. Find the value of 880 articles at £3, 16*s* 8*d*. per hundred.

Sol.

	£.	*s*.	*d*.				
	3	16	8	= value of 100 articles.			
			8				
	20	13	4	= ,,	,,	800	,,
50 = ½ of 100	1	18	4	= ,,	,,	50	,,
25 = ½ of 50	0	19	2	= ,,	,,	25	,,
5 = ⅕ of 25	0	3	10	= ,,	,,	5	,,
	33	14	8	= ,,	,,	880	,,

EXERCISE 66.

Find, by Practice, the value of :—

1. 3 mds. 10 sr. at Rs. 3. 5*a*. 4*p*. per md.
2. 4 mds. 15 sr. at Rs. 5, 10*a*. 8*p*. per md.
3. 6 mds. 25 sr. at Rs. 4, 13*a*. 4*p*. per md.
4. 8 mds. 27 sr. 8 chk. at Rs. 5, 12*a*. 6*p*. per md.
5. 10 mds. 37 sr. 8 chk. at Rs. 8, 6*a*. 8*p*. per md.
6. 7 mds. 28 sr. 8 chk. at Rs. 6, 14*a*. 8*p*. per md.
7. 8 mds. 11 sr. 7 chk. at Rs. 6, 10*a*. 8*p*. per md.
8. 12 mds. 25 sr. 12 chk. at Rs. 2, 3*a*. 4*p*. per sr.
9. 8 mds. 17 sr. 8 chk. at Rs. 3, 9*a*. 6*p*. per sr.
10. 7 mds. 14 sr. 14 chk. at Rs. 5, 6*a*. 8*p*. per md.
11. 5 mds. 10 sr. 10 chk. at Rs. 9, 10*a*. 8*p*. per md.
12. 27 mds. 18 sr. 9 chk. at Rs. 12, 13*a*. 4*p*. per md.
13. 5 cwt. 2 qr. 14 lb. at Rs. 16, 10*a*. 8*p*. per cwt.
14. 7 cwt. 1 qr. 21 lb. at Rs. 12, 12*a*. 6*p*. per cwt.
15. 23 tons 15 cwt. 2 qr. 7 lb. at £3, 13*s*. 4*d*. per ton.
16. 5 tons 12 cwt. 2 qr. 16 lb. at £5, 16*s*. 8*d*. per ton.
17. 16 yds. 2 ft. 3 in. at Rs. 6, 9*a*. 8*p*. per yd.
18. 43 yds. 1 ft. 4 in. at Rs. 5, 13*a*. 4*p*. per yd.
19. 17 tolas 8 mashas 4 ratis at Rs. 22, 10*a*. 4*p*. per tola.
20. 27 tolas 9 mashas 5 ratis at Rs. 26, 13*a*. 4*p*. per tola.
21. 24 tolas 10 mashas 4 ratis at Rs. 25, 4*a*. 6*p*. per tola.

22. Find the rent of 586 acres 1 rood 31 sq. poles at £4, 1*s*. 10½*d*. per acre.

23. Find the value of 1 ton 5 cwt. 2 qr. 14 lb. at £3, 15*s*. 7*d*. per cwt.

24. Find the value of 5 acres 3 roods 7 sq. poles 5½ sq. yards of land at £161, 6*s*. 8*d*. per acre.

25. Find the cost of 7 miles 5 fur. 165 yds. at Rs. 682, 7*a*. 4*p*. per mile.

26. Find the rent of 156 acres 3 roods 24 sq. poles 11 sq. yards at Rs. 25, 3*a*. 4*p*. per acre.

27. Find the price of 15 bags of wheat weighing 2 mds. 27 sr. 8 chk. at Rs. 6, 10*a*. 8*p*. per md.

28. Calculate the price of 25 bags of sugar weighing 3 mds. 35 sr. 8 chk. each at Rs. 16, 13*a*. 4*p*. per md.

29. What is the cost of 40 bales of cotton weighing 4 mds. 27 sr. 8 chk. each at Rs. 25, 8*a*. per md. ?

30. Find the price of 35 boxes of tea at Rs. 9, 2*a*, 8*p*. per md., the weight of each box being 2 mds. 15 sr. 8 chk.

31. Find the price, in English coin, of 45 bags of sugar weighing 2 mds. 15 sr. 8 chk. each at Rs. 20, 10*a*. 8*p*. per md. (Re. 1=2*s*. 2*d*.)

32. If a man's debts amount to Rs. 15789, 4*a*., and he can pay only 12*a*. 3⅜*p*. for each rupee, how much do his creditors get ?

33. Find the price of 10 lbs. 11 oz. 16 dwts. 16 grs. of gold at £3, 17*s*. 10½*d*. per oz.

34. Find the value of 2 tons 15 cwt. 1 qr. 7 lb. at £13, 6*s*. 8*d*. per ton.

Find the value of:—

35. 5 tons 5 cwt. 2 qr. 17½ lb. at £3, 6*s*. 8*d*. per ton.

36. 2 tons 7 cwt. 3 qr. 11 lb. at £21, 12*s*. 6*d*. per cwt.

37. 1347 cwt. 3 qr. 21 lb. at £3, 17*s*. 10¼*d*. per cwt.

38. 1565 cwt. of coal at Rs. 125, 12*a*. 6*p*. per 100 cwt.

39. 7 tons 2 cwt. 2 qr. at Rs. 3, 2*a*. per maund, assuming that one ton is equal to 27¼ maunds.

40. Find the cost of a fence of length 5 miles 1104 yds. 2 ft. at £13, 15*s.* per mile.

41. Find the rent of 24 acres 3 roods 26 sq. poles at £3, 18*s.* 4*d.* per acre.

42. Find the cost of 5 tons 3 cwt. 3 qr. 27 lb. 12 oz. at £14 per cwt.

43. Find the rent for 3 months 3 weeks 4 days from January 1 at Rs. 106, 12*a.* per month.

44. Find to the nearest penny the value of 11 tons 17 cwt. 3 qr. 21 lbs. at £4, 17*s.* 6*d.* a ton. *(Most easily done by the decimal system.)*

INVOICE OR BILL.

§4. **An invoice** is a detailed description of the goods together with their price, quality and quantity supplied by a seller to the buyer.

If the payment is made then and there, the invoice is receipted, *i.e.*, the seller writers the words "Received payment" puts his signature and date. If the total amount of the bill is over Rs. 20, one anna stamp must be affixed. [See page 193].

Note Each separate amount in an invoice is called an item

§5. **Specimen of an account.**

LAHORE
June 4, 1944.

The Headmaster, Jain High School, Indore. Dr.
To the Mathematical House, Lahore.

Date	Description.	Rs.	A.	P.
April 13, 1944.	To goods as per invoice No. 231	338	6	0
May 15, 1944.	To goods as per invoice No. 352	56	7	3
May 29, 1944.	To goods as per invoice No. 402	36	3	6
	Total Rs.	431	0	9

§6. Specimen of an invoice or bill.

LAHORE.

No. 231. APRIL 13, 1944.

The Headmaster, Jain High School, Indore.

Bought of The Mathematical House.

Quantity	Description	Rate.	Rs.	A.	P.	Rs.	A.	P.
40	"Algebra Made Simple" by Ghosh and Dhawan ...	3/-/-	120	...	...			
30	"Arithmetic Made Easy" by Ghosh and Dhawan ...	3/-/-	90	...	...			
31	Standard Geometry by M. K. K. Pillay ...	3/-/-	93	...	...			
			303	...	...			
	Deduct discount at 15 p. c.		45	7	...	257	9	0
36	Selected Exercises in Geometry with solutions ...	1/4/-	45	0	0			
42	20 Model Papers in Matriculation Mathematics A with solutions by K. M. Ghosh, M. A. ...	1/8/-	63	0	0			
50	Ataliq Hissab by L. R. Dhawan ...	1/10/-	81	4	0			
			189	4	0			
	Deduct discount at 20 p.c.		37	13	6	151	6	6
			Total Rs.			408	15	6

E. & O. E.

(Sd.) SATYA PAL.

For *The Mathematical House*

Received Payment
S. P. Dhawan
April 13, 1944.

EXERCISE 67.

Make the following invoices, giving name, date etc. :—

1. 20 slates at 7*a.* per slate ; 50 boxes of Geometrical instruments at Re. 1, 2*a.* per box; 2 gross of pencils at 3*a.* 6*p.* a dozen ; 15 colour boxes at Re. 1, 7*a.* a box.

2. 40 copies of Revision Book on Arithmetic by L. R. Dhawan at Re. 1 per copy ; 30 copies of Domestic Arithmetic and Household Accounts Part I by L. R. Dhawan at Rs. 2, 4*a.* per copy ; 20 copies of "Algebra Made Simple" by Ghosh and Dhawan at Rs. 3 per copy. (Deduct discount at 15 p. c.)

3. 10 yards of linen at Re. 1, 3*a.* per yard ; 20 yards flannel at Rs. 5, 8*a.* per yard ; 28½ yards of velvet at Rs. 3, 12*a.* per yard ; 20 pairs of sarees at Rs. 7, 8*a.* per pair ; two dozen of towels at 9*a.* per towel.

4. 20 seers of sugar at 8*a.* 9*p.* per seer ; 1¾ maunds of rice at Rs. 5, 8*a.* per maund ; 15 seers of ghee at Rs. 2, 3*a.* per seer ; 21 lbs. of butter at 8¾*a.* per lb. ; 3¾ maunds of flour at Rs. 5, 7*a.* per maund.

5. 20 pairs of stocking at 8*a.* 9*p.* per stocking ; 30 handkerchiefs at 2*a.* 9*p.* per handkerchief ; 40 umbrellas at Re. 1, 6*a.* 6*p.* per umbrella ; 20 boxes of soap at 11*a.* 3*p.* per box ; 1½ dozen of gloves at Re. 1, 3*a.* 9*p.* per pair.

6. 10 lbs. of tea at 1*s.* 1½*d.* per lb., 15 lbs. of coffee at 1*s.* 2¾*d.* per lb. ; 30 lbs. of chicory at 1*s.* 9½*d.* per lb. ; 20 lbs. of sugar at 9¾*d.* per lb.

7. One gross of pen-holder at 4*a.* 6*p.* per dozen ; 300 envelopes at 12*a.* 3*p.* per hundred ; 23 exercise books at 2*a.* 9*p.* per copy ; 5 dozen quill pens at 2*a.* 6*p.* per dozen ; 4½ reams of foolscape paper at 6*a.* per quire ; ½ gross inkpots at 7*a.* 6*p.* per dozen.

8. 15½ yds. of linen at 14*a.* per yd. ; 21½ yds. of flannel at Rs. 2, 4*a.* per yd. ; 45 yds. of carpet at 5*a.* 4*p.* per yd. ; 3 pairs of socks at 3*a.* 6*p.* per pair ; 10½ yds. of sheeting at Re. 1, 10*a.* 8*p.* per yd.

9. 50 mds. of wheat at Rs. 5, 6*a.* per md. ; 40 mds. of gram at Rs. 6, 3*a.* per md. ; 75 mds. of rice at Rs. 4, 2*a.* per

md. ; 21½ mds. of sugar at Rs. 15, 10*a.* per md. ; 30½ mds. of coal at Rs. 3, 6*a* 6*p.* per md.

10. 21 lbs. of tea at 1*s.* 3*d.* per lb. ; 13 lbs. of coffee at 1*s.* 5*d.* per lb. ; 24½ lbs. of chicory at 1*s.* 2*d.* per lb. ; 54 lbs. of sugar at 9*d.* per lb., 17½ lbs. of green tea at 1*s.* 1*d.* per lb.

11. 20 copies of Arithmetic Made Easy by Ghosh and Dhawan at Rs. 3 per copy ; 24 copies of Algebra Made Simple by Ghosh and Dhawn at Rs. 3 per copy ; 32 copies of 20 Model Papers in Matriculation Mathematics A by Prof. Ghosh and Atma Ram at Re. 1 per copy ; 26 copies of Typical Questions in Arithmetic by L. R. Dhawn at 3*a.* per copy ; 27 copies of Typical Questions in Algebra by L. R. Dhawn at 3*a.* per copy ; 2 copies of Key to Arithmetic Made Easy at Rs. 2, 8*a.* per copy ; 25 copies of Standard Geometry revised by M. K. K. Pillay at Rs. 2, 8*a.* per copy. Deduct discount at 12½%.

12. 20 copies of Youngman's English Composition for middle classes by Malhotra and Katyal at Re. 1, 8*a.* per copy ; 25 copies of Youngman's English Composition for High Classes by Raj and Sharma at Rs. 2, 8*a.* per copy ; 30 copies of Selected Exercises in Geometry (solved) at Re. 1 per copy ; 25 copies of Revision Book on Arithmetic (solved) by L. R. Dhawan at Re. 1 per copy, 50 copies of Youngman's History of England Hindi Edition at Rs, 2, 4*a.* per copy ; 40 copies Urdu edition of the same at Rs. 2 per copy ; 30 copies of Youngman's Matriculation Geography in the form of questions and answers by Ram Lal Katyal, in Hindi at Rs. 2 per copy. 40 copies of the same Urdu edition at Rs. 2 per copy. The Mathematical House allows 15% discount on these books. Make out the bill and also the receipt of payment issued by the firm to the customer.

CHAPTER XI

INVOLUTION AND EVOLUTION

1. INVOLUTION

§1. **A power** of a number is the product which is obtained from successive multiplication by itself; the operation by which it is obtained is called **Involution.** Thus the 1st power of 2 is 2; the second power is $2\times2=4$; the third power is $2\times2\times2=8$ and so on. For the sake of convenience we denote these operations by the help of indices or small figures placed above the number a little to the right; thus, we may write as

$$2^1=2,\ 2^2=4,\ 2^3=8,\ \text{etc., etc.}$$

§2. The second and third powers of numbers are called their squares and cubes respectively.

§3. Study the following table of the numbers and their squares:

Numbers	1	2	3	4	5	6	7	8	9	10
Squares	1	4	9	16	25	36	49	64	81	100

Notice carefully that no square number ends in 2, 3, 7 or 8; hence it follows that *number which ends in* 2, 3, 7 *or* 8 *is not a perfect square.*

§4. **Abbreviated methods of finding the square of any number.**

Rule 1. *Split up the number in two parts and apply the algebraical formula* $(a+b)^2=a^2+b^2+2ab$.

Example 1. Find the square of 64.

Sol.
$$64^2=(60+4)^2$$
$$=3600+16+2\times60\times4$$
$$=4096. \quad \textbf{Ans.}$$

Rule 2. *Add and subtract the unit figure of the given number, to and from the number itself. Multiply the sum and difference thus obtained and then add the square of the unit's figure to the product.*

Example 2. Find the square of 64 by this rule.

Sol. $64^2=(64+4)(64-4)+(4)^2$.

$=68\times60+16$
$=4080+16=4096.$ **Ans.**

This method may also be proved by the algebraical formula

$\because a^2=a^2-b^2+b^2$
$\therefore a^2=(a+b)(a-b)+b^2.$

Rule 3. *If the number contains more than 2 digits, then more than 1 digit on the right of the number may be taken instead of unit's figure.* Thus

Example 3. Find the square of 526.

Sol. $526^2=(526+26)\ (526-26)+((26)^2$
$=552\times500+676$
$=276000+676=276676.$ **Ans.**

Rule 4. *If the number ends in figure 5, then neglect the five and multiply the remaining figure by the next higher figure and affix 25 to the right.* Thus

Example 4. Find the square of 65 and 135.

Sol. (*i*) $65^2=6\times7$ with 25 affixed$=4225.$ **Ans.**

(*ii*) $135^2=13\times14$ with 25 affixed$=18225.$ **Ans.**

Rule 5. *If the number ends in 25, then neglect the twenty-five and multiply the remaining figure by the remaining figure with a 5 to its right, and affix 625 to the right* Thus

Example 5. Find the squares of 525 and 1625.

Sol. (*i*) $525^2=5\times55$ with 625 affixed$=275625.$ **Ans.**

(*ii*) $1625^2=16\times165$ with 625 affixed$=2640625.$ **Ans.**

EXERCISE 68.

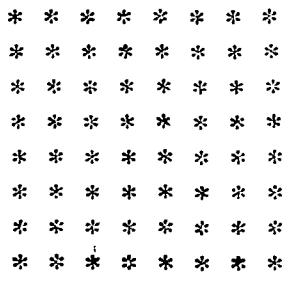
Write down the squares of the following numbers :—

1. 44	2. 57.	3. 36.	4. 48.
5. 49.	6. 46.	7. 59.	8. 88.
9. 53.	10. 47.	11. 99.	12. 56.
13. 128.	14. 156.	15. 166.	16. 106.
17. 65	18. 75.	19. 145.	20. 155.
21. 165.	22. 205	23. 245.	24. 305.
25. 625.	26. 929.	27 1325.	28. 2125.

§15. A Solid Square and a Hollow Square.

A Solid Square is one in which the number of rows is equal to the number of men in each row. In diagram (1) there are 8 rows, each containing 8 men. The total number of men is 8^2 or 64.

(1)



(2)

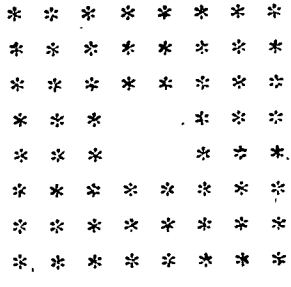


Diagram (2) is **of a hollow square three deep** having 8 men in the front rank. It is three deep because the number of complete rows counted from any front is three. The number of men in this hollow square $=8^2-(8-2\times3)^2=60$

Example. Find the least number of soldiers which can be drawn up in a hollow square 6, 8, 12 and 15 deep and also in a solid square.

Sol. We have to find the least number which is exactly divisible by 6×4, 8×4, 12×4, 15×4.

The L. C. M. of 24, 32, 48, 60$=480=16\times30$

$\therefore$ least square number$=16\times30\times30=14400$ **Ans.**

II. EVOLUTION

§6. **Evolution** is the inverse process of involution. Whereas involution is the process of raising a number to any power so evolution is the process of extracting a required root of a given number. Thus

$5^2=5\times5=25$. Here 25 is the square of 5 and

5 is the second root or square root of 25, also

$5^3=5\times5\times5=125$. Here 125 is the cube of 5 and

5 is the third root of 125. Similarly

5 is the fourth root of $5\times5\times5\times5$ or 625 and so on.

§7: Square Root.

The square root of a given number is that number which when multiplied by itself will produce the given number. Thus 5 is the square root of 25 because 5 multiplied by 5 produces the given number 25.

§8. The square root of a given number is indicated by the symbol $\surd$ placed before it. It is sometimes denoted by placing the fraction $\frac{1}{2}$ above the number towards the right. Thus the square root of 25 is denoted either by $\surd 25$ or $(25)^{\frac{1}{2}}$

§9. Square root of 9 is 3 but the square root of 3 cannot be accurately determined. We may only write an approximate answer in decimals. Such an incomplete root is called **surd root** and the one of which the root is accurately determined is called **a rational root.** Thus the square root of 9 is 3; 3 is therefore a rational root; but the square root of 3 cannot be accurately ascertained; therefore $\surd 3$ is a surd root.

§10. **To ascertain the number of digits in the square root of a given whole number.**

Since the square root of 1 is 1;
" " " " " 100 is 10;
" " " " " 10000 is 100 &c. &c.

It follows at once that the square root of a number consisting of less than 3 digits will consist of only one digit, that of a number of 3 and 4 digits will consist of 2 digits and so on. Or if a dot (·) be placed over every alternate digit beginning with the digit in the unit's place of a number, *i, e.*, separate the digits of the given number into periods of two, the number of gots thus placed, *i. e.*, the number of periods thus marked will show the number of digits in the square root of the number. Thus 8972 will be divided into two periods and 48972 into three, *e. g.*, $89\dot{7}\dot{2}$ and $4\dot{8}9\dot{7}\dot{2}$. In the first case 72 is the first period and 89 the second. In the second case 72 is the first period, 89 the second and 4 the third. It may be noted that if the given number consists of an odd number of digits the last period will be of one figure only.

§11. To Find the square root of a number by factors.

Example 1. Extract the sruare root of 11025.

Sol. $\sqrt{11025}=7\times7\times5\times5\times3\times3$

$=7^2\times5^2\times3^2$

$\therefore$ the square root $=7\times5\times3=105$. **Ans.**

Note. It is not necessary to break the given number into prime factors ; we conveniently break it into equal pairs of factors.

EXERCISE 69.

Extract the square root of :—

1. $4\times4\times5\times5$.
2. $7\times7\times3\times3$.
3. $11\times11\times3\times3\times2\times2$.
4. $13\times13\times5\times5\times2\times2$.
5. 1225.
6. 2025.
7. 9216.
8. 17424.
9. 19600.
10. 20736.
11. 63504.
12. 81796.

13. Find the *least* numbers, which when multiplied by the following numbers, will make the products perfect squares— 75 ; 180 ; 1456 ; 750750.

14. What is the least square number which is divisible by 4, 8, 15, 108 or 125 ?

---

§12 To find the square root of a whole number.

When the factors of a given number are not conveniently traceable we use the method corresponding to the algebraical formula $\sqrt{a^2+2ab+b^2}=a+b$.

Before we explain how to extract the square root of a given number let us analyse the operation by finding the squares of $a+b$ and $20+5$ when 20 stands for a and 5 for b.

$a+b$	$20+5$	Or	400
$a+b$	$20+5$		200
a^2+ab	$400+20\times5$		25
$+ab+b^2$	$+20\times5+25$		625
$a^2+2ab+b^2$	$400+2\times20\times5+25$		
	$=400+5(2\times20+5)$		

Now reverse the operation and find the square root of 625. First mark off the number into periods as explained in Art. 10. As there are two periods in the number the square root will consist of two digits. We also find that the second period is

less than 9 and greater than 4. Therefore the left-hand side digit in the square root will be 2 and the square root will be greater than 20 and less than 30. We shall now proceed thus:—

20	$\dot{6}\dot{2}\dot{5}$	(20+5
	400	
40	225	
	200	
	25	

Exp. We have subtracted the square of 20 from 625 and have obtained remainder 225. Now doubling the 20, we get 40. Divide the remainder 225 by 40. It goes 5 times, leaving the remainder 25 which is the square of 5. We conclude therefore that in order to make up the remainder 225 we have to add 5 to 40 and then multiply the sum by 5. We shall write the process thus :

(I)
$$\begin{array}{r} 20)\dot{6}\dot{2}\dot{5}(20+5=25 \\ 400 \\ \hline 40+5=45)225 \\ 225 \\ \hline \end{array}$$

(2) Or briefly thus

2	$\dot{6}\dot{2}\dot{5}$	(25
	4	
45	225	
	225	

Note. In the 2nd process zeroes have been omitted.

The above process has been modelled on that of algebra. For the sake of reference we write the process below:—

a	$a^2+2ab+b^2$	$(a+b$
	a^2	
$2a+b$	$2ab+b^2$	
	$2ab+b^2$	

§3. A Graphical Explanation.

We have read in algebra that the difference of the squares of two numbers is equal to the product of the sum and difference of those numbers ; as

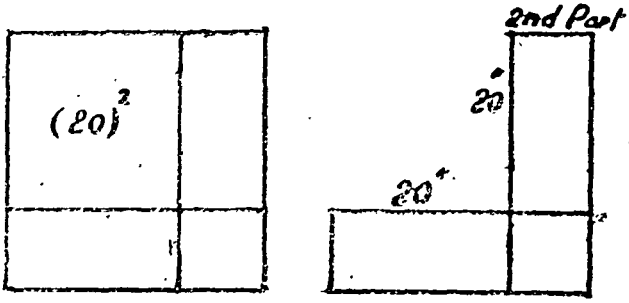
$$\begin{aligned}(27)^2-(20)^2&=(27+20)(27-20)\\&=47\times 7\\&=(\text{twice } 20+7)+7.\end{aligned}$$

We conclude therefore that if from the square of a number we subtract the square of a part of it, the remainder = (twice that part + the second part) × the second part.

With the help of this result let us find the square root of 729. It will be divided into two periods, the first period being 29 and the second 7 ; 7 is greater than 4 and less than

9 : therefore the left hand digit in the square root must be 2, and the square root will be between 20 and 30. Let us now draw a square supposing its area to be equal to 729 square inches

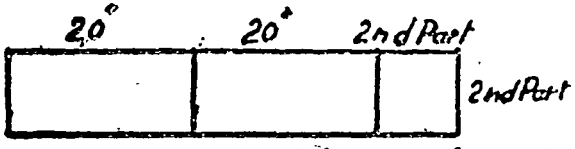
Fig. I. Fig. II.



Cut off a square of 20 inches from it, the remaining figure as shown in Fig. II is equal to 729—400, *i.e.*, 329 square inches in area.

Figure II can be put like the following rectangle, the area of which is equal to

(Twice the first part + the second part) × the second part = 329 square inches.



Now we have to find out the second part so that by adding it to twice 20 and multiplying the sum by it we may get 329.

Twice 20 = 40 ; when we divide 329 by 40 it goes 8 times : adding 8 to 40 and multiplying the sum by 8 we get the figure in the unit's place 4 instead of 9, we therefore reject this number and try another. Let us try 7, adding 7 to 40 we get 47 and multiplying 47 by 7, we get 329. Therefore the second part is 7 and the side of the square is 20 + 7, *i.e.*, 27 inches.

20	$\dot{7}2\dot{9}$ (20+7	2	$\dot{7}2\dot{9}$ (27
	400		4
20×2=40	329	47	329
40+7=47	329		329

From the above explanations we can extract the following

Rule. *Mark off the given number into periods as explained in Art. 10. Subtract the square of the greatest*

possible number (putting the number as the first figure of the root) from the first period at the left. Bring down the remainder and the next period. Divide the first two or three digits of the dividend thus formed by twice the part of the root already obtained (called trial divisor). Place the quotient as the second digit in the root and also to the right of the trial divisor. Now multiply the newly formed divisor by the second digit of the root and put it as the subtrahend. Continue the process till the periods are exhausted.

Example 2. Extract the square root of 1452025.

Sol.	1	$\dot{1}45\dot{2}0\dot{2}5$ (1205. **Ans.**
		1
	22	45
		44
	2405	12025
		12025

Expl. Mark off the periods; there being 4 periods we find that the square root will consist of 4 digits. First period at the left is 1 and the square root of it is 1; subtracting 1 from 1, the remainder is zero. Bring down the next period 45 as the next dividend. The trial divisor is 1×2 or 2 and the first figure in the trial dividend is 4. The quotient will be 2. Place this 2 as the second digit of the root, and annex it to the trial divisor as well making it 22. Multiply 22 by 2 and put the product 44 as subtrahend and then subtract it from 45. The remainder is 1. Bring down the next period 20. Now the trial divisor 12×2, *i. e.*, 24 cannot divide the first two digits 12 in the trial dividend 120; so the next digit in the root is zero. Place this zero in the root and also at the end of the divisor. Bring down the next period 25 and now we get 5 as the last digit of the required root.

Example 3. Find the least number which must be subtracted from or added to 89023 to make it a perfect square.

Sol.	2	$\dot{8}9\dot{0}2\dot{3}$ (298
		4
	49	490
		441
	588	4923
		4704
		219

Hence, (*i*) 219 is the least number to be subtracted from and (*ii*) $(299)^2 - 89023 = 378$ is the least number to be added to 89023 to make it a perfect square.

EXERCISE 70.

Find the square root of:—

1. 25600.	2. 28900.	3. 44100.
4. 50625.	5. 73441.	6. 85849.
7. 839056.	8. 819025.	9. 978121.
10. 2298256.	11. 4515625.	12. 9641025.
13. 80496784.	14. 72471169.	15. 81162081.
16. 244328161.	17. 234733041.	18. 464661136.

19. 26422502500 20. 850058184196.

21. 2250075000625. 22. 49000350000625.

23. A certain number of men subscribed as many pies each as there were subscribers, the whole subscription being Rs. 16, 5*a*. 4*p*. How many subscribers were there ?

24. A number of persons subscribed as many pice each as there were persons. The total subscription is Rs. 206, 10*a*. 3*p*.; find the number of persons.

25. The subscription to a certain fund amounted to Rs. 976. 9*a*., and each person subscribed as many annas as there were subscribers altogether. Find the number of subscribers.

26. A General arranged his men numbering 53824 into a solid square. How many men were there in the front ?

27. A General wishing to arrange his men numbering 276674 into a solid square found that there were two men less, How many men were there in the front ?

28. A General wishing to draw up his 251132 men into a solid square found that he had 131 men over ; find the number of men in the front.

29. Find the least numbers which must be subtracted from 8972 and added to 12728 to make them perfect squares.

30. Find the number whose third part multiplied by its seventh part gives 756.

31. Find the least number of men in a regiment which can be drawn up in a hollow square 5, 8, 10 or 12 deep and also into a solid square.

§14. **To find the Square Root of a Decimal Fraction.**

$\because$ ·1 × ·1 = ·01 $\therefore$ $\sqrt{\cdot 01}$ = ·1.

$\because$ ·01 × ·01 = ·0001 $\therefore$ $\sqrt{\cdot 0001}$ = ·01.

$\because$ ·001 × ·001 = ·000001 $\therefore$ $\sqrt{\cdot 000001}$ = ·001

From the above examples we note the following points :—

(1) If any decimal fraction is squared there is always an even number of decimal places in the result. Consequently a decimal fraction must contain an *even* number of decimal places if it is a perfect square *i. e.*, if its square root can be exactly found. If the number does not contain an even number of digits a zero may be annexed.

(2) The pointing must begin from the place of *units* towards the right-hand over every alternate figure as before and the number of such points will be the same as the number of decimal places in the square root.

(3) The number of decimal places in the square root is always half the number of decimal places in the given number.

(4) The square root is always greater than the given number.

§15. We shall deduce the rule for extracting the square root of a decimal fraction from the process of the following simple examples :—

Example 1. Find the square root of 23.

Note. It is clear that the square root of 23 cannot be accurately determined. An approximate answer in decimal may be given. We shall therefore put the decimal point and annex some pairs of zeroes and proceed as before,

Sol.

4	23·000000	4·795... **Ans.**
	16	
87	700	
	609	
949	9100	
	8541	
9585	55900	
	47925	
	7975	

The process of extracting the square root of 23·000000 is

really the process of extracting square root of number consisting partly of a whole number and partly of a decimal.

Example 2. Find the square root of ·0023562.

Note. The number of decimal places in the given number is not even, in order to make them even we annex a zero.

Thus ·0023562=·00235620.

Sol.

0	·00̇23̇56̇20̇	(·0485...**Ans.**
4	16	
88	756	
	704	
965	5220	
	4825	
	395	

Rule. *Make the number of decimal places even by affixing a zero if necessary. Place a dot over the unit's figure and then over every alternate figure to its right and left to get periods of two and then proceed as in the case of a whole number. The number of periods in the integral part of the given number will show the number of digits in the integral part of the root and the number of periods in the decimal part will show the number of decimal places in the root.*

Example 3. The product of two numbers is 105·625 and their quotient is 2·5 ; find the numbers.

Sol. ∵ Product÷Quotient=(smaller No.)^2

$\therefore$ smaller No. $=\sqrt{105{\cdot}625 \div 2{\cdot}5}$

$=\sqrt{1056{\cdot}25 \div 25}$

$=\sqrt{42{\cdot}25}$

$=6{\cdot}5$

and the greater No. $=105{\cdot}625 \div 6{\cdot}5$

$=1056{\cdot}25 \div 65$

$=16{\cdot}25$

The required numbers are 16·25 and 6·5. **Ans.**

Example 4. The products obtained by multiplying together each pair of three given numbers are 8, 11·25, 14·4 respectively ; find the numbers.

Sol. Product of the three numbers $=\sqrt{8\times 11{\cdot}25\times 14{\cdot}4}$

$=\sqrt{8\times 162}$

$=\sqrt{4\times 2\times 2\times 9\times 9}$

$=36.$

∴ Nos. are (*i*) $36 \div 8 = 4{\cdot}5$
(*ii*) $36 \div 11{\cdot}25 = 3{\cdot}2$
(*iii*) $36 \div 14{\cdot}4 = 2{\cdot}5$ } Ans.

EXERCISE 71.

Find the square root of :—

1.	·09	2.	·0064	3.	·0081
4.	7·29	5.	16·81	6.	34·81
7.	9·3025	8.	84·8241	9.	37·0881
10.	150·0625	11.	477·4225	12.	225·6004
13.	52·649536	14.	·015625	15.	·813604
16.	251953·8025	17.	227·798649.		

Find to four decimal places the square root of :—

18.	·1.	19.	·00001.	20.	·005.
21.	15·013.	22.	121·3654.	23.	256·1034.

Find to five decimal places the square root of :—

24.	11.	25.	19.	26.	31.	27.	101.
28.	501.	29.	605.	30.	117.		

Find the square root of :—

31. ·005329. 32. ·00053361. 33. ·00000049112064.

34. 40000·400001.

35. The product of two numbers is ·078125 and their quotient is 5 ; find the numbers.

36. The products obtained by multiplying together each pair of three given numbers are 7·14, 11·76, 19·04 respectively ; find the numbers.

---

§16 **To find the square root of a vulgar fraction.**

Observe the following examples :—

(*i*) $\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$

$\sqrt{2\frac{7}{9}} = \sqrt{\frac{25}{9}} = \frac{\sqrt{25}}{\sqrt{9}} = \frac{5}{3} = 1\frac{2}{3}$

(*iii*) $\sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{\sqrt{9}} = \frac{1{\cdot}414}{3} = {\cdot}471\ldots$

(*iv*) $\sqrt{\frac{2}{7}} = \sqrt{{\cdot}428571} = {\cdot}654\ldots\ldots$

Or $\sqrt{\frac{3}{7}} = \frac{\sqrt{3\times7}}{\sqrt{7\times7}} = \frac{\sqrt{21}}{\sqrt{49}} = \frac{5{\cdot}582}{7} = {\cdot}654\ldots\ldots$

(*v*) $\sqrt{\frac{8}{9}} = \frac{\sqrt{8\times4}}{\sqrt{9\times4}} = \frac{\sqrt{32}}{\sqrt{36}} = \frac{5{\cdot}656}{6} = {\cdot}942\ldots\ldots$

The above examples give us the following rule :—

Rule. *(i) If the numerator and the denominator of a fraction and of a mixed number when reduced to an improper fraction be perfect squares, then take the square root of the numerator for a new numerator and the square root of the denominator for a new denominator. (Exs. i, ii.)*

(ii) If only the denominator of a fraction be a perfect square, the square root in that case is obtained by dividing the square root of the numerator by the square root of the denominator. (Ex. iii.)

(iii) If the numerator and the denominator be both imperfect squares, then convert the fraction into a decimal fraction and extract its square root. (Ex. iv.)

Or *Multiply both the numerator and the denominator by such a number as to make the denominator a perfect square and then proceed as in. (Exs. iv, v.)*

Example. Extract the square root of $\frac{3{\cdot}75}{8{\cdot}75}$

Sol. $\frac{3{\cdot}75}{8{\cdot}75} = \frac{375}{875} = \frac{3}{7}$ $\therefore$ $\sqrt{\frac{3{\cdot}75}{8{\cdot}75}} = \sqrt{\frac{3}{7}} = {\cdot}654$. **Ans.**

EXERCISE 72.

Find the square root of :—

1. $\frac{64}{81}$.
2. $\frac{121}{144}$.
3. $\frac{225}{361}$.
4. $\frac{256}{625}$.
5. $6\frac{1}{4}$.
6. $7\frac{1}{9}$.
7. $5\frac{1}{16}$.
8. $5\frac{19}{25}$.
9. $34\frac{15}{49}$.
10. $84\frac{37}{121}$.
11. $71\frac{2}{49}$.
12. $29\frac{33}{49}$.
13. $10\frac{151}{225}$.
14. $21\frac{51}{169}$.
15. $\frac{1{\cdot}44}{1{\cdot}69}$.
16. $\frac{16{\cdot}9}{62{\cdot}5}$

Find to three decimal places, the square root of :—

17. $\frac{5}{16}$. 18. $\frac{12}{25}$. 19. $\frac{7}{36}$. 20. $\frac{15}{49}$.

21. $2\frac{1}{9}$. 22. $5\frac{3}{4}$. 23. $3\frac{5}{16}$. 24. $5\frac{1}{4}$.

25. $\frac{5}{6}$. 26. $\frac{7}{8}$. 27. $\frac{5}{12}$. 28. $2\frac{1}{12}$.

29. $287\frac{5}{8}$. 30. $367\frac{2}{7}$. 31. $\frac{6{\cdot}05}{8{\cdot}47}$. 32. $\frac{316{\cdot}8}{10{\cdot}08}$.

33. The product of two numbers is 50 and their quotient is $1\frac{1}{8}$. Find the numbers.

34. The products obtained by multiplying each pair of the three given numbers are $7\frac{1}{12}$, $12\frac{1}{18}$, $16\frac{2}{24}$, respectively; find the numbers.

---

§17. **To find the square root of recurring decimals.**

If a recurring decimal be a *perfect square*, it is convenient to reduce it to a vulgar fraction and proceed as in Art. 12 (*i*); thus, $\sqrt{2{\cdot}\dot{7}}=\sqrt{2\frac{7}{9}}=\sqrt{\frac{25}{9}}=1{\cdot}\dot{6}$.

§18. But if the recurring decimal be *not a perfect square* we may extend the recurring part by repeating its period and then proceed as in decimal questions.

Thus to extract the square root of $4{\cdot}2\dot{7}\dot{3}$ to 3 places of decimals, we can extend the recurring part 73 as long as to enable us to get 3 decimal places in the square root, *e. g.*, we may write $4{\cdot}2\dot{7}\dot{3}$ as 4·273737...and then proceed as in Art. 11.

EXERCISE 73.

Find the square root of :—

1. $\cdot\dot{0}2\dot{7}$. 2. $28\dot{4}$. 3. $4738{\cdot}02\dot{7}$.

4. $\cdot13\dot{4}$. 5. $\cdot\dot{0}49382716\dot{}$. 6. $2{\cdot}3\dot{6}1\dot{}$.

7. $\cdot\dot{0}\dot{7}$. 8. $4{\cdot}02981\dot{}$. 9. $\cdot00562\dot{}$.

Cube Root

§19. The **cube root** of a number is that number which when multiplied by itself three times will produce the given number. Thus 4 is the cube root of 64, since 64 is the cube of 4.

§20. The cube root of a number is indicated by the symbol $\sqrt[3]{}$ placed before it. It is sometimes denoted by the sombol $\frac{1}{3}$ placed above the number towards the right. Thus teh cube root of 64 is denoted by $\sqrt[3]{64}$ or $(64^{\frac{1}{3}})$.

§21. A number is said to be a **perfect cube** when its cube root can be exactly found *e. g.*, 125 is a perfect cube but not 135 since there is no number, integral or fractional, whose cube is exactly 135.

§22. Study the following table of the natural numbers and their cubes :—

Numbers...	1	2	3	4	5	6	7	8	9
Cubes ...	1	8	27	64	125	216	343	512	729

Notice carefully that a cube may end with any digit even or odd. It is not so in the case of a square number.

It may also be noted that when a cube ends in, 1, 8, 7, 4, 5, 6, 3, 2, 9, its cube root ends in 1, 2, 3, 4, 5, 6, 7, 8, 9 respectively.

§23. **To ascertain the number of digits in the cube root of a given integral number.**

Since the cube root of 1 is 1 ;
the cube root of 1000 is 10 ;
the cube root of 1000000 is 100 ; etc., etc.

It follows at once that cube root of a number lying between 1 and 1000 must be between 1 and 10, *i. e.*, must be a number of one digit. Similarly the cube root of a number lying between 1000 and 1000000 must lie between 10 and 100, *i.e.*, must be a number of two digits, and so on. Hence it follows that if a dot (.) be placed over the unit's figure in the number and thence over every third digit to the left, the number of dots will evidently be the number of digits in the cube root of the number. This rule may also be applied to decimals.

§24. **To find the cube root of a given number.**

Rule. *Place a dot over the unit's figure in the given number and then over every third figure to its left and also to its right if the number contains any decimal portion; adding zeroes if necessary to get periods of three digits. It may be noted that each period shall consist of three figures, except the first which may contain 1, 2 or 3 figures.*

Now find the number whose cube is either equal to, or next less than the first period on the left hand and write it as the first figure of the required root. Subtract its cube from the first period and write down the remainder with the next period following it.

Multiply the square of the figure in the root just obtained by 300 and take the product as a **trial** *divisor. Then find how many times this divisor is contained in the dividend,—the quotient thus obtained will be the* **next** *figure in the cube root. Then, multiply this quotient by the product of previous figure in the root by 30 and place this result below the trial divisor. Below these, write the square of this last quotient and the sum of all the three will be regarded as a* **complete** *divisor. Multiply this complete divisor by the second figure of the root and subtract. Write down the remainder with the next period following it, which will form the next dividend.*

Multiply the square of the root already obtained by 300 and take tte product as a trial divisor and then find as before, how many times this trial divisor is contained in the dividend,—the quotient thus obtained will be the next, i. e., the third figure in the root. Then multiply the figures of the cube root already obtained by 30 and the product by the last quotient and place this product below the trial divisor. Below these, write the square of the last quotient and add all the three together—the sum being taken as a complete divisor. Multiply this divisor by the last figure in the root and subtract. Write down the remainder with the next period following it, which will form the next dividend. Proceed in this way till all the periods are exhausted.

Note. If at any stage of the process dividend is less than the trial divisor put a zero in the root, two zeroes to the trial divisor and bring down the next period.

Example 1. Find the cube root of 46656.

Sol. 46656 (36 **Ans.**
27

$3^2 \times 300 = 2700$ |	19656
$3 \times 30 \times 6 = 540$ |	
$6^2 = 36$ |	
3276	19656

Exp. Here, first divide into periods beginning with 6; the first period on the left contains only 2 digits *viz*, 46. The trial divisor 2700 goes into the dividend 19656, 6 times.

Example 2. Find the cube root of 64481201.

Sol. $\dot{6}4\dot{4}81\dot{2}0\dot{1}$ (40·1 **Ans.**

64

	481201
$40^2 \times 300 = 480000$	
$40 \times 30 \times 1 = 1200$	
$1^2 = 1$	
481201	481201

Exp. The first trial divisor 4800 does not go into the dividend 481 and hence a zero is put after 4 in the root.

§25. If the given number be not a perfect cube, its cube root may be obtained to any number of decimal places by placing zeroes and bringing down periods of three zeroes each.

Example 3. Find the cube root of ·3 to 3 decimal places.

Sol. $\cdot 300\dot{0}00\dot{0}00$ (·669 **Ans.**

216

	84000
$6^2 \times 300 = 10800$	
$6 \times 30 \times 6 = 1080$	
$6^2 = 36$	
11916	71496
$\times 300 = 1306800$	12504000
$30 \times 9 = 17820$	
$9^2 = 81$	11922309
1324701	581691

Exp. Since the root is to be found to 3 decimal places there must be 3 periods of 3 figures each in the decimal part, hence we affix 8 zeroes to 3.

§26. When one more than a half of the digits in the required root is obtained by the ordinary method, the rest can be found by contracted division as explained in Art. 7 Chapter IX.

§27. **Cube root of a fraction.**

If we are to find the cube root of a fraction, the denominator of which is a *perfect cube*, we may find the cube roots of the numerator and denominator separately; as in

(*i*) $\sqrt[3]{\frac{64}{729}} = \frac{\sqrt[3]{64}}{\sqrt[3]{729}} = \frac{4}{9}$ **Ans.**

(*ii*) $\sqrt[3]{\frac{29}{125}} = \frac{\sqrt[3]{29}}{\sqrt[3]{125}} = \frac{3{\cdot}0723\ldots\ldots}{5} = {\cdot}6144$ ..... **Ans.**

But if the denominator be not a *perfect cube*, then we may either reduce the given fraction to an equivalent fraction whose denominator is a perfect cube and then proceed as in previous case or we may convert the given fraction to a decimal and proceed in the ordinary way as in

$$\sqrt[3]{3\tfrac{4}{7}}=\sqrt[3]{\tfrac{25}{7}}=\sqrt[3]{\tfrac{25}{7}\times\tfrac{49}{49}}$$

$$=\sqrt[3]{\frac{1225}{343}}=\frac{\sqrt[3]{1225}}{\sqrt[3]{343}}=\frac{10{\cdot}6}{7}=1{\cdot}5\ldots\textbf{Ans.}$$

Or $\sqrt[3]{3\frac{4}{7}}=\sqrt[3]{3{\cdot}571428}=1{\cdot}5\ldots$ **Ans.**

EXERCISE 74.

Find the cube root of:—

1.	1331.	2.	21952.	3.	2048383.
4.	19034163.	5.	105823817.	6.	702121283072
7.	12812·904.	8.	103·823	9.	·000512.
10.	·000012326391.	11.	·876467493.	12.	$\frac{5}{7}\frac{1}{2}\frac{2}{9}$.
13.	$465\frac{31}{64}$.	14.	$\frac{132{\cdot}651}{64}$	15.	18·609625.
16.	·00103030$\dot{1}$.	17.	·0$\dot{3}\dot{7}$.	18.	1587·9$\dot{6}\dot{2}$.

Find the cube root of the following up to 3 places of decimals :—

19.	$\frac{5}{7}$.	20.	$18\frac{7}{12}$.	21.	·003.
22.	$\frac{1}{10}$.	23.	7·52.	24.	$5\frac{4}{9}$.

25. Find the cube root of 137174210836762689026063l.

---

§28. **Extraction of some other roots.**

The **Fourth** root of a number is found by finding the square root of the sqare root of the number, *e.g.*,

$\sqrt[4]{81}=\sqrt{9}=3$. **Ans.**

§29. The **Sixth** root of a number is found either by findidg the cube root of the square root of the number or by finding the square root of the cube root of the number, *e.g.*,

$\sqrt[6]{4096}=\sqrt[3]{64}=4$; **or** $\sqrt[6]{4096}=\sqrt{16}=4$. **Ans.**

§30. The **Eighth** root of a number is found by finding the square root of the square root of the square root of the number; *i.e.*, the process of finding the square root is to be done thrice, *e.g.*

$\sqrt[8]{256}=\sqrt{\sqrt{\sqrt{16}}}=\sqrt{\sqrt{4}}=\sqrt{2}=2$. **Ans.**

§31. **The Ninth** root of a number is found by finding the cube root of the cube root of the number, *e.g.*,

$$\sqrt[9]{19683}=\sqrt[3]{\sqrt[3]{27}}=\sqrt[3]{3}=3. \quad \textbf{Ans.}$$

EXERCISE 75.

Find the fourth root of :—

1. 1679616.
2. 1575·2961.

Find the sixth root of :—

3. 531441.
4. 308·915776.

Find the eighth root of :—

5. 214358881.
6. 429981696.

Find the ninth root of :—

7. 1953125.
8. 134217728.

CHAPTER XII.

METRIC SYSTEM AND DECIMAL COINAGE.

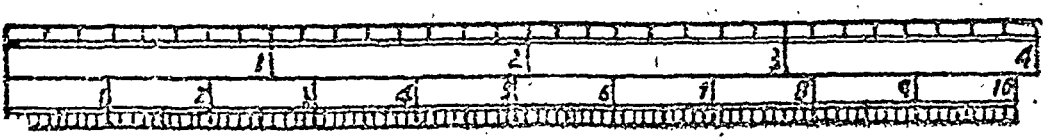
§1. A Board of Mathematicians recommended a new system of weight and measure called the **Metric System** after the **French Revolution** of 1789. It was first originated in France and then was introduced in almost all the countries of Europe. This system, the fundamental unit of which is a **Metre**, whence it derives its name, proceeds upon the principle of decimal division. It is, therefore, being an easy calculation, largely used in scientific measurements in all parts of the world.

§2. The unit of length is **Metre** which is equal to 39·3707 inches nearly.

TABLE.

10 millimetres (mm.)	= 1	centimetre (cm.)
10 centimetres	= 1	decimetre (dm.)
10 decimetres	= 1	Metre (m.)
10 metres	= 1	decametre (Dm.)
10 decametres	= 1	hectometre (Hm.)
10 hectometres	= 1	kilometre (Km.)
10 kilometres	= 1	myriametre (Mm.)

4 inches.



$\frac{1}{10}$ m. or 1 dm. or 10 cm. or 100 mm.

The root words in the Metric System are derived from the Greek and Latin.

Greek		Latin	
Deca means	10 times.	Deci means	one-tenth of.
Hecto ,,	100 ,,	Centi ,,	one-hundreth of.
Kilo ,,	1000 ,,	Milli ,,	one-thousandth of.
Myria ,,	10000 ,,		

The following diagram will best illustrate the above Metric table:—

Diagram.

Myriametre.	Kilometre.	Hectometre.	Decametre.	**Metre.**	Decimetre.	Centimetre.	Millimetre.

It will be seen that the value to the left of **Metre** increases tenfold and that to the right decreases tenfold.

Note. Long distances are generally quoted in Kilometres : 1 kilometre=$\frac{5}{8}$ English mile nearly.

§3. **Reduction.** It is clear from the above diagram that problems of reduction may be solved by mutliplying or dividing by 10 as in the decimal system of measurement.

The following examples will best illustrate the method of work:—

Example 1. Express 9·05 metres in millimetres.

Sol. 9·05 metres=90·5 decimetres
=905 centimetres
=9050 millimetres. **Ans.**

Example 2. Express 5915·5 millimetres in metres.

Sol. 5915·5 millimetres=591·55 centimetres.
=59·155 decimetres.
=5·9155 metres.

Evidently this answer may be read as
5 m. 9 dm. 1 cm. 5·5 mm. **Ans.**

Example 3. Express 8972·2 metres in kilometres,

Sol. 8972·2 metres=897·22 decametres
=89·722 hectometres
=8·9722 kilometres.
=8 Km. 9 Hm. 7 Dm. 2 m. 2 dm. **Ans.**

Example 4. Read as quickly as possible 25·1653894 myriametres into kilometres, hectometres, etc.

25·1653894 Mm.=25 Mm. 1 Km. 6 Hm. 5 Dm. 3 m. 8 dm. 9 cm. 4 mm. **Ans.**

Example 5. Read as quickly as possible 893756 millimetres into centimetres, etc.

893756 mm.=8 Hm. 9 Dm. 3 m. 7 dm. 5 cm, 6 mm. **Ans.**

Example 6. Multiply 2·26 metres by ·05 and read the product in centimetres, etc.

Sol.

$$\begin{array}{r} 2{\cdot}26 \\ \cdot 05 \\ \hline \cdot 1130 \end{array}$$

The product= ·1130 metres.

=1 dm. 1 cm. 3 mm. **Ans.**

Example 7. Divide 1262·5 kilometres by 32 and read the quotient in kilometres, hectometres, etc.

Sol.

$$\begin{array}{rr} 2) & 1262{\cdot}5 \\ \hline 16) & 631{\cdot}25 \\ \hline \end{array}$$

The quotient= 39·453125 Km.

=39 Km. 4 Hm. 5 Dm. 3 m. 1 dm. 2 cm. 5 mm. **Ans.**

EXERCISE 76.

Reduce :—

1. 179 millimetres to decimetres, etc.
2. 1215 centimetres to decametres, etc.
3. 1679 centimetres to metres, etc.
4. ·15 decametres to millimetres.
5. 15·5 kilometres to millimetres.

Read the numbers in the following :—

6. 5·075 metres.
7. 9·7018 kilometres.
8. 515·4 centimetres.
9. 8·972 metres.

Simplify (10—12) and express the answer in metres, etc. :—

10. 2125 millimetres+5123 centimetres+1215 decimetres +21 metres.

11. 156·035 metres—·149002 kilometres.

12. ·176005 kilometres—142·056 metres.

13. Multiply 1·15 metres by ·003 and express the result in millimetres.

14. Multiply 13·15 decimetres by 2·5 and express the product in metres, etc.

15. Divide 1501·308 kilometres by 9 and express the quotient in millimetres.

16. Divide 151300 millimetres by 1250 and express the quotient in metres.

17. The circumference of a wheel is 48 dms. and it makes $2\frac{1}{2}$ revolutions per second. How long will it take to travel 48 miles?

---

§4. **Surface.** The unit of surface is the *square metre.* In measuring land, the unit used is a *square decametre.* This is called an **are.**

TABLE

10 centiares	make 1 deciare	= 11·96033	sq. yds.
10 deciares	,, 1 are	= 119·6033	sq. yds.
10 ares	,, 1 dekare	= 1196·033	sq. yds.
10 dekares	,, 1 hectare	= 11960·33	sq. yds.
		= $2\frac{1}{2}$	Eng. acres nearly.

Note. Large surfaces are generally quoted in hectares.

Example. 16506 centiares = 1650·6 deciares
= 165·06 ares
= 16·506 dekares
= 1·6506 hectares
= 1 hectare, 5 dekares, 5 ares, 5 centiares. **Ans.**

§5. **Volume.** The unit of volume is the *cubic metre.* In measuring wood this is called a **stere.**

TABLE

10 decisteres	make 1 stere	= 35·317 cub. ft.
10 steres	,, 1 dekastere	= 353·17 cub. ft.

Example 1685 decisteres = 168·5 steres.
= 16·85 dekasteres.
= 16 dekasteres, 8 steres, 5 decisteres. **Ans.**

§6. **Capacity.** The unit of capacity for liquid and dry goods is the **Litre.** It is equal to a cubic decimetre.

TABLE

10 centilitres (cl.)	make 1 decilitre (dl.)	= 1·714704 chataks
10 decilitres	,, 1 **Litre** (lit.)	= 1·07169 srs.

10 litres make 1 dekalitre (Dl.)=10·7169 srs.
10 dekalitres ,, 1 hectolitre (Hl.)=2·679 mds.
10 hectolitres ,, 1 kilolitre (Kl.)=26·79 mds.

Note. 1 litre=1¾ English pint nearly.

Example. 15016·5 litres=1501·65 decalitres (Dl.)
=150·165 hectolitres (Hl.)
=15·0165 kilolitres (Kl.)
=15 Kl. 1 Dl. 5 lit. 5 Dl. Ans.

§7. **Weight.** The unit of weight is the *gramme* which is the weight of a cubic centimetre of distilled water at a certain fixed temperature. The weight of a *litre* of such water is 1000 grammes, or one **kilogram**.

TABLE

10 milligrams (mg.) make 1 centigram (cg.)
10 centigrams ,, 1 decigram (dg.)
10 decigrams ,, 1 **gram** (gr.)=1·028823 mashas.
10 grams ,, 1 decagram (Dg.)=·8573527 tolas
10 decagrams ,, 1 hectogram (Hg.)=8·573527 ,,
10 hectograms ,, 1 kilogram (kg.)=1·07169 seers.
10 kilograms ,, 1 myriagram (mg.)=10·7169 ,,
10 myriagrams ,, 1 quintal.
10 quintals ,, 1 millier.

§8. **To sum up.** We now give below the principal measures in the metric system with their approximate English equivalents and *vice versa*.

(i) **Measures of Length.**

1 metre =39·3708 inches=1·0936 yds.
1 kilometre =·6214 mile =1094 yds.

---

1 inch = 2·54 cm. | 1 yard =·9144 metres.
1 foot = 3·05 dm. | 1 mile =1·61 kilometres.

Hence, a **kilometre** is nearly 5 furlongs or $\frac{5}{8}$ of a mile and a metre is nearly $1\frac{1}{11}$ yards.

(ii) **Measures of Area.**

1 sq. metre = 10·7643 sq. ft.
= 1·1961 sq. yds.

and

1 sq. inch = 6·4515 sq. cm.
1 acre = ·4047 hectares.
Hence an **hectare** is a little less than $2\frac{1}{2}$ acres.

(*iii*) **Measures of Capacity.**

1 litre = ·0353 cu. ft. = 1·7608 pints.
1 cu. metre = 220·0967 gallons.
and
1 cu. inch = 16·3852 cu. cm.
1 pint = ·9465 litres
1 gal. = 4·5435 litres.
Hence a **litre** contains a little more than $1\frac{3}{4}$ pints.

(*iv*) **Measures of Weight.**

1 gram = 15·43 grains.
and
1 grain = ·0648 grams.
Hence a gram is nearly $15\frac{1}{2}$ grains.

§9. **Money.** The unit of money is the Franc. It is about $9\frac{3}{4}d.$ English.

TABLE

10 centimes (c.) Make 1 decime.
10 decimes ,, 1 franc (fr.)

Accounts are kept in francs and centimes only: thus "20·56 francs" is read as 20 fr. and 56c. The franc is a silver coin composed of 9 parts of silver and one part of copper and weighs 5 grams. The *Napoleon* is a gold coin = 20 francs.

§10. **The Advantages of metric system.**

The great advantages of this system, as we observe from the above tables, are that (*i*) a compound quantity can be reduced to a single quantity and *vice versa* without any process (*ii*) that the relative size of the different units are very easily understood.

Note. In the examples given below, the specific gravity of a substance means the ratio which the weight of any volume of the substance bears to that of an equal volume of water.

EXERCISE 77.

1. Find the weight of 37 hectolitres of water.

2. How many fields each containing 2 hectares, 47 ares can be made from a farm of 313 hectares, 69 ares?

3. Find the weight of 20 cu. cm. of mercury, whose specific gravity = 13·5.

4. If a man walks 11 metres in 5 secs., how many kilometres can he walk in an hour.

5. Find the weight in kilos of a bar of gold 10 cm. × 30 mm. × 25 mm., its specific gravity being 19·36.

6. A gentleman's income is reduced from 56085 fr. 50 c. to 52720 fr. 37 c. by income-tax (1) How much tax does he pay in a franc? (2) How much tax does he pay in the £, taking £1 = 25 francs?

7. After paying an income-tax of 15 c. in a franc, a gentleman's income is 365318 fr. 10 c. Express the income-tax paid by him as a fraction of his net income.

8. If the specific gravity of alcohol be ·8, how many kilograms of alcohol will fill one-third of a tank whose measurements are 1·50 metres by 3·20 metres by 80 cm.?

9. If the circumference of a wheel be 1 dekam. 25 decim. and if it makes $2\frac{1}{2}$ revolutions in 1 second, how long will it take to travel 50 miles, taking 1 mile = $1\frac{3}{5}$ kilom.

10. If a sq. metre = 1550·031 sq. inches, find to five decimal places the number of sq. cm. in a sq. inch.

CHAPTER XIII

MEASUREMENT OF AREA.

Note: Some questions in this chapter are solved by the method of atic andalligation. They may be omitted till these methods are learnt.

1. Observe the following figures :—

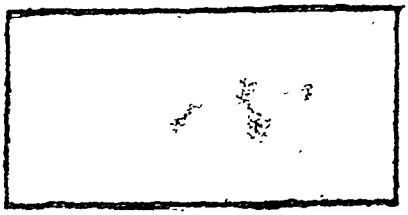


Fig. I.

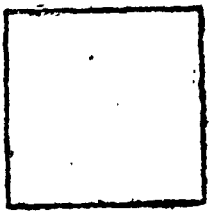


Fig. II.

Figure I is a **rectangle** and figure II is a **square.**

Their surfaces are bounded by four straight lines making every angle a right angle. The oppsite sides of the rectangle are equal while the square has all its sides equal.

The **Perimeter** of a rectangle or a square is the sum of all its sides. Hence the

Perimeter of a rectangle=2×(Length+breadth) and
Perimeter of a square =4×side

Note. The length and breadth of a rectangle are called its dimensions.

2. Observe the following two more figures :

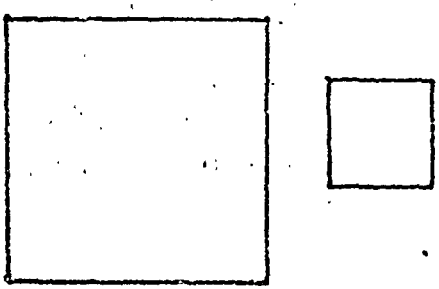


Fig. I Fig. II.

Each side in figure I is 1 inch and each side in figure II, 1 centimetre. The first figure, therefore, represents one square inch and the second figure one square centimetre. Thus a square the side of which measures 1 foot represents one square foot and so on.

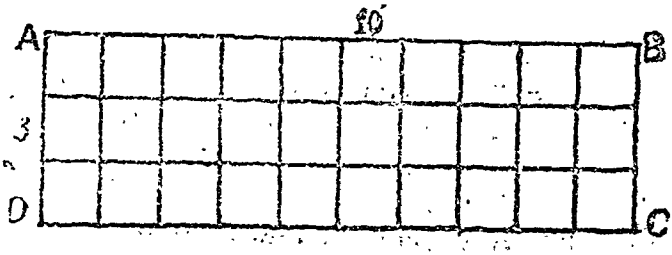
The area of figure means the amount of surface enclosed by its bounding lines in square measures.

§3. **To find the area of a rectangle.**

The following examples will illustrate the rule of finding the area of a rectangle.

Example 1. Find the area of a rectangle whose length is 10 inches and breadth 3 inches.

Let $ABCD$ be a rectangle of which the length AB is 10 inches and the breadth AD is 3 inches. Divide AB and AD



into 10 and 3 equal parts respectively and through the points of division draw lines parallel to AD and AB respectively. The rectangle has thus been divided into 30 equal squares each having one square inch surface. Hence the area of the rectangle is 30 square inches. Note that $10 \times 3 = 30$.

Learn, therefore the following formulae :—

Area of a rectangle = Length × Breadth.
Whence Length = area ÷ breadth.
and Breadth = area ÷ length.

§4. **To find the area of a square.**

A square, is also, as you have noticed by the figure, a kind of rectangle. The only difference is that it has all its sides equal.

Hence the area of a square = (side)^2
Whence the side of a square = $\sqrt{\text{area}}$

Example 2. Find the area of a square whose side is 3 ft.

Sol. $\because$ area of a square = (side)^2

$\therefore$ the reqd. area $= 3 \times 3 = 9$ sq. ft. **Ans.**

Note. "9 square feet" denote an area 9 times as large as a square foot. But if we write "9 feet square" it will denote the area of a square whose side is 9 ft.

Example 3. The length of a room is 15 ft. 6 inches and width 12 ft. 6 in. Find its area.

Sol. area = length × breadth.

$= 15\frac{1}{2} \times 12\frac{1}{2}$.

$= \frac{31}{2} \times \frac{25}{2} = \frac{775}{4}$ sq. ft.

$= 193$ sq. ft. 108 sq. in. **Ans.**

Example 4. The area of a room is 372 sq. ft. ; the breadth is 15 ft. 6 in., find the length.

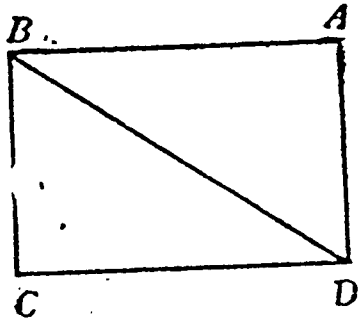
Sol. Length = area ÷ breadth
$= 372 \div 15\frac{1}{2}$
$= 372 \times \frac{2}{31} = 24$ ft. **Ans.**

Example 5. The area of the floor of a roon is 36 sq. yds, 6 sq. ft. 112 sq in. ; the breadth is 19 ft. 1 in. ; find its length.

Sol. Length = area ÷ breadth
= 36 sq. yds. 6 sq. ft. 112 sq. in. ÷ 19 ft. 1 in.
= 330 sq. ft. 112 sq. in. ÷ 19 ft. 1 in.
$= \frac{2977}{9}$ sq. ft. $\div \frac{229}{12}$ ft.
$= \frac{2977}{9} \times \frac{12}{229}$ ft. $= 5\frac{2}{3}$ ft. = 17 ft. 4 in. **Ans.**

§5. Diagonal of a rectangle and a square.

(*i*) $ABCD$ is a rectangle and BD is the diagonal.



Now BCD is a right-angled $\triangle$

$\therefore BD = \sqrt{(BC)^2 + (CD)^2}$

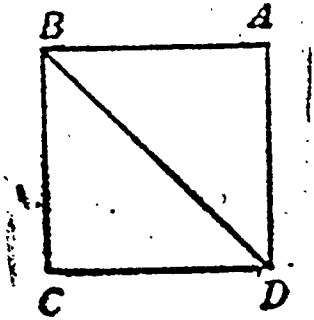
If $BC = 5$ ft. and $CD = 12$ ft.

Then $BD = \sqrt{(5)^2 + (12)^2}$

$= \sqrt{25 + 144} = \sqrt{169}$

$= 13$ ft. **Ans.**

(*ii*) $ABCD$ is a square and BD is the diagonal.



Now BCD is a right-angled $\triangle$

$\therefore BD = \sqrt{(BC)^2 + (CD)^2}$.

If $BC = CD = 5$ ft.

Then $BD = \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25 + 25} = \sqrt{50}$

$= \sqrt{25 \times 2} = 5\sqrt{2}$

$= BC\sqrt{2}$.

Since we see that the area of the square $= 5 \times 5 = 25$ sq. ft. and the diagonal $= \sqrt{25 \times 2}$ $\therefore$ we conclude that

(i) Diagonal $= \sqrt{2}$ area or side $\times \sqrt{2}$
(ii) Area = half the square of its diagonal.

Example 6. A square recreation ground containing 33 acres 3765 sq. yards 4 square feet is to be enclosed with barbed wire placed at heights 2, 4, 6 feet above the ground. Find the length of the wire if its length required for each round is $3\frac{1}{3}\%$ greater than the perimeter of the ground.

Sol. Area = 33 acres 3765 sq. yds. 4 sq. ft.
= 1471369 sq. ft

$\therefore$ perimeter $= \sqrt{1471369} \times 4$ ft.
$= 1213 \times 4 = 4852$ ft.

$\because$ the wire is to make 3 rounds and the length of each round $= 4852 \times \frac{310}{300}$ ft.

$\therefore$ The reqd. length $= 4852 \times \frac{310}{300} \times 3$ ft.
$= 15041{\cdot}2$ ft. **Ans.**

EXERCISE 78.

Find the area of the rectangles having the following dimensions :—

1. 18. ft. by 10 ft.
2. 12 ft. 8 in. by 10 ft. 6 in.
3. 16 ft. 6 in. by 8 ft. 8 in.

Find the area of the following in sq. yards, sq. feet and sq. inches :—

4. A room 20 ft. 8 in. long, 12 ft. 6 in. broad.
5. A courtyard 9 ft. 1 in. long, 17 ft. 4 in. broad.
6. A courtyard 45 ft. 6 inches long, 28 ft. 8 inches broad.
7. Find the area of a rectangular field 440 yards long and 220 yards broad in acres.
8. A square field is 550 yards long, find its area in acres.
9. How many acres of land are there in a field 550 yards long, 220 yards broad ?
10. How many sq. yards of matting will be required for a rectangular room 45 ft. long, 18 ft. 4 in. broad ?
11. Find the breadth of a room, the length of which is 35 ft. and the area is 46 sq. yards 6 sq. ft.
12. A room contains 48 sq. yards 6 sq. ft. What will be its length when the breadth is 18 ft. ?

13. A rectangular field contains 2 acres. What will be its length when the breadth is 22 yards ?

14. A square field contains 10 acres ; find its side.

15. A room is 20 metres in length and 10 metres in breadth. Find the number of square yards in the area of the floor, taking a metre as equal to 39·37 inches.

16. The length of a field containing 20 acres is twice its breadth. Find the length of the field.

17. What is the area of a square whose diagonal is 21 ft. ?

18. The area of a square field is 162 sq. ft. Find its diagonal.

19. How long will a man take to run round the boundary of a sq. field containing 250 acres at the rate of 3 miles per hour ?

20. What length of wire will be required to enclose a square garden containing 62·5 acres placed at hights 1, 2, 3, 4 ft. above the ground, each circuit being 4% longer than the perimeter of the garden ?

---

§6. Carpeting or paving rooms, etc.

The area of the carpet required to cover the floor of a room or the area of stones or bricks required for paving the floor of a room or courtyard is evidently equal to the area of its floor.

$$\therefore \text{length of the carpet} = \frac{\text{area of the floor}}{\text{breadth of the carpet}}$$

$$\text{and breadth of the carpet} = \frac{\text{area of the floor}}{\text{length of the carpet}}$$

$$\text{No. of bricks or stones} = \frac{\text{area of the floor}}{\text{area of one brick or stone}}$$

Example 1. Find the cost of carpeting the floor of a room 35 ft. long and 20 ft. broad at 2*a*. per sq. ft.

Sol. Area of the floor $= 35 \times 20 = 700$ sq. ft.

$\because$ area of the carpet = area of the floor

$\therefore$ " " " " $= 700$ sq. ft.

$\therefore$ cost " " " $= \text{Rs. } \frac{1}{8} \times 700$

$= \text{Rs. } 87, 8a.$ **Ans.**

Example 2. Find the cost of carpeting a room 30 ft. long and 24 ft. broad with carpet 18 inches wide at 2*a*. 6*p*. per yard.

Sol. Area of the carpet $=30\times24=720$ sq. ft.
$\therefore$ length of the carpet $=720\div\frac{18}{12}=480$ ft. $=160$ yds.
$\therefore$ cost of the carpet $=\text{Rs. }\frac{5}{32}\times160$
$=\text{Rs. }25.$ **Ans.**

Example 3. How many bricks 9 in. by 4 in. will be required for paving a room 36 ft. by 21 ft. ?

Sol. Area of the floor $=36\times21=756$ sq. ft.
and area of one brick $=9\times4=36$ sq. in.
$=\frac{1}{4}$ sq. ft.
$\therefore$ No. of bricks reqd. $=\dfrac{\text{area of the floor}}{\text{area of the brick}}$
$=756\div\frac{1}{4}=3024.$ **Ans.**

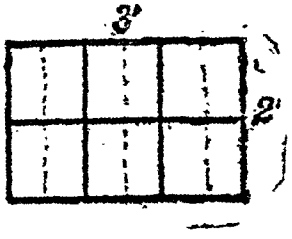
Example 4. A room 26 ft. 3 in. long and 23 ft. 9 in. broad is to be paved with equal square tiles ; find the largest size of each tile and the number of tiles required.

Sol. 26 ft. 3 in. $=315$ in.
23 ft. 9 in. $=285$ in.
G. C. M. of 315 and 285 $=15$.
$\therefore$ the size of the tile is 15 in. square
and No. of tiles $=\dfrac{315\times285}{15\times15}=399.$ } **Ans.**

Example 5. The length and breadth of a room are as 3 : 2, and the cost of matting it at 5*a*. per sq. yard is Rs. 7, 8*a*. Find its dimensions.

Sol. Re. $\frac{5}{16}$ is the cost of 1 sq. yard ;
$\therefore$ Re. 1 is the cost of $\frac{16}{5}$ sq. yards ;
$\therefore$ Rs. $\frac{15}{2}$ is the cost of $\frac{16}{5}\times\frac{15}{2}=24$ sq. yards.

Now the area of the floor of the room is 24 sq. yards, or 216 sq. ft. Consider a rectangle with dimensions 3 ft. and 2 ft. It will contain 6 squares each of 1 sq. ft.



$\therefore$ the area of one square $=216\div6$
$=36$ sq. ft.
$\therefore$ the side of the square $=6$ ft.
$\therefore$ the length of the room $=6\times3=18$ ft.
and the breadth $=6\times2=12$ ft. } **Ans.**

Example 6. The cost of carpeting a room is Rs. 187, 8*a*. Had the width of the room been 3 ft. less the cost of the carpet would have been Rs. 150 only. Find the width of the room.

Sol. Diff. between the costs = Rs. $187\frac{1}{2}$ — Rs. 150 = Rs. $37\frac{1}{2}$.

i. e., Rs. $37\frac{1}{2}$ is the cost of a carpet of a room 3 ft. wide.

$\therefore$ Re. 1 ,, ,, ,, ,, ,, $3\times\frac{2}{75}$ ft.

$\therefore$ Re. $187\frac{1}{2}$,, ,, ,, ,, ,, $3\times\frac{2}{75}\times\frac{375}{2}$ ft.

$\therefore$ width reqd. $=3\times\frac{2}{75}\times\frac{375}{2}$ ft. $=15$ ft. **Ans.**

Example 7. A room is 20 ft. long and 18 ft. wide. Had its length been 5 ft. more and width 2 ft. more the cost of the carpet would have increased by Rs. 4, 6*a*. Find the cost of the carpet.

Sol. Area of the floor $=20\times18=360$ sq. ft.

Area in the 2nd case $=(20+5)(18+2)$ sq. ft.

$=500$ sq. ft.

Diff. in the areas $=500-360=140$ sq. ft.

The cost of carpet 140 sq. ft. in area = Rs. $4\frac{3}{8}$.

$\therefore$,, ,, 1 ,, ,, $=$ Rs. $\frac{35}{8}\times\frac{1}{140}$

$\therefore$,, ,, 360 ,, ,, $=\frac{35}{8}\times\frac{1}{140}\times360$

$=$ Rs. 11, 4*a*. **Ans.**

Example 8. The length of a room is 8 ft. more than its breadth and its perimeser is 80 ft. Find the cost of carpeting it at 8*a*. 6*p*. per sq. yard.

Sol. Perimeter $=$ (width + width + 8 ft.) $\times 2 = 80$ ft.

$\therefore$ 4 width + 16 ft. $= 80$ ft.

$\therefore$ 4 width $= 64$ ft.

$\therefore$ width $= 16$ ft.

and length $= 16+8 = 24$ ft.

Now solve the question further.

Example 9. The sum of the length and breadth of a room is 27 ft. and the area is 180 sq. ft. Find its dimensions.

Sol. Consider any room whose dimensions are known to you and verify the following formulae.

$$(\text{length}+\text{breadth})^2-(\text{area}\times4)=(\text{length}-\text{breadth})^2$$

Hence $(27^2-180\times4)=(\text{length}-\text{breadth})^2$

$\therefore$ $729-720 = (\text{length}-\text{breadth})^2$

$\therefore$ Diff. of the sides $=\sqrt{9}=3$ ft.

but sum of the sides $=27$ ft.

$\therefore$ length $=(27+3)\div2=15$ ft.
and breadth $=(27-3)\div2=12$ ft. } **Ans.**

EXERCISE 79.

Find the length of carpets required for the floor of :—

1. A room 15 ft. long, 12 ft. broad ; carpet 2 ft. wide.
2. A room 21 ft. long, 18 ft. broad ; carpet 3 ft. wide.
3. A room 18½ ft. long, 13½ ft. broad ; carpet 27 in. wide.
4. A room 25½ ft. long, 16½ ft. broad ; carpet 44 in. wide.
5. How many pieces of carpet each 6 ft. long, 3 ft. wide will cover the floor of a room 21 ft. by 12 ft. ?
6. How many pieces of carpet each 10 ft. long, 27 in. wide will cover the floor of a room 16 ft. 8 in. by 13 ft. 6 in. ?
7. How many paving stones each 2 ft. long 1½ ft. wide would be required to pave a rectangular courtyard 45 ft. by 25 ft. ?
8. How many bricks each 9 in. long and 6 in. wide would be required to pave the floor of a room 18 ft. 6 in. by 13 ft. 6 in. ?
9. How many paving stones each 1½ feet long, 9 in. wide would be required to pave a square courtyard whose side is 30 ft. ?
10. Find the cost of carpeting a room 15 ft. 4 in. by 14 ft. 3 in. at 4*a*. per square foot.
11. Find the cost of matting a room 24 ft. 9 in. by 16 ft. at 2*a*. per sq. yard.
12. What will be the expenses of paving a courtyard 18½ ft. by 14½ ft. at 3*a*. per sq. yard ?
13. Find the cost of carpeting a room 18 ft. 8 in. long, 16 ft. 6 in. broad with carpet 2 ft. 4 in. wide at 8*a*. per yard.
14. Find the expenses of carpeting the floor of a room 21 ft. 4 in. by 14 ft. 3 in. with carpet 2 ft. wide at 12*a*. per yard.
15. What will be the expenses of paving a courtyard 50 ft. by 33 ft. with paving stones 1½ ft. by 6 in. at Rs. 16 per hundred ?
16. A hall is 48 ft. long and the cost of carpeting it at 5*a*. 4*p*. per square yard amounts to Rs. 53, 5*a*. 4*p*. Find its breadth.
17. A hall is 45 ft. long and the expenses of carpeting it. at 8*a*. per square yard amount to Rs. 100. Find its breadth.

18. The cost of paving and varnishing the floor of a square room at 2*s*. 6*d*. per square ft. is £78, 2*s*. 6*d*. find its side.

19. The length of a room is three times its breadth and the cost of matting it at 2*a*. 3*p*. per square yard is Rs. 6, 12*a*. Find its dimensions.

20. The length and breadth of a room are as 3 : 2 and the cost of carpeting it at 2*a*. 6*p*. per square yard is Rs. 23, 7*a*. Find its dimensions.

21. The length and breadth of a room are in the ratio of 5 : 4 and the cost of carpeting it at 2*a*. 3*p*. per sq. yard is Rs. 31, 4*a*. Find its dimensions.

22. A room is 25 ft. by 21 ft. and the cost of carpeting it at 4*a*. per yard is Rs. 21. Find the width of the carpet.

23. A room is 24 ft. by 16 ft. and the cost of carpeting it with carpet 18 inches wide is Rs. 24. Find the cost of the carpet per yard.

24. A courtyard is 40 feet long and the cost of carpeting it at Rs. 18 per 100 square yards is Rs. 20. Find the cost of fencing it round at 12*a*. per yard.

25. The expenses of carpeting a square courtyard at 1*a*. per square ft. are Rs. 64. Find the cost of fencing it round at 4*a*. per ft.

26. A room 38 ft. 9 in. by 30 ft. is to be paved with equal square tiles, find the largest tile which will exactly fit and also find the number of tiles required.

27. A courtyard 37 ft. 6 in. by 31 ft. 6 in. is to be paved with equal square tiles; find the largest tile which will exactly fit and also find the number of tiles required.

28. The area of a rectangular field whose breadth is 500 yards is 100 acres. Find the cost of cultivating it at Rs. 3, 2*a*. 8*p*. per 100 sq. yard and also the cost of fencing it round at Rs. 2, 8*a*. per yard.

29. The length of a rectangular field is twice its breadth. If the rent of the field at £3, 7*s*. 6*d*. an acre be £151, 17*s*. 6*d*. find the cost of surrounding it with a fence at $4\frac{3}{4}$*d*. per yard.

30. A rectangular courtyard the sides of which are as 5 : 11 costs Rs. 144, 6*a*. for paving at 10*a*. 6*p*. per square yard. Find the length of its sides.

31. The cost of carpeting a room is Rs. 106, 10*a*. 8*p*. If the width of the room had been 4 ft. less, the cost of carpeting would have decreased by Rs. 26, 10*a*. 8*p*. ; find the width of the room.

32. A room is 16 ft. long and 12 ft. broad. Had its length and breadth been 2 ft. and 1 ft. more respectively the cost of carpeting would have increased by Rs. 5, 4*a*. Find the cost of carpeting it.

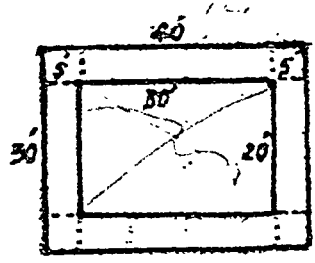
33. A room is $16\frac{1}{2}$ ft. long and $12\frac{1}{2}$ ft. broad. Had the length been $3\frac{1}{2}$ ft. more and breadth $1\frac{1}{2}$ ft. less, the cost of carpeting it would have increased by Rs. 3, 7*a*. Find the cost of carpeting it.

34. The length of a room is 10 ft. more than its breadth and its perimetre is 100 ft. find the cost of carpeting it at 7*a*. 6*p*. per sq. yard.

35. The sum of the length and breadth of a room is 41 ft. and its area is 400 sq. ft. ; find its dimensions.

Verandah round a room and path round a garden.

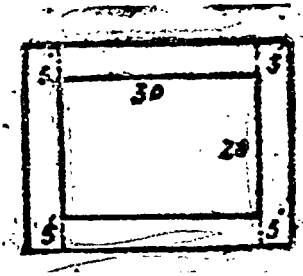
Example 1. A room 30 ft. long, 20 ft. broad is surrounded by a verandah 5 ft. wide. Find the area of the verandah and also the cost of paving it at 2*a*. per sq. ft.

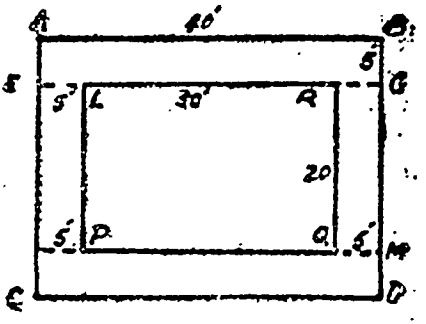


Sol. It is clear from the diagram that area of the verandah is equal to the area of the outer rectangle *minus* the area of the inner rectangle.

Area of the outer rectangle $=40\times30=1200$ sq. ft.
" " inner " $=30\times20=600$ sq. ft.
∴ area of the verandah $=1200-600=600$ sq. ft.
Hence cost $=\text{Re. }\frac{1}{8}\times600=\text{Rs. }75$. **Ans.**

Aliter. The length of the verandah
$=30+30+(20+10)+(20+10)$
$=120$ ft. ; breadth $=5$ ft.
∴ area $=120\times5=600$ sq. ft.
Hence cost $=\text{Rs. }\frac{1}{8}\times600$
$=\text{Rs. }75$. **Ans.**



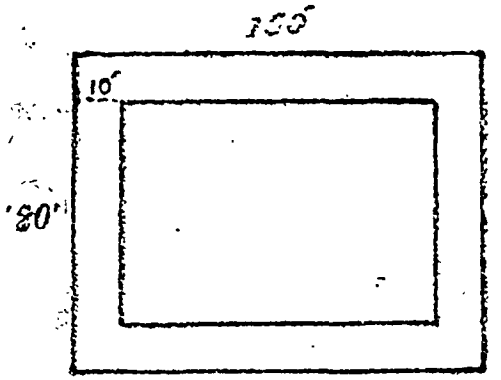


Aliter

Area of $ABGE = 40 \times 5 = 200$ sq. ft.
Area of $FMDC = 40 \times 5 = 200$ sq. ft.
Area of $ELPF = 20 \times 5 = 100$ sq. ft.
Area of $GRQM = 20 \times 5 = 100$ sq. ft.
Total area $= 200 + 200 + 100 + 100$
$= 600$ sq. ft.
Cost $=$ Re. $\frac{1}{8} \times 600 =$ Rs. 75. **Ans.**

Example 2. A lawn 150 ft. long 120 ft. broad has a path 10 ft. wide inside running round it. Find the cost of covering the path with flag stones at 2*s*. 6*d*. per square yard.

Sol.



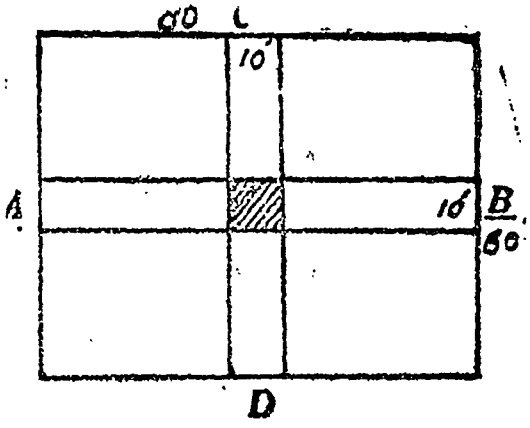
Area of the outer rectangle $= 150 \times 120 = 18000$ sq. ft.
" " inner rectangle $= 130 \times 100 = 13000$ sq. ft.
$\therefore$ area of the path $= 18000 - 13000 = 5000$ sq. ft.
$\therefore$ cost $= £\frac{1}{8} \times \frac{5000}{9} = £\frac{625}{9}$
$= £69,\ 8s.\ 10\frac{2}{3}d.$ **Ans.**

The second and third methods are left for the student as exercises.

Note. In the first example twice the width of verandah was added to the length and breadth of the room because the verandah was outside it

In the second example twice the width of the path was subtracted from the length and breadth of the lawn because the path was running inside it.

Example 3. A rectangular lawn 80 ft. by 60 ft. has two roads each 10 ft. wide running in the middle of it, one parallel to the length and and the other parallel to the breahth. Find the cost of gravelling them at 3*s*. per square yard.



Area of the path AB $=80\times10$ $=800$ sq. ft.

Area of the path CD $=60\times10$ $=600$ sq. ft.

Total Area $=800+600=1400$ sq. ft.

But area of the common portion $=10\times10$ $=100$ sq. ft.

$\because$ area of the roads to be gravelled

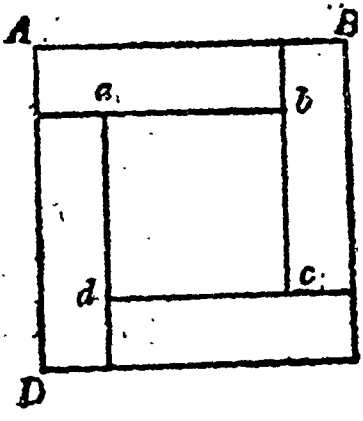
$=1400-100=1300$ sq. ft.

$\therefore$ cost $=\text{Re. } \frac{3}{16}\times\frac{1300}{9}=\text{Rs. } \frac{325}{12}$

$=\text{Rs. } 27, 1a.\ 4p.$ **Ans.**

Example 4. A path 9 ft. wide, running all round within a square garden has an area of 3 acres. Find the area of the part of the garden enclosed by the path.

Sol. $ABCD$ is a garden; $abcd$ is the part enclosed by the path. Produce the sides as shown in the diagram.



Now area of the four equal rectangles $=3$ acres

$\therefore$ area of one rect $=\frac{3\times4840}{4}$ q. yds.

$=3630$ sq. yds.

$\because$ breadth of the rectangle $=3$ yds.

$\therefore$ length of the rectangle $=3630\div3$

$=1210$ yds.

$ab=1210-3=1207$ yds.

$\therefore$ the area of the sq. $abcd = 1207 \times 1207$ sq. yds.

$= 1456849$ sq. yds.

$= 301$ acres 9 sq. yds. **Ans.**

Example 5. A path of uniform width running *outside* a rectangular garden contains 2800 sq. ft. If the dimensions of the garden be 150 ft. and 120 ft., find the width of the path.

Sol. In example 9 of Art. 5 we have stated a formula that $(\text{length}+\text{breadth})^2-(\text{area}\times 4)=(\text{length}-\text{breadth})^2$ whence

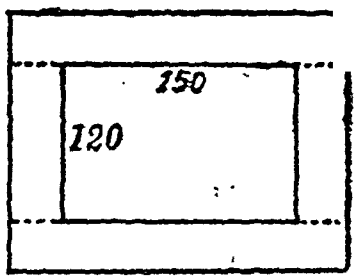
(*i*) $(\text{length}-\text{breadth})^2+\text{area}\times 4=(\text{length}+\text{breadth})^2$

and (*ii*) $(\text{length}+\text{breadth})^2-(\text{length}-\text{breadth})^2=\text{area}\times 4$.

It is also evident that the difference between the sides of the outer rectangle must be the same as the difference of the sides of the inner rectangle

$\because$ the difference between the sides of the inner rectangle is 30 ft.

$\therefore$ the difference between the sides of the outer rectangle is also 30 ft.



also area of the outer rectangle $=(150\times 120)+2800$ sq. ft

$=20800$ sq. ft.

Now apply formula (*i*)

$\because (30)^2+20800\times 4=(\text{length}+\text{breadth})^2$

Or $\sqrt{900+83200}=\text{length}+\text{breadth}$

Or $\text{length}+\text{breadth}=290$ ft.

but $\text{length}-\text{breadth}=30$ ft.

$\therefore \text{length}=(290+30)\div 2=160$ ft.

i.e., the length of the outer side $=160$ ft.

$\therefore \quad \text{path}\times 2=160-150=10$ ft.

$\therefore \quad \text{path}=5$ ft. **Ans.**

Aliter. Since the difference between the length and breadth of the inner rectangle is 30 ft. therefore the difference in the dimensions of the outer rectangle is also 30 ft. The area of outer rect. is $(120\times 150)+2800=20800$ sq. ft.

Now find out two factors of 20800 whose difference is 30

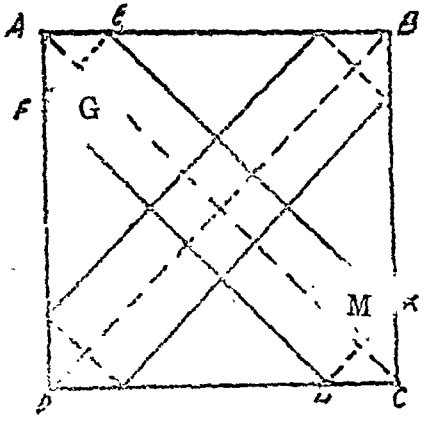
$20800 = 160 \times 130$

$\therefore$ length of the outer rectangle $= 160$ ft.

$\therefore$ twice the width of the path $= 160 - 150 = 10$ ft.

$\therefore$ width " " $= 5$ ft. **Ans.**

Example 6. A square garden $ABCD$ contains 1·8 acres. It is to be crossed by two paths along the diagonals AC, BD. Each path is to be 8 yards wide with its centre line along the diagonal. Find the cost of making and paving the paths at 6s. 3*d*. per square yard.



Sol. $\because$ Area $= 1{\cdot}8$ acres

$\therefore$ $AC^2 = 1{\cdot}8 \times 2$ acs.

$= 3{\cdot}6 \times 4840$ sq. yds.

$\therefore$ $AC = \sqrt{3{\cdot}6 \times 4840} = \sqrt{36 \times 484}$

$= 6 \times 22$ or 132 yds.

Now $EF = 8$ yds. and G is the mid. pt. of EF,

$\therefore$ $AG = GE = GF = 4$ yards.

[AG bisects rt. $\angle A$.

Similary, CM = 4 yards.

$\therefore$ GM or $EK = 132 - (4+4)$ or 124 yds.

$\therefore$ Area of the path $AEKCHF = \frac{1}{2} \times 8 \times 4 \times 2 + 124 \times 8$

or 1024 sq. yds.

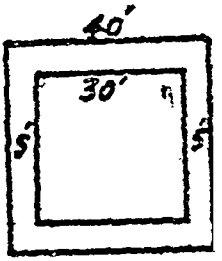
Likewise area of the other path = 1024 sq. yards.

Area of the centre sq. $= 8 \times 8$ or 64 sq. yards.

$\therefore$ Area reqd. for paving $= 1024 + 1024 - 64 = 1984$ sq. yds.

Hence cost $= \text{£}\frac{5}{16} \times 1984 = \text{£}620$. **Ans.**

Example 7. In the centre of the room 40 ft. square there is a square carpet and the rest of the floor is covered with cloth. The price of the carpet is 12*a*. per sq. yard and that of the cloth 4*a*. 6*p*. per sq. yard and the total cost of both is Rs. 96. 14*a*. ; find the width of the cloth.



Sol. Area of the floor $= 40 \times 40 = 1600$ sq. ft.

Had there been only carpet on the floor the cost would have been $= \text{Rs. } \frac{3}{4} \times \frac{1600}{9}$

$= \text{Rs. } 133.\ 5a.\ 4p.$

i.e., the cost would have been increased by Rs. 133, 5*a*. 4*p*.—Rs. 96, 14*a*.$=$Rs. 36, 7*a*. 4*p*.

$= \text{Rs. } \frac{875}{24}$.

Again, if there had been only cloth on the floor the cost would have been Rs. $\frac{9}{32}\times\frac{1600}{9}$=Rs. 50.

i. e., the cost would have been decreased by Rs. 96, 14*a.*—Rs. 50=Rs. 46. 14*a.*=Rs. $\frac{375}{8}$

Hence the ratio between the areas of the carpet and the cloth would be inverse, *i,e.*, $\frac{375}{8}:\frac{875}{4}$=1125 : 875=9 : 7.

∴ the area of the carpet=$1600\times\frac{9}{16}$=900 sq. ft.

∴ The length of the carpet=$\sqrt{900}$=30 ft. But the length of the room=40 ft.

∴ twice the width of the cloth=10 ft.

∴ width of the cloth=5 ft. **Ans.**

Aliter. The area of the room=$\frac{1600}{9}$ sq. yards.

The mean cost per sq. yard $= \frac{\text{Rs. } 96,\ 14a.\times 9}{1600} = \left(\frac{279}{32}\right)a.$

carpet		cloth
12*a.*		$\frac{9}{2}a.$
	$\frac{279}{32}a.$	
$\frac{279}{32}-\frac{9}{2}$	:	$12-\frac{279}{32}$
or $\frac{135}{32}$	:	$\frac{105}{32}$
or 9	:	7

By the Alligation Rule we got the ratio between the area of the carpet and the area of the cloth. Now proceed further.

EXERCISE 80.

1. A rectangular courtyard 100 ft. long, 80 ft. wide has within it a gravel path 8 ft. wide running round it. Find the area of the path and the cost of gravelling it at 5*a.* 3*p.* per square yard.

2. The area of a square garden is 10 acres. On the inside of the garden and along the four sides of it there is a gravel path 5 ft. wide. Find the cost of constructing the path at 1*a.* 6*p.* per sq. yard.

3. Find the cost of paving a pathway 6 ft. wide round and immediately outside a flower garden 21 yards long and 10 yards broad, at $5\frac{1}{7}$ pies per sq. yard.

4. A field is 110 yards long and 90 yards broad. On the inside of it along the four sides there is a path 5 yards wide. The cost of gravelling the path is 4*a.* per sq. yard and the cost of planting grass on the remaining field is Re. 1, 8*a.* per 100 sq. yards. Find the total cost.

5. A field contains one acre ; its length is 110 yards ; on the inside of it along the four sides a road 5 ft. wide has been constructed and the remaining field has been planted with grass at 2*a.* 3*p.* per sq. yard. Find the cost.

6. A room is 24 ft. 4 in. long and 14 ft. 3 in. broad. It is surrounded by a verandah 4 feet wide. Find the cost of carpeting it at 13*a*. 6*p*. per square yard.

7. A garden is 350 yds. long and 250 yds. broad. It has two roads each 8 feet wide running in the middle of it, one parallel to the length and the other parallel to the breadth. Find the cost of gravelling them at 1*a*. 3*p*. per sq. yard.

8. A garden 220 yards by 110 yards, has two roads 6 ft. and 4 ft. wide running in the middle of it. The first road runs parallel to the length and the second parallel to the breadth. Find the area of the roads.

9. How many paving stones, each of them 1 ft. long, 9 in. wide, will be required for paving a street 30 ft. wide, surrounding the outside of a square grass plot, the area of the grass plot being 10 acres ?

10. A road 6 feet wide has been constructed in the middle of a garden parallel to the length. The area of the road is 420 square yards and the area of the garden is 31500 square yards ; find the dimensions of the garden.

11. A garden contains 2 acres and its breadth is 88 yards. A road has been constructed in the middle of it parallel to the length of the garden. If the area of the road be 1650 square ft. find its breadth.

12. A rectangular court is 50 yards long and 30 yards broad. It has a path 6 ft. in breadth joining the middle points of the opposite sides and also a path of the same breadth running all round it. The remainder is covered with grass. If the cost of pavement be 1*s*. 8*d*. per sq. ft. and of the turf 3*s*. per sq. yard, find the cost of laying out the court.

13. A marginal walk all round the *inside* of a rectangular park 48 ft. by 36 ft. occupies 608 sq. ft. ; find the width of the path.

14. A path of uniform width running *outside* a rectangular garden 60 yards by 40 yards contains 3100 sq. ft. ; find the width of the path.

15. In the centre of a room 32 feet square, there is a square carpet costing 11*a*. per sq. yd. and the rest of the floo

is covered with a cloth at 4*a.* 6*p.* per sq. yard. The total cost of the carpet and the cloth is Rs. 58 ; find the width of the cloth.

16. A room is 25 ft. square. In the centre of it there is a square carpet and the rest of the floor is covered with cloth. The carpet and cloth cost respectively Re. 1, 2*a.* and 7*a.* per sq. yard, and the total cost of both is Rs. 60, 15*a.* ; find the length and breadth of the cloth.

17. A path 15 ft. wide running inside all round a square garden has an area of one acre. Find the area of that part of the garden enclosed by the path and also find the cost of garvelling it at Re. 1, 9*a.* per 100 sq. yards.

18. A path 5 ft. wide runs inside all round a square park and the cost of turning the remaining into a grassy plot at 1*a.* per sq. yard is Rs. 100. Find the cost of surrounding it with a fence at Rs. 12, 8*a.* per 100 ft.

19. A room measuring 42 ft. 6 in. by 22 ft. 9 in. inside, with walls 2 ft. 3 in. thick is surrounded by a verandah 10 ft. 6 in. wide. Find the cost of paving this verandah with tiles measuring 4½ in. by 3 in. and costing 6 pies each.

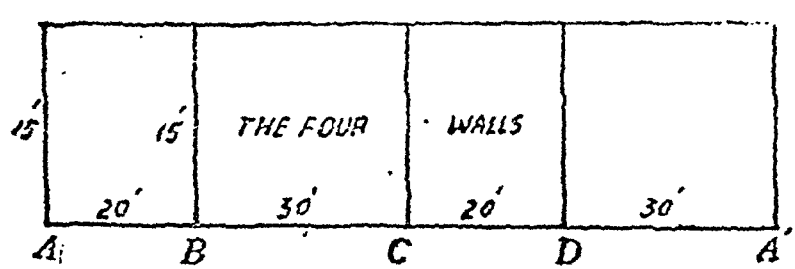
20. A hall 70 ft. long and 36 ft. broad is enclosed by walls 18 in. thick and all round the outside there is a verandah 13½ ft. deep. What will be the cost of paving this verandah at the rate of 12*a.* per sq. yard ?

21. A square piece of ground ABCD has an area of 10 acres. It is to be crossed by two paths along the diagonals AC, BD. Each path is to be 6 yards wide with its centre line along the diagonal. Find to the nearest rupee the cost of making the path at 2*a.* per square yard.

§8. Area of the four walls of a room.

Example 1. Find the area of the four walls of a room 30 ft. long, 20 ft. broad and 15 ft. high.

Exp. If a diagram of the four walls of a room be drawn side by side, it will form a rectangle as shown below:—



6. 18 ft. 5 in. long, 15 ft. 7 in. broad, and 12½ ft. high; paper 25 in. wide.

Find the cost of papering the walls of a room (7—8):—

7. 21 ft. 6 in. long, 16 ft. 6 in. broad and 12 ft. 6 in. high; paper 1½ ft. wide at 4*a.* per yard.

8. 23 ft. 4 in. long, 17 ft. 6 in. broad and 18 ft. high; paper 33 in. wide at 1*a.* 8*p.* per yard, allowing 358 sq. ft. for doors and windows.

9. The length, the breadth and the height of a room are 25 ft. 7 in., 20 ft. 5 in. and 14 ft. respectively. Its walls are papered at 3*s.* 6*d.* a sq. yard and its ceiling painted at 1*s.* 2*d.* a sq. ft. Find the total cost.

10. Find the cost of white-washing a room 22½ ft. by 12 ft. and 11 ft. high at one anna per square yard, making allowance for four windows each 4 ft. × 2½ ft. and two doors each 8½ ft. × 4 ft.

11. The cost of papering the walls of a room 22 ft. long, 18 ft. broad, and 14½ ft. high at 4*a.* per yard is Rs. 40. Find the width of the paper.

12. A room is 19 ft. 6 in. long. 17 ft. broad and 13 ft. high. The cost of papering the walls with paper 26 in. wide is Rs. 73. Find the cost of the paper per yard.

13. The cost of papering and white-washing the walls of a room 16 ft. long and 14 ft. broad at 5*a.* per square yard is Rs. 25. Find the height of the room.

14. A room is 17 ft. 6 in. long and 14 ft. high, the cost of papering and white-washing its walls at 3*a.* per sq. ft. is Rs. 170, 10*a.* Find the breadth of the room.

15. Find the area of the four walls of a room, the length of which is 15 ft. and the height 13 ft., if the cost of matting its floor at 2*a.* per square yard is Rs. 2, 8*a.*

16. The cost of matting a room 16 ft. broad and 12 ft. high at 3*a.* per square yard is Rs. 7, 9*a.* 4*p.* What will be the cost of papering its walls at the same rate, allowing for 6 doors each 6 ft. by 3 ft. ?

17. The length of a room is 32½ ft. The cost of painting the walls at Rs. 1, 14*a.* per square yard is Rs. 308, 2*a.* and the cost of carpeting the room at Rs. 2. 4*a.* per square yard is Rs. 150, 5*a.* Find the height and width of the room.

18. A room is 25 ft. high and its length is thrice its breadth. The cost of white-washing its four walls at 5*a.* per

sq. yard is Rs. 62, 8*a*. find the cost of carpeting it at 12*a*. per sq. yard.

19. The length and breadth of a room are in the ratio of 5 : 4. The cost of papering its four walls with paper 27 in. wide at $1\frac{1}{2}a$. per yard is Rs. 20 and the cost of carpeting it at 7*a*. 6*p*. per square yard is Rs. 26, 8*p*. ; find its length, breadth and height.

20. The length, breadth and height of a room are in the ratio of 5 : 4 : 3 and the area of its four walls is 384 sq. yards. Find its length, breadth and height.

MISCELLANEOUS EXERCISE 82.

1. A room is 15 ft. 6 in. long, 12 ft. 6 in. broad and 14 ft. high. Find the cost of white-washing its walls at 1*a*. per square yard.

2. A room is 21 ft. long, 16 ft. 6 in. broad and 15 ft. high. Find the cost of papering its walls at 1*a*. 6*p*. per sq. yard allowing 72 sq. ft. for doors and windows.

3. A room is 17 ft. 8 inches long, 15 ft. 4 in. broad and 16 ft. high. Find the cost of papering its walls with paper 18 inches wide at 4*a*. 6*p*. per piece of 10 feet.

4. My room is in a very bad condition. It is 16 ft. long 12 ft. broad and 15 ft. high. It has two doors each 6 ft. by 3 ft., two windows each 4 ft. by $2\frac{1}{2}$ ft. I intend to have the doors and windows painted and the walls white-washed, the cost of painting is 1*a*. 6*p*. per sq. ft. and that of white-washing 2*a*. 3*p*. per square yard. Calculate the total cost.

5. A room of my house is 21 feet 6 inches long and 15 ft. broad. The cost of painting and repairing the four walls at 4*a*. 6*p*. per sq. yard amounts to Rs. 36, 8*a*. Find the height of the room.

6. The cost of white-washing a room 17 ft. broad and 13 ft. 8 in. high at 2*a*. per sq. yd. amounts to Rs. 13, 10*a*. 8*p*. Find the length of the room.

7. The length of a room is twice its breadth and its height is 15 ft. The cost of white-washing its four walls at 1*a*. 3*p*. per sq. yard is Rs. 9, 6*a*. Find the length of the room.

8. A room is 25 ft. by 15 ft. It has 4 doors each 6 ft. by $3\frac{1}{2}$ ft., 4 windows each 4 ft. by $2\frac{1}{2}$ ft., one fire-place 2 ft. by 1 ft. Allowing for doors, etc., the cost of white-washing at 1*a*. 3*p*. per square yard is Rs. 11, 6*a*. 6*p*. Find the height of the room.

9. The length and breadth of a room are 3 : 2, and the height is 21 ft. The cost of papering the four walls at 1*a*. per sq. yard is Rs. 11, 10*a*. 8*p*. Find the dimensions of the room.

10. A room is 23 ft. 8 in. long and 19 ft. 4 in. broad. The cost of papering its walls after allowing 109 sq. ft. for windows, etc., with paper $1\frac{1}{2}$ ft. wide at 4*a*. per yard amounts to Rs. 89, 8*a*. Find the height of the room.

11. Find the number of bricks, each 9 in. by 4 in., required for paving a courtyard 36 ft. by 21 ft.

12. A room is 13 ft. 6 in. broad and the cost of carpeting it at 12*a*. per sq. yard is Rs. 36. Find the length of the room.

13. 2640 yards by 2 ft. paper is required for papering the walls of a room the length of which is twice its breadth. How many yards of carpet 4 feet wide will be required for the floor of the same room, the height of the room being 33 ft. ?

14. The length of a room is twice its breadth and the height is 20 ft. The cost of papering the walls with paper $2\frac{1}{2}$ feet wide at 4*a*. per yard amounts to Rs. 64. Also the carpet required for the floor is 42 yards 2 ft. Find the width of the carpet.

15. The breadth of a room is 15 ft. The cost of papering its walls at 2*p*. per square ft. is Rs. 16, 10*a*. 8*p*. and the cost of carpeting it at 2*a*. per sq. ft. is Rs. 46, 14*a*. Find the length and the height of the room.

16. The length of a room is 30 ft. The cost of carpeting it at 2*a*. 3*p*. per sq. yard is Rs. 11, 11*a*. 6*p*. and the cost of repairing its walls at 9*a*. per sq. yard is Rs. 137, 8*a*. Find the breadth and the height of the room.

17. The cost of carpeting the floor of a room is Rs. 75. If the breadth of the room had been 3 ft. less, the cost would have been Rs. 60. Find the breadth of the room.

18. The cost of carpeting a room is Rs. 125. If the length of the room had been 4 ft. less, the cost would have been Rs. 100. Find the length of the room.

19. The cost of carpeting a room is £7, 4*s*. and of papering the same room with paper at $2\frac{1}{2}$*d*. per square ft. is £10, 12*s*.

6*d*. The length of the room is 18 ft. and if the width had been 4 ft. less, the cost of the carpet would have been £1, 16*s*. less. Find the height of the room.

20. A room is 16 ft. by 12 ft. Had it been 2 ft. longer and 1 ft. wider the cost of carpeting it would have been increased by Rs. 5, 4*a*. Find the cost of the carpet.

21. A room is 18 ft. by 16 ft. Had it been 2 ft. longer and 1 ft. less wide the cost of carpeting it would have been increased by Rs. 2, 4*a*. Find the cost of the carpet.

22. The length of a room exceeds its breadth by 12 ft. and its perimeter is 76 ft. Find the cost of carpeting it at 4*a*. per sq. ft.

23. The length of a room is greater than its breadth by 8 ft. and its perimeter is 64 ft. Find the cost of carpeting it at 12*a*. per sq. yard.

24. A room is 18 feet broad. Leaving a passage 2 ft. wide along the sides, a carpet has been spread, the cost of carpeting at 2*a*. per square ft. being Rs. 28 ; find the length of the room.

25. A room is 30 ft. long. Leaving a passage 2½ ft. wide along the sides a carpet worth Rs. 14; 13*a*. 6*p*. at 4*a*. 6*p*. per square yard has been spread. Find the cost of papering the ceiling at 1*a*. 3*p*. per sq. yard.

26. A garden is 220 ft. by 160 ft. A road 5 ft. wide has been constructed within along the sides. Find the cost of gravelling it at 9*a*. per 100 sq. ft.

27. A garden is 300 ft. by 250 ft. It is surrounded by a path 10 ft. wide. Find the cost of gravelling this path at 3*a*. per sq. yard.

28. A field is 150 yards by 120 yards. It has two roads each 10 ft. wide running in the middle of it, one parallel to the length and other parallel to the breadth. Find the area of these roads, also find the cost of planting grass at 7*a*. per 100 sq. ft. on the remaining portion.

29. A path 6 ft. wide, running all round a square garden has an area of 2 acres. Find the area of that part of the garden enclosed by the path.

30. Find the cost of papering the walls of a room 22 ft. long, 18 ft. wide and 20 ft. high with rolls of paper 21 inches wide, at Rs. 2, 10*a*. per roll of 12 yards.

31. The area of a rectangular field whose breadth is 500 yards is 100 acres. Find the cost of cultivating it at Rs. 3, 2*a*. 8*p*. per 100 sq. yards and also the cost of fencing it round at Rs. 2, 8*a*. per yard.

32. The cost of painting a room 9 ft, 6 in. high, 15 ft. 3 in. long and 10 ft. broad is Rs. 47, 8*a*. What must be the height of another room whose length and breadth are respectively 18 ft. 2 in. and 11 ft. 7 in. if the painting of the walls at the same cost per sq. yard amounts to Rs. 74, 6*a*. ?

33. The sum of the length and breadth of a room is 26 ft. and its area is 160 sq. ft. ; find its dimensions.

34. A rectangular field is 80 ft. long 60 ft. broad. A path of uniform width whose area is 1300 sq. ft. has been constructed *within* along the four sides ; find its width.

35. A path of uniform width *outside* a rectangular garden contains 3200 sq. ft. If the length and breadth of the garden be 80 ft. and 60 ft. respectively, find the width of the path.

36. Find the cost of lining with zinc, at 6*s*. 9*d*. per sq. yard the sides and bottom of a cistern whose dimensions are : length 7 ft. 10 in. breadth 5 ft. 4 in. and depth 1 ft. 6 in. [*Burma 1923*]

37. A room is 21 ft. 4 in. long, 15 ft. 9 in. broad ; the doors and windows together occupy 65 sq. ft. The cost of papering the remaining part of the surface of the walls with paper 25 in. wide at 3*s*. 9*d*. per piece of 12 yards is £2, 8*s*. 8*d*. Find the height of the room. [*Burma 1924*]

38. A lawn tennis ground is half as long again as it is wide. The cost of levelling it at 5*a*. per square yard is Rs. 1740; find the cost of enclosing it with an iron railing at Rs. 4 per yard.

39. The length of a room is 20 ft. ; the cost of papering the walls with paper $2\frac{1}{2}$ ft. wide at 4*a*. per yard is Rs. 30, 6*a*. 8*p*. ; and that of carpeting the room at Rs. 3, 5*a*. 4*p*. per sq. yard is Rs. 122, 3*a*. $6\frac{2}{3}$*p*. ; find the height of the room.

40. The length of a square plot of land is 110 yards. It has two paths each $2\frac{1}{2}$ yards with its centre line along the diagonal. Find to the nearest penny the cost of covering the remaining plot with grass at 2*s*. per square yard.

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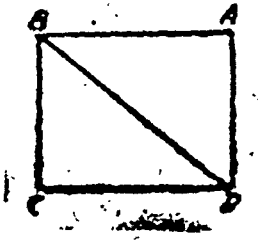
CHAPTER XIV

AREA OF SOME SIMPLE FIGURES

§1. In the previous chapter we have explained how to find out the area of a rectangle and a square. Now in this chapter we shall explain how to find out the area of some other simple figures.

§2. **Area of a right-angle triangle.**

Let $ABCD$ be a rectangle with BD as its diagonal. It is clear from the diagram that the diagonal divides the rectangle into two equal right-angled $\triangle$s BCD and ABD.



$\because$ the area of the rectangle $= BC \times CD$

$\therefore$ the area of the right-angled $\triangle BCD = \frac{BC \times CD}{2}$

where BC and CD are the perpendicular and the base of the right-angled triangle respectively.

$\therefore$ the area of a right-angled triangle $= \frac{\text{perpendicular} \times \text{base}}{2}$

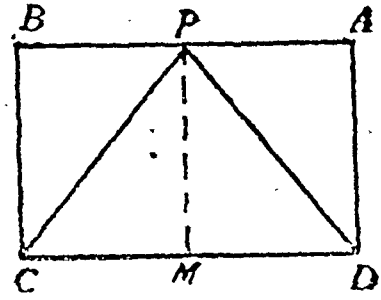
whence perpendicular $= \frac{\text{area} \times 2}{\text{base}}$ and base $= \frac{\text{area} \times 2}{\text{perpendicular}}$

In the right-angled $\triangle$ the side opposite to the right-angle is called the Hypotenuse.

It may be noted that hypotenuse $= \sqrt{(\text{perp.})^2 + (\text{base})^2}$

§3. **Area of any triangle.**

Let $ABCD$ be a rectangle. Take a point P on AB and join CP and DP Thus we have a triangle CDP within the rectangle. PM is the height of the triangle and CD the base. It is clear from the diagram that the area of the triangle is half the area of the rectangle.



Since the area of the rectangle $= CD \times AD$;

$\therefore$ the area of the triangle $= \frac{CD \times AD}{2} = \frac{CD \times PM}{2}$

We shall express this result in words as follows:—

$$\text{area of a triangle} = \frac{\text{base} \times \text{height}}{2}$$

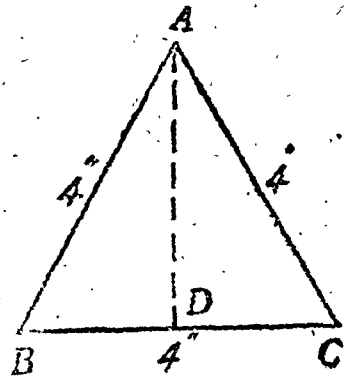
whence $\text{Base} = \frac{\text{area} \times 2}{\text{height}}$ and $\text{height} = \frac{\text{area} \times 2}{\text{base}}$

Aliter. $\text{Area} = \sqrt{s(s-a)\ (s-b)\ (s-c)}$:

where s=semi-perimeter and a, b, c, the sides of the triangle. The formula is proved in geometry.

§4: Area of an equilateral triangle.

An equilateral triangle is one which has all its sides equal. Let ABC be an equilateral triangle whose side is equal to 4 inches. By application of the second formula of Art. 3 we shall proceed thus:



Sem.-perimetre $= \frac{1}{2}(4+4+4) = 6$

$\therefore$ area $= \sqrt{6 \times (6-4)\ (6-4)\ (6-4)}$

$= \sqrt{6 \times 2 \times 2 \times 2}$

$= \sqrt{2 \times 2 \times 2 \times 2 \times 3}$

$= \sqrt{4 \times 4 \times 3}$

$= 4 \times 1{\cdot}732$

$* = 4 \times 4 \times {\cdot}433.$

Also height $AD = \frac{\text{area} \times 2}{\text{base}}$

$= \frac{4 \times 4 \times {\cdot}433 \times 2}{4}$

$* = 4 \times {\cdot}866.$

From the asterisked lines we conclude the following formulae:

area of an equilateral triangle = side × side × ·433

Height or altitude = side × ·866

§5 Area of a quadrilateral.

Quadrilateral is a plane figure bounded by four sides.

(*a*) If four sides and one diagonal be given we shall find out the areas of the two triangles thus formed by the formula $\text{Area} = \sqrt{s(s-a)\ (s-b)\ (s-c)}$ and then add.

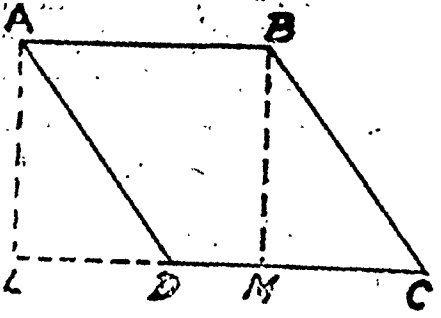
(*b*) If one diagonal and the two off sets from this diagonal to the opposite corners be given the area of the quadrilateral is found by the formula $\frac{1}{2}d.\ (P_1 + P_2)$

Example. Find the area of the quadrilateral whose diagonal is 213 ft. ; off sets 97 and 103 ft.

Sol. Area $=\frac{1}{2}\times 213(97+103)=21300$ sq. ft. **Ans.**

§6. **Area of a parallelogram.**

Parallelogram is a four-sided figure having its opposite sides equal and parallel.



Let $ABCD$ be a parallelogram

Produce CD and draw AL and BM perpendiculars on CD.

Now $ALMB$ is a rectangle the area of which is equal to the area of the given parallelogram, since $\triangle ADL$ is equal to $\triangle BCM$ in all respects.

Now area of the rectangle $ALMB = AL \times LM$

$\therefore$ area of the parallelogram $= AL \times LM$

$= AL \times DC$ [$\because LM = DC$

We express this result in words thus:—

Area of a parallelogram = height × base.

whence height or altitude = area ÷ base.

and base = area ÷ height.

§7. **Area of a rhombus.**

Rhombus is a four sided figure, having all sides equal and the diagonals of which bisect each other at right-angles.

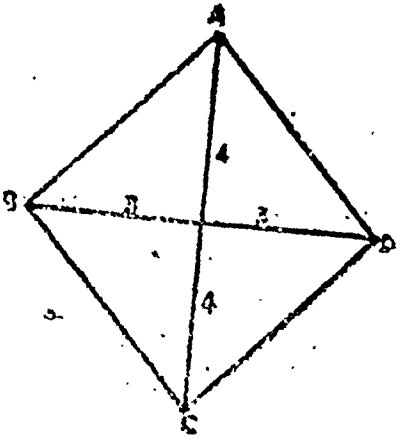
Let $ABCD$ be a rhombus with diagonals 6″ and 8″, *i.e.*, $BD = 6''$ and $AC = 8''$.

$\therefore$ area of $\triangle ABC = \frac{8\times 3}{2}$ sq. in.

and area of $\triangle ACD = \frac{8\times 3}{2}$ sq. in.

$\therefore$ area of the rhombus

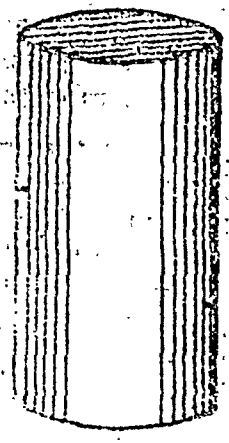
$= \frac{8\times 3\times 2}{2}$ sq. in. $= \frac{8\times 6}{2}$ sq. in.



The result may be expressed in words thus:—

$$\text{Area of a rhombus} = \frac{\text{diagonal} \times \text{diagonal}}{2}$$

§8 Area of a cylinder.



Take a rectangular piece of paper and join the edges of its breadth. It will represent the figure of a cylinder. The length of the rectangular paper will represent its circumference and breadth, its height.

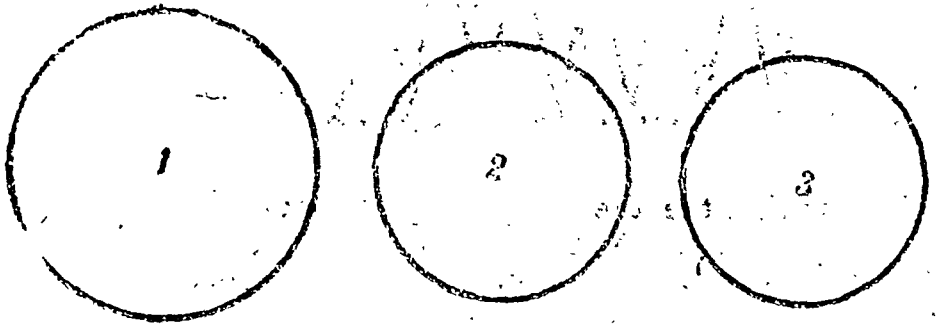
∵ the area of a rectangle = length × breadth

∴ Area of a cylinder = circumference × height.

whence circumference = area ÷ height
and height = area ÷ circumference.

§9. Circle.

Draw some circles of different radii. Mark points on the circumferences near each other and measure the circumference with threads as accurately as possible from one point to another till the whole circumference is measured. Now measure the thread



used in each circle in inches and fill up the following table :—

No.	Circumference in inches.	Diameter in inches.	Ratio between circumference and diameter.
1			
2			
3			

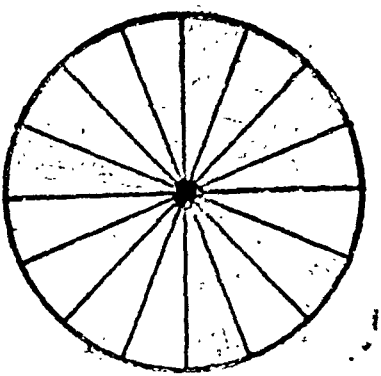
You will now find that the circumference in each case is $\frac{22}{7}$ times the diameter, that is to say

circumference $=$ **diameter** $\times \frac{22}{7}$ **and**

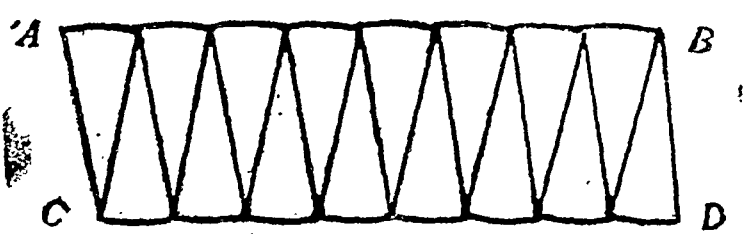
semi-circumference $=$ **radius** $\times \frac{22}{7}$.

§10. Area of a circle.

Draw a circle on a thin card board. Draw diameters so as to divide the circle into 16 equal parts. Cut off these parts and arrange them in a figure as shown below:—



These parts make a figure similar to a parallelogram. Had the circle been divided into 32 equal parts, lines AB and CD would have been more straight.



It is clear that the area of the parallelogram is equal to the area of the circle. We also observe that the base of the perallelogram is equal to the semi-circumference of the circle and height is equal to the radius.

$\therefore$ Area of the circle $=$ semi-circumference $\times$ radius.

But semi-circumference $=$ radius $\times \frac{22}{7}$ [Art. 9.

$\therefore$ **Area of a circle** $=$ **radius** $\times$ **radius** $\times \frac{22}{7}$

EXERCISE 83.

Note. The first ten questions to be done mentally.

1. The base of a right-angled triangle is 8 yards and perpendicular is 6 yards ; find the area.
2. The base and the height of a traingle are 7 and 3 yards respectively ; find the area.
3. The base of a parallelogram is 15 ft. and height is 5 ft. ; find its area.
4. The circumference of a cylinder is $5\frac{1}{2}$ yards and height is 8 yards ; find its area.

5. The diagonals of a rhombus are 15 ft. and 12 ft.; find its area.

6. What would be the area of a rhombus the diagonals of which are $4\frac{1}{2}$ ft. and 6 ft. ?

7. The circumference of a circle is 88 ft. ; find its diameter.

8. The circumference of a circle is 154 ft. ; find its radius.

9. The radius of a circle is 7 ft. ; find its area.

10. The side of an equilateral triangle is 4 ft.; find its area.

11. The area of a right-angled triangle is 46 sq. ft. 96 sq. inches and the base is 8 ft. 9 in. ; find its height.

12. The area of a right-angled triangle is 18 sq. ft. 48 sq. inches and its height is 3 ft. 4 in. ; find its base.

13. The hypotenuse of a right-angled triangle is 120 ft. and one side is 24 yards ; find the other side.

14. The sides of a triangle are 15, 25 30 ft. ; find its area to 3 decimal places.

15. The base of a triangle is 35 ft. and the height is 124 ft. ; find its area.

16. The area of a triangular field is 140 square yards and the base is 63 ft. ; find its altitude.

17. The sides of a triangle are 16 yards 2 ft., 26 yds. and 37 yds. 1 ft. respectively ; find the perpendiculars drawn from the angles on the opposite sides.

18. The side of an equilateral triangle is 50 ft.; find its area.

19. The area of an equilateral triangle is 270 sq. ft. 90 sq. inches ; find its side.

20. The side of an equilateral triangle is 12 ft. 6 in. ; find its altitude.

21. The circumference of a cylinder is 15 ft. and the height is $3\frac{1}{2}$ ft. ; find its area.

22. The circumference of a cylinder is 21 ft. and the height is 3 ft. 4 in. ; find its area.

23. The area of a cylinder is 52 sq. ft. 72 sq. inches and the height is 4 ft. 8 inches ; find its circumference.

24. The base of a parallelogram is 21 ft. 4 in. and the height is 7 ft. 6 in. ; find its area.

25. The area of a parallelogram is 148 sq. ft. 32 sq. in. and the height is 9 ft. 8 in.; find its base.

26. The diagonals of a rhombus are 6 ft. 8 in. and 5 ft. 3 in. respectively ; find its area.

27. The area of a rhombus is 50 sq. ft. ; one of the diagonals is 6 ft. 8 in ; find the other diagonal.

28. The diametre of a circle is 10 ft. 6 in. ; find its circumference.

29. The radius of a circle is 7 ft. ; find its circumference.

30. The circumference of a circle is 88 ft. ; find its radius.

31. The circumference of a circle is 14 yards 2 ft. ; find its diameter.

32. The radius of a circle is $3\frac{1}{2}$ ft. ; find its area.

33. The circumference of a circle is 220 yards ; find its radius.

34. The diagonals of a rhombus are 6 ft. and 8 ft. ; find its side.

35. The area of a rhombus is 2400 sq. ft. and the side is 50 ft. ; find the diagonals.

36. The side of a rhombus is 20 ft., and one of its diagonals is 24 ft. ; find the other diagonal.

37. Find the area of a ring the outer and inner radii of which are $3\frac{1}{2}$ ft. and $2\frac{1}{2}$ ft.

38. The area of a circle is equal to that of a square. Compare their perimeters.

39. In a quadrilateral $ABCD$, the diagonal AC measures 2 ft. 9 in. and the offsets from this diagonal to B and D measure 1 ft. 7 in. and 11 in. respectively ; find the area of the quadrilateral.

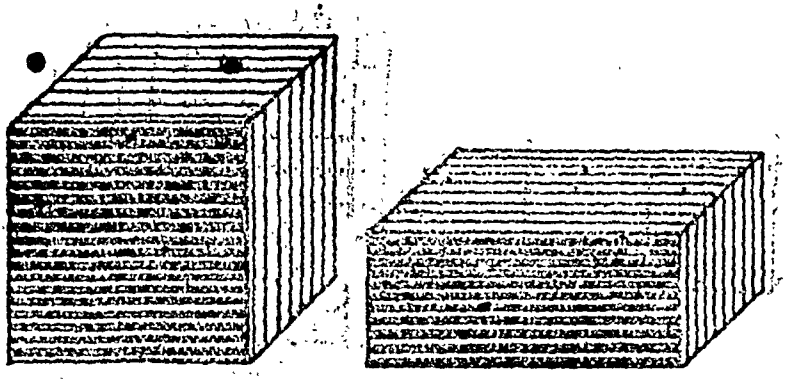
40. In a quadrilateral $ABCD$, the sides AB, BC, CD, DA, measure 20, 13, 17, 10 links respectively, and the diagonal AC, 21 links ; find the area of the quadrilateral.

CHAPTER XV.

MEASUREMENT OF VOLUME

§1. A thing which occupies some space is called **a Solid** and the space which a solid occupies is called its **Volume Capacity or Cubic Content.**

§2. Observe the following two figures. You have often seen such solids and you can note that both the solids are bounded by six faces. The only difference you can find is that in one soild each face is a square and in the other each face is



a rectangle. The one which has square faces is called a **cube** and the one which has rectangular faces is called a **rectangular solid** or **a cuboid.**

§3. A rectangular solid measuring an inch each way is called a **cubic inch,** measuring a foot each way is called a **cubic foot** ; and so on.

Note. A rectangular solid is said to have three dimensions viz length, breadth and thickness (or height or depth).

§4. **To prove that**

one cubic yard=27 cubic feet and
one cubic foot=1728 cubic inches.

Take 27 wooden rectangular solids, each measuring a cubic-foot, and place them as shown on the next page in fig. 1.

It will be seen that the model thus prepared is a cubic yard. It has three layers, each layer has three lines and each line has three wooden rectangular solids. Hence it is proved that a cubic yard $=3\times3\times3=27$ cubic feet.

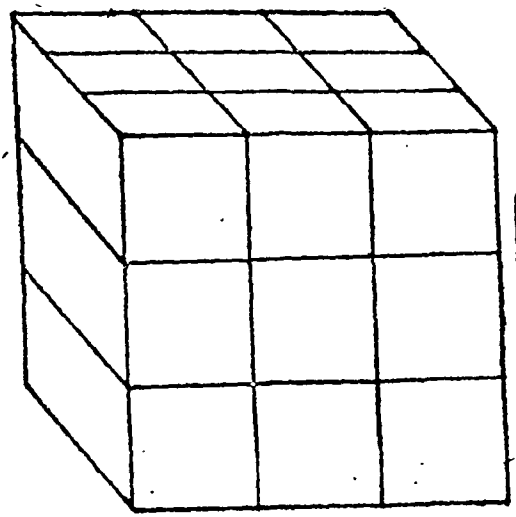


Fig. 1.

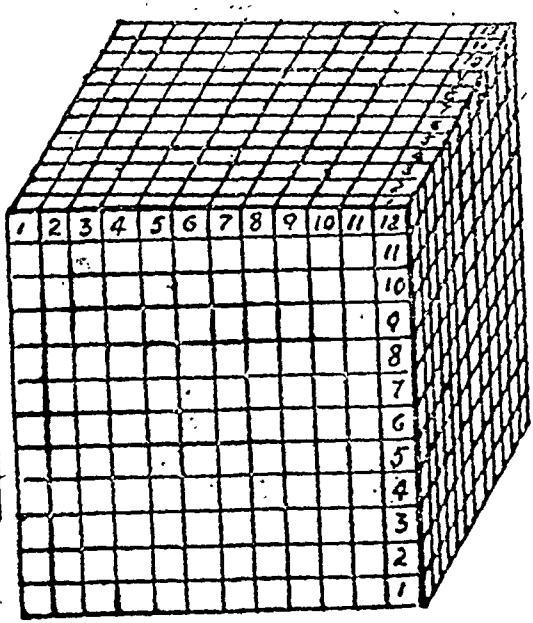
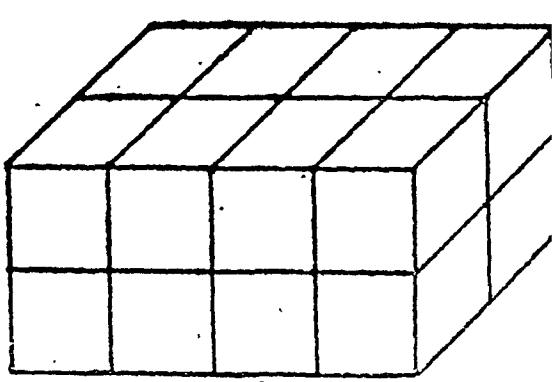


Fig. 2.

Similarly by the aid of the diagram No. 2, it will be seen that a cubic foot contains $12\times12\times12=1728$ cubic inches.

§6 To Find the volume of a cuboid and a cube.

(*i*) Let us take some wooden rectangular solids again each measuring 1 cubic ft. Make a layer containing 4 solids in its length and 3 in its breadth. Make another similar layer and place it on the first layer as shown in the annexed figure. The model now has two layers, each layer has three lines and each line has four wooden rectangular solids. It is clear, therefore, that the model contains $4\times3\times2=24$ cubic ft. where 4′ is the length,



3′ is the breadth and 2′ is the height. Hence the

Volume of a cuboid = length × breadth × height or briefly

$$v = l \times b \times h$$

(*ii*) Since in a cube, length, breadth and height are equal,

∴ **Volume of a cube = (length)^3 or (edge)^3.**

Rule. *Express the length, breadth and height in units of the same denomination, their product will give the volume in cubic units of the same denomination.*

§6. **Diagonal of a cuboid and a cube.**

(*i*) Diagonal of a cuboid $= \sqrt{l^2+b^2+h^2}$

Let $l = 5$ yards, $b = 4$ yards, and $h = 3$ yards, then

$$\text{diagonal} = \sqrt{5^2+4^2+3^2}$$
$$= \sqrt{25+16+9}$$
$$= \sqrt{50} = 7{\cdot}07 \text{ yds.} \quad \textbf{Ans.}$$

Note. This is the longest rod that can be placed in a room having dimension 5, 4 and 3 yds.

(*ii*) Diagonal of a cube $= \sqrt{l^2+l^2+l^2}$

$$= \sqrt{3l^2} \quad l\sqrt{3}$$

Let the side of a cube be 5 in., then

the diagonal $= 5\sqrt{3}$

$$= 5 \times 1{\cdot}732 = 8{\cdot}66 \text{ inches.} \quad \textbf{Ans.}$$

§**To find the surface of a cuboid and a cube.**

(*i*) Let the dimensions of a cuboid be 5, 4, 3 in. respectively. Since the cuboid has six rectangular faces and the two opposite ones are equal

∴ the surface of the cuboid $= 2(5\times4+5\times3+4\times3)$

$$= 94 \text{ sq. in.}$$

i. e., surface $= 2(lb+lh+bh)$

(*ii*) Since a cube has six equal faces

∴ the surface of a cube $= 6 \times (\text{edge})^2$

Example 1. A log is 15 ft. long, 2 ft. 3 in. broad and 1 ft. 6 in. thick ; find its volume and its surface.

Sol. (*i*) Volume = length × breadth × thickness

$$= 15 \times \tfrac{9}{4} \times \tfrac{3}{2} = \tfrac{405}{8}$$
$$= 50 \text{ cubic ft. } 1080 \text{ cubic in. } \textbf{Ans.}$$

(*ii*) Surface $= 2(lb+lh+bh)$

$$= 2(15\times\tfrac{9}{4}+15\times\tfrac{3}{2}+\tfrac{9}{4}\times\tfrac{3}{2}) \text{ sq. ft.}$$
$$= 2(\tfrac{135}{4}+\tfrac{45}{2}+\tfrac{27}{8}) = \tfrac{477}{4} \text{ sq. ft.}$$
$$= 119 \text{ sq. ft. } 36 \text{ sq. in } \textbf{Ans.}$$

Example 2. A box measures 3 ft. 6 in. each way ; find its volume and also find the cost of varnishing it at 4*p*. per sq. ft.

Sol. (*i*) Volume = length × breadth × height.

$= \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} = \frac{343}{8}$ cubic ft.

= 42 cubic ft. 1512 cubic in. **Ans.**

(*ii*) Since the box has six surfaces and each surface is a sq.

∴ the area of one surface $= \frac{7}{2} \times \frac{7}{2} = \frac{49}{4}$ sq. ft.

∴ ,, ,, six surfaces $= \frac{49}{4} \times 6$ sq. ft.

hence cost $= \text{Re. } \frac{49}{4} \times 6 \times \frac{4}{192} = \text{Re. } \frac{49}{32}$

= Re. 1, 8*a*. 6*p*. **Ans.**

Example 3. A platform 8 ft. long, $5\frac{1}{2}$ ft. broad and 3 ft. high is intended to be constructed ; find the cost of its construction at 2*a*. per cubic ft. and also find how many bricks each 9 in. long, 4 in. broad and 3 in. thick will be required for it.

Sol. Volume = length × breadth × height

$= 8 \times \frac{11}{2} \times 3 = 132$ cubic ft.

(*i*) Cost = Rs. $132 \times \frac{1}{8}$ = Rs 16. 8*a*.

(*ii*) No. of bricks $= \frac{132 \times 1728}{9 \times 4 \times 3} = 2122.$ } **Ans.**

Example 4. A tank is 30 ft. long, 12 ft. broad and $4\frac{1}{2}$ ft. deep ; how much water will it contain if the weight of one cubic ft. of water is 1000 ounces ?

Sol. Volume of the tank = length × breadth × height.

$= 30 \times 12 \times \frac{9}{2} = 1620$ cubic ft.

∴ weight of water $= \frac{1620 \times 1000}{16} = 101250$ lbs.

= 45 tons 4 cwt. 2 lbs. **Ans.**

Example 5. A box with a lid measuring on the outside 3 ft. 2 in. in length, 2 ft. 2 in. in breadth and 1 ft. 8 in. in height is made of wood 1 in. thick ; find how many cubic ft. of air it contains and also find the weight of the box if a cubic foot of the wood weighs 27 lbs.

Sol. The wood being 1 in. thick the inner dimensions are 3 ft., 2 ft. and $1\frac{1}{2}$ ft. respectively.

(*i*) ∴ the volume of the box occupied by the air.

$= 3 \times 2 \times \frac{3}{2} = 9$ cubic ft. **Ans.**

(*ii*) The volume of the box $= 3\frac{1}{6}$ ft. $\times 2\frac{1}{6}$ ft. $\times \frac{5}{3}$ ft.

$= \frac{19}{6} \times \frac{13}{6} \times \frac{5}{3} = \frac{1235}{108}$ cubic ft.

$\therefore$ the volume of the wood $=(\frac{1235}{108}-9)=\frac{263}{108}$ cubic ft.
and $\therefore$ weight of the box $=\frac{263}{108}\times 27$ lb. $=\frac{263}{4}$ lb.
$=65\frac{3}{4}$ lb. **Ans.**

Example 6. In the above example, find the expenses of painting the box inside and outside at 5*a*. 4*p*. per square yard.

Sol. The exterior surface = area of the four walls + area of floor + area of roof
$=2(\frac{19}{6}+\frac{13}{6})\times\frac{5}{3}+\frac{19}{6}\times\frac{13}{6}\times 2$
$=2\times\frac{32}{6}\times\frac{5}{3}+\frac{19}{6}\times\frac{13}{6}\times 2$
$=\frac{160}{9}+\frac{247}{18}=\frac{63}{2}$ sq. ft.
Similarly interior surface $=2(3+2)\times\frac{3}{2}+3\times 2\times 2$
$=2\times 5\times\frac{3}{2}+3\times 2\times 2$
$=15+12=27$ sq. ft.
$\therefore$ the whole area to be painted $=\frac{63}{2}+27$
$=\frac{117}{2}$ sq. ft. $=\frac{13}{2}$ sq. yards.
$\therefore$ the cost required $=$Rs. $\frac{13}{2}\times\frac{1}{3}=$Rs. $\frac{13}{6}$.
$=$Rs. 2, 2*a*. 8*p*. **Ans.**

Example 7. Water flows into a reservoir 24 ft. long 15 ft. broad and 11 ft. deep through a pipe of 9 sq. in. at the rate of 4 miles an hour. How long will it take to fill the reservoir ?

Sol. Volume of the reservoir $=24\times 15\times 11$ cubic ft.
Volume of water that flows into the reservoir.
$=4\times 1760\times 3\times\frac{9}{144}=1320$ cubic ft. per hour.
$\therefore$ time reqd. $=\frac{24\times 15\times 11}{1320}=3$ hours. **Ans.**

Example 8. A reservoir measuring inside 8 ft. in length, 5 ft. in breadth and 4 ft. in depth contains 60 cubic ft. of water. Bricks each measuring 9″ by 6″ by $2\frac{2}{3}$″ are put in the reservoir till it is brimful ; find the number of bricks put in it, supposing each brick absorbs $\frac{1}{26}$th of its own volume of water.

Sol. Volume of the reservoir $=8\times 5\times 4=160$ cubic ft.
Volume of water $=60$ cubic ft.
Therefore bricks can occupy $160-60=100$ cubic ft. of space if they do not absorb any water. But as the bricks absorb $\frac{1}{26}$th of their own volume of water, consequently there will be more than 100 cubic ft. of space for them.
$\therefore$ volume occupied by bricks $=100\times\frac{26}{25}=104$ cubic ft.
But volume of a brick $=\frac{9}{12}\times\frac{6}{12}\times\frac{8}{36}=\frac{1}{12}$ cubic ft.
$\therefore$ number of bricks $=104\div\frac{1}{12}=1248$. **Ans.**

EXERCISE 84.

Note. The first 11 questions are to be done mentally.

Find the cubic contents of the rectangular solids having the following dimensions:—

1. 4 ft., 3 ft., $1\frac{1}{2}$ ft.
2. 3 ft., 2 ft., 1 ft.
3. 9 in., 6 in., 2 in.
4. 10 in., 5 in., $1\frac{1}{2}$ in.
5. 3 ft., $2\frac{1}{2}$ ft., $1\frac{1}{2}$ ft.
6. 8 cm., 6 cm., 5 cm.
7. 18 in., 12 in., 8 in.
8. 4 ft., 3 ft., $2\frac{1}{4}$ ft.
9. 5 yds., 4 yds., $1\frac{1}{2}$ ft.
10. $5\frac{1}{3}$ yds., 2 ft., $1\frac{1}{2}$ ft.

11. Find the volume of a cube whose edge is
(*a*) 4 inches. (*b*) 6 inches, (*c*) 3 yds.

12. Find the longest rod that can be placed in a room having dimensions 16 ft., $12\frac{1}{2}$ ft. and 18 ft.

13. Find the diagonal of a cube whose edge is 9 ft.

Find the surface of the rectangular solids in examples 14 and 15, whose dimensions are the following:—

14. Length 1 ft. 3 in., breadth 11 in. and height 8 inches.
15. Length 4 ft. 6 in., breadth 2 ft. 4 in. and height 1 ft. 3 inches.
16. The edge of a wooden cube is 2 ft. 2 in. Find its surface.
17. Find the surface of a cube whose edge is 4 ft. 3 in.
18. A solid is measuring 3 ft. 4 in. each way, find its capacity.
19. A room is 18 ft. long, 12 ft. broad and 9 ft. high ; how many cubic yds. of air does it contain ?
20. A room is 16 ft. 8 in. long, 12 ft. 4 in. broad and 9 ft. high ; how many cubic yards and cubic feet of air does it contain ?
21. How many bricks each 9 in. by 5 in. by $1\frac{1}{2}$ in., are required for a wall 18 ft. long, 15 ft. high and $2\frac{1}{2}$ ft. thick ?
22. How many bricks, each 8 in. by 6 in. by 2 in. are required for a platform 21 ft. long, 12 ft. broad and $4\frac{1}{2}$ ft. high ?
23. How many bricks, each 8 in. by 4 in. by 2 in. are required for a wall 15 ft. long, 12 ft. high, and $2\frac{1}{2}$ ft. thick, leaving in it a doorway 6 ft. by 4 ft. ?

24. Find (correct to the nearest pie) the cost of painting the surface of a box, each edge of which is 3 ft. 4 in. at 8*p*. per sq. ft.

25. Find the cost of painting the surface of a box the dimensions of which are, length 4 ft., breadth 3 ft. and height $2\frac{1}{2}$ ft. at 6*p*. per sq. ft.

26. A log of wood 16 ft. long, $2\frac{1}{2}$ ft. broad and 1 ft. thick is purchased at 2*a*. 3*p*. per cubic ft. and the cost of varnishing it is 4*p*. per sq. ft. Find the total cost.

27. A reservoir 21 ft. long 15 ft. broad and 6 ft. deep contains water. Find the weight of the water when a cubic ft. of water weighs 1000 ounces.

28. A cubic foot of water weighs 60 lbs. Find in tons, etc., the weight of a rainfall of 2 inches over a field 16 yards long and 8 yards wide.

29. A cubic foot of water weighs $6\frac{1}{2}$ gallons. Find in gallons the water of a rainfall of $1\frac{1}{2}$ inches over a park 18 ft. by 16 ft.

30. A ditch is 12 ft. long, 8 ft. wide and 4 ft. deep. Find the cost of filling it with earth when 48 cubic ft. of earth cost Rs. 5, 12*a*.

31. A box with a lid measuring on the outside 3 ft. $1\frac{1}{2}$ in. in length, 2 ft. $7\frac{1}{2}$ in. in breadth and 1 ft. $5\frac{1}{2}$ inches in height is made of wood $\frac{3}{4}$ in. thick ; how many cubic ft. of air does it contain ?

32. The interior dimension of a box made of wood of 1 in. in thickness are 3 ft. 4 in. long, 2 ft. 10 in. broad and 2 ft. 4 in. high. Find the weight of the box if a cubic ft. of the wood weighs 36 lbs.

33. The exterior dimensions of a box made of wood of $1\frac{1}{2}$ in. in thickness are 4 ft. long, 3 ft. broad and $2\frac{1}{2}$ ft. high. Find how many books 6 in. long, 3 in. broad and $\frac{3}{4}$ in. thick are required to fill it.

34. A cistern 22 yds. long, 15 yds. wide and 2 yds. deep is to be filled by a pipe whose sectional area is 24 sq. in. How long will it take to fill it when the water flows 5 miles an hour ?

35. A reservoir 33 ft. long, 21 ft. wide, $7\frac{1}{2}$ ft. deep is filled in 2 hours 37 min. 30 sec. by, pipe whose bore is 4 in. $\times$ 3 in. How fast does the water flow in the pipe ?

36. A cistern 6 ft. long, 3 ft. wide and 4 ft. deep contains 42 cubic ft. of water. How many bricks 8 in. by 4 in. by 3 in. can be put in it, when a brick is supposed to absorb $\frac{1}{7}$ of its own volume of water ?

37. A crow wishing to quench its thirst came to a vessel which contained 28 cubic inches of water. The crow being unable to reach the water picked up several small stones, each three quarters of a cubic inch in size and let them drop into the vessel until the water came to the top of it. If the size of the vessel was such that it could exactly hold 73 cubic inches of water, find the number of stones dropped in the vessel by the clever crow.

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§8. In the foregoing exercises we have thoroughly practised the formulae:—

Length × breadth × height = volume.

whence Length = volume ÷ (breadth × height.)

Breadth = Volume ÷ (length × height).

and Height = volume ÷ (length × breadth.)

Example 1. The volume of a couboid is 61 cubic ft. 432 cubic in. ; its length is 5 ft. 5 in. and the thickness is 2 ft. 6 in. Find the breadth.

Sol. ∵ breadth = volume ÷ (length × thickness)

∴ reqd. breadth $= 61\frac{1}{4} \div (5\frac{1}{4} \times 2\frac{1}{2})$ ft.

$= \frac{245}{4} \times \frac{4}{21} \times \frac{2}{5}$ ft.

$= \frac{14}{3}$ ft. or 4 ft. 8 in. **Ans.**

Example 2. A rectangular solid costs Rs. 5. 10*a*. at 2*a*. 3*p*. per cubic ft. Find its thickness if its length be 8 ft. and breadth 2 ft. 6 in.

Sol. $\frac{9}{4}a$. is the cost of 1 cubic ft.

∴ 1*a*. ,, ,, $\frac{4}{9}$ cubic ft.

∴ 90*a*. ,, ,, $\frac{4}{9} \times 90$ or 40 cubic ft.

Now 40 cubic ft. is the volume of the rectangular solid.

∵ thickness = volume ÷ (length × breadth)

∴ reqd. thickness $= \frac{40 \times 2}{8 \times 5} = 2$ ft. **Ans.**

EXERCISE 85

1. The cubic content of a rectangular solid is 30 cubic ft.; the length is 4 ft. and the breadth 3 ft. Find its height.

2. The cubic content of a rectangular piece of wood is 40 cubic ft.; the length is 5 ft. and the breadth is 4 ft. Find its thickness.

3. What will be the width of a rectangular piece of wood the volume of which is 15 cubic ft., the length 5 ft. and the thickness $1\frac{1}{2}$ ft.?

4. Find the thickness of a rectangular stone the capacity of which is 25 cubic ft., the length 5 ft. and the breadth 3 ft.

5. The length and the thickness of a rectangular stone are 8 ft. and $3\frac{1}{2}$ ft. respectively. Find its breadth when its volume is 112 cubic ft.

6. The cubic content of a log of wood is 64 cubic ft. 1008 cubic in., its length is 15 ft. 6 in. and the breadth 2 ft. 6 in. Find its thickness.

7. The volume of a heap of coal is 16 cubic yards 6 cubic ft. 1296 cubic in. What will be its height if the length be 15 ft. and breadth $6\frac{1}{2}$ ft.?

8. The volume of a heap of coal 16 ft. long $8\frac{1}{2}$ ft. broad is 27 cubic yds. 19 cubic ft. Find its breadth.

9. The cubic content of a log of wood 28 ft. long and $1\frac{1}{2}$ ft. thick is 105 cubic ft. Find its breadth.

10. A cistern 10 ft. long and 6 ft. 8 in. broad contains 14 cubic yds. 22 cubic ft. of water. Find its depth.

11. A log of wood whose edge is a square contains 36 cubic ft. Find its width, if the length be 16 ft.

12. A log of wood whose edge is a square contains 28 cubic ft. 216 cubic in.; its length is 18 ft. Find its thickness.

13. A log of wood whose edge is a square of 1 ft. 4 in. side contains 1 cubic yd. 5 cubic ft. Find its length.

14. A solid stack 5 ft. by 5 ft. by 3 ft. contains 1728 bricks, each 10 in. long and 5 in. broad; find the thickness of each brick.

15. A solid stack 8 ft. by 4 ft. contains 2560 bricks, each 9 in. $\times$ 4 in. $\times$ $4\frac{1}{2}$ in.; find the height of the stack.

16. A cistern 24 ft. long, 15 ft. broad is filled in 3 hours by a pipe whose sectional area is 9 sq. in. The water flows in the cistern at 4 miles an hour; find the depth of the cistern.

17. A piece of ground is 21 yds. long and 18 yds. wide. To what uniform height must earth be spread upon it, that it may cost the owner Rs. 210 at Rs. 3, 5*a*. 4*p*. per cubic yard ?

18. A cistern containing 600 gallons measures externally 7 ft. by 2 ft. 11 in. by 5 ft. 6 in. ; the sides are $1\frac{1}{2}$ in. thick ; find the thickness of the bottom, supposing $6\frac{1}{4}$ gallons = 1 cubic ft.

19. In a box which measures internally 4 ft. by 3 ft. I can pack 2880 books each 6 in. long, 3 in. wide and $1\frac{1}{2}$ in. thick. Find the depth of the box.

20. In a box measuring internally $3\frac{1}{2}$ ft. by 3 ft. by $2\frac{1}{2}$ ft. I can pack 2016 books each 6 in. long and 3 in. wide. Find the thickness of each book.

§9. Volume of a prism.

Definition. A right prism is a solid bounded by plane faces, the two ends of which are congruent papallel figures and the side faces are rectangles.

If we take a wooden cuboid with dimensions—length 5", breadth 4" and height 3" and draw two diagonals parallel to each other, one on the lower surface and the other on the upper surface of it and with a saw cut the cuboid into two equal parts across these diagonals, each part will then represent a right prism, the base of which will be a right-angled triangle.

Since the volume of the cuboid $= 5 \times 4 \times 3$ cub. inches

$\therefore$ " " " prism $= \frac{5 \times 4 \times 3}{2} = 10 \times 3$ cub. in.

where 10 sq. in. is the area of the right-angled triangle or the area of the base of the prism. Therefore

Volume of a prism = Area of the base × height.

The base of the right prism may be an equilateral triangle, a trapezium, a pentagon, a parallelogram, etc., etc., but the method of finding the volume in each case is the same as mentioned above.

§10. Volume of a right circular cylinder.

Since the upper and lower surfaces of a right circular cylinder are equal and parallel to each other.

$\therefore$ Volume of a cylinder = Area of the base × length and area of the curved surface = circumference × height.

Example 1. The area of the base of a right prism is 9 sq. in. and its height is 5 inches. Find the volume.

Sol. Volume = area of the base × height.
$= 9 \times 5 = 45$ cubic inches. **Ans.**

Example 2. The base of a right prism is an equilateral triangle whose side is 2 ft. If the height be 5 ft., find the volume.

Sol. Volume = area of the base × height
$= (2 \times 2 \times \cdot 433) \times 5$ cub. ft.
$= 1 \cdot 732 \times 5$ cub. ft.
$= 8 \cdot 66$ cub. ft. **Ans.**

Example 3. Find the volume and the curved surface of a cylinder, the height of which is 8 inches and a base of diameter 1 in.

Sol. (*i*) Volume = area of the base × height
$= (\frac{1}{2} \times \frac{1}{2} \times \frac{22}{7}) \times 8$ cub. in.
$= \frac{11}{14} \times 8 = \frac{44}{7}$ or $6\frac{2}{7}$ cub. in. **Ans.**

(*ii*) Curved surface = circumference × height
$= (1 \times \frac{22}{7}) \times 8$ sq. in.
$= \frac{22}{7} \times 8$ or $\frac{176}{7}$ sq. in. $= 25\frac{1}{7}$ sq. in. **Ans**

Example 4. The external diameter of a hollow cylindrical tube made of iron $\frac{1}{2}$ in. thick is 3 inches. If the length of the tube be 21 ft.; find the number of cubic inches of iron in it.

Sol. External radius of the tube $= 1\frac{1}{2}$ in.
Internal ,, ,, ,, $= 1\frac{1}{2} - \frac{1}{2} = 1$ in.
∴ volume of the tube $= \frac{3}{2} \times \frac{3}{2} \times \frac{22}{7} \times 21 \times 12$ cub. in.
$= 1782$ cub. in.
and internal volume of the tube $= 1 \times 1 \times \frac{22}{7} \times 21 \times 12$ cub. in.
$= 792$ cub. in.
∴ volume of iron $= 1782 - 792 = 990$ cub. in. **Ans.**

Example 5. Two cylindrical iron pipes, each open at both ends, have equal internal volumes. The external and internal diameters of one are 11 and 10 inches, and those of the other are $5\frac{3}{4}$ and 5 inches. Compare the quantity of iron used in each pipe.

Sol. Let h and H inches be the respective length of the pipes.

Then the internal volume of the 1st $=25\pi h$

and the internal volume of the 2nd $=\frac{25}{4}\pi H$

$\therefore \frac{25}{4}\pi H = 25\pi h$ [Given.

or $H = 4h$.

$\therefore$ quantity of iron of the 1st $=[(\frac{11}{2})^2-(5)^2]\pi h$

and quantity of iron in the 2nd $=[(\frac{23}{8})^2-(\frac{5}{2})^2]\times\pi H$

$\therefore$ 1st : 2nd $=\frac{21}{4}\pi h : \frac{129}{64}\pi H$

$=28 : 43$. [$H=4h$.

EXERCISE 86.

Note. The first 5 questions to be done mentally.

1. The area of the base of a right prism is 21 sq. ft. and height is 5 ft. Find the volume.

2. The base of a right prism is a parallelogram whose adjacent sides are 12 ft. and 8 ft. Find the volume of the prism if its height be 10 ft.

3. The area of the base of a cylinder is 15 sq. in. and its height is 10 in. Find the volume.

4. Find the volume of a cylinder which has a height of 10 in. and a base of radius 7 in.

5. Find the volume of a cylinder the height of which is 14 in. and the base of radius 1 in.

6. The base of a right prism is a regular hexagon, whose area is 3·125 sq. in. If the height is 8 in. find the volume.

7. The area of the base of a right prism is 12 sq. ft. 1.125 sq. in. and its height is 8 in. Find the volume.

8. The base of a right prism is a rectangle whose adjacent sides are 12 ft. and 9 ft. If the height of the prism be 6 ft., find the volume.

9. The base of a right prism is a rhombus whose diagonals are 10 ft. and 8 ft. If the height of the prism be 6 ft. find the volume.

10. The base of a right prism is a right-angled triangle; the sides making the right angle are 4 ft. and 3 ft. If the height of the prism be 5 ft., find the volume.

11. The radius of a cylindrical tube is $\frac{3}{4}$ in. and its height is 28 ft. Find the quantity of water it can contain.

12. The diameter of a cylindrical tube is 2 in. and its height is 21 ft.; how much water can it contain?

13. The radius of a well is 7 ft. and its depth 30 ft. How much earth was taken out when it was excavated ?

14. The radius of a well is $4\frac{3}{8}$ ft. How much water does it contain if the depth of the water is 32 ft. ?

15. A well contains 1760 cubic ft. of water and its radius is 4 ft. Find the depth of the water.

16. The depth of water in a well is 25 ft. Find its radius if it contains $962\frac{1}{2}$ cubic ft. of water

17. The radius of a well is $3\frac{1}{2}$ ft. and its wall is 1 ft. thick. If its depth be 28 ft. find the volume of its wall.

18. A well, $4\frac{1}{2}$ ft. inside radius is to be sunk 28 ft. deep with a brick lining 1 ft. thick. How many bricks each 9 in. by 4 in. by 3 in. are required to construct it ?

19. A hollow cylindrical tube open at both ends is made of iron 13 in. thick. If the external diameter be 2 ft. 3 in. and the length of the tube be 14 ft., find the number of cubic feet and inches of iron in it.

20. The external diameter of a hollow cylindrical tube (open at both ends) made of iron 2 in. thick is 1 ft. 10 in. The length of the tube is 21 ft., find the number of cubic ft., etc., of iron in it.

21. A cylindrical vessel contains $111\frac{3}{8}$ cubic inches of water and its radius is $2\frac{1}{4}$ in. Find the depth of the vessel.

22. A cylindrical vessel contains 11 lbs. of water and its radius is $3\frac{1}{2}$ in. Find the depth of the vessel if a cubic ft. of water weighs $61\frac{5}{7}$ lbs.

23. A tube is 110 yards long and its radius is 8 in. How many gallons of water can it contain when a cubic ft. of water $=5\frac{1}{4}$ gallons ?

24. A well 7 ft. inside diameter has been sunk 21 ft. deep. Earth taken out of it forms an embankment of uniform width of $10\frac{1}{2}$ ft. round the well. Find the height of the embankment.

25. A well has been excavated 10 ft. deep. The earth taken out from it has been spread all round it to a uniform width of 7 ft. to form an embankment. Find the height of the embankment when the diameter of the well is 14 ft.

26. The external diameter of an open cylindrical pipe made of iron $\frac{1}{2}$ inch thick is 13 inches and the internal diameter of another made of the same metal $\frac{3}{8}$ inch thick is 6 inches ; compare the quantities of metal in the pipes if they have equal internal volumes.

§9. We now close this chapter with the recapitulation of the mensurational formulae.

Mensurational Formulae.

1. **Rectangle.**

(*i*) Area = length × breadth.
(*ii*) Length = area ÷ breadth.
(*iii*) Breadth = area ÷ length.

2. **Square.**

(*i*) Area = $(\text{side})^2$ or half the square of its diagonal.
(*ii*) Side = $\sqrt{\text{area}}$.
(*iii*) Diagonal = side $\sqrt{2}$ or $\sqrt{2 \text{ area}}$.

3. **Parallelogram.**

(*i*) Area = base × height.
(*ii*) Base = area ÷ height.
(*iii*) Height = area ÷ base.

4. **Triangle.**

(*a*) Right-angled △:—

(*i*) $\text{Area} = \frac{\text{base} \times \text{perpendicular}}{2}$.

(*ii*) $\text{Base} = (a)\frac{\text{area} \times 2}{\text{perpendicular}}$

(*b*) $\sqrt{(\text{hypotenuse})^2 - (\text{perpendicular})^2}$.

(*iii*) $\text{Perpendicular} = (a)\frac{\text{area} \times 2}{\text{base}}$

(*b*) $\sqrt{(\text{hypotenuse})^2 - (\text{base})^2}$

(*iv*) $\text{Hypotenuce} = \sqrt{(\text{base})^2 + (\text{perpendicular})^2}$.

(*b*) Isosceles Right-angled △ :—

(*i*) $\text{Base} = \frac{\text{hypotenuse}}{\sqrt{2}}$.

(*ii*) $\text{Perpendicular} = \frac{\text{hypotenuse}}{\sqrt{2}}$

(*c*) Equilateral △ :—

(*i*) Area = side × side × ·433
(*ii*) Height = side × ·866.
(*iii*) Side = height ÷ ·866.

(*d*) Any triangle :—

Area = (*i*) $\frac{\text{base} \times \text{height}}{2}$

(*ii*) $\sqrt{s(s-a)(s-b)(s-c)}$, where s denotes semi-perimeter and a, b, c, the sides of the $\triangle$.

5. Rhombus.

(*i*) Area = (*a*) $\frac{1}{2}$ × product of its diagonals.
(*b*) Base × height.

(*ii*) Diagonal = $\frac{2 \times \text{area}}{\text{other diagonal}}$

6. Quadrilateral.

(*i*) Area = $\frac{1}{2}$ diagonal × (sum of its offsets).

(*ii*) Diagonal = $\frac{2 \times \text{area}}{\text{sum of its offsets}}$

(*iii*) Area of a quadrilateral inscribed in a circle $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$, where s is the semi-sum of sides a, b, c, d.

7. Trapezoid.

(*i*) Area = $\frac{1}{2}$ × sum of parallel sides × height.

(*ii*) Height = $\frac{2 \times \text{area}}{\text{sum of parallel sides}}$

8. Regular Polygon.

(*i*) Area = $\frac{\text{No. of sides}}{2}$ × side × radius of inscribed circle.

(*ii*) Side = $\frac{2 \times \text{area}}{\text{No. of sides} \times \text{radius of inscribed circle}}$

(*iii*) Area of Hexagon = $\frac{3a^2\sqrt{3}}{2}$ where a is the side.

(*iv*) Area of Octagon = $2a^2(1+\sqrt{2})$,, ,,

9. Circle.

(*i*) Area = radius^2 × π ; [$\pi = \frac{22}{7}$

(*ii*) Radius = $\sqrt{\text{area} \times \frac{7}{22}}$.

(*iii*) Diameter = circumference × $\frac{7}{22}$.

(*iv*) Circumference = diameter × $\frac{22}{7}$.

10. Arc.

Arc of D° = $\frac{D}{360}$ × circumference.

11. Sector of D^0

(*i*) Area $= \frac{D}{360} \times$ area of circle;

(*ii*) Area $= \frac{1}{2}$ arc $\times$ radius.

12. Segment.

Area = sector − triangle.

13. Cuboid.

(*i*) Surface $= 2(lb + lh + bh)$; where l denotes length, b breadth and h height.

(*ii*) Volume $= l \times b \times h$.

14. Cube.

(*i*) Surface $= 6(\text{edge})^2$;

(*ii*) Volume $= (\text{edge})^3$.

15. Prism.

(*i*) Lateral surface = (perimeter of base) $\times$ height.

(*ii*) Volume = area of base $\times$ height,

16. Cylinder.

(*i*) Curved surface $= 2r\pi h$.

(*ii*) Volume $= r^2\pi h$.

17. Cone.

(*i*) Volume $= \frac{1}{3}$ area of base $\times$ height $= \frac{1}{3}r^2\pi h$.

(*ii*) Curved surface $= \frac{1}{2}$ perimeter of base $\times$ slant height $= \pi rl$, where l is slant height.

(*iii*) Whole surface $= \pi r l + r^2\pi = \pi r(l + r)$.

Again ; if h, the height of the cone, be given

$$l = \sqrt{h^2 + r^2}$$

$\therefore$ whole surface $= r\pi(\sqrt{h^2 + r^2} + r)$.

18. Sphere. (*i*) Surface $= 4\pi r^2$, (*ii*) Volume $= \frac{4}{3}\pi r^3$.

19. Pyramid. Volume $= \frac{1}{3}$ area of base $\times$ height.

20. If the sides opposite the angles A, B, C of the $\triangle$ ABC be a, b, c ; $\triangle$ its area ; R circumradius ; r inradius ; r_1, r_2, r_3 ex-radii, then

(*i*) $\triangle = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \frac{1}{2}(a+b+c)$;

(*ii*) $R = \frac{abc}{4\triangle}$ (*iii*) $r = \frac{\triangle}{s}$

(*iv*) $r_1 = \frac{\triangle}{s-a}$; $r_2 = \frac{\triangle}{s-b}$; $r_3 = \frac{\triangle}{s-c}$

CHAPTER XVI

UNITARY METHOD AND CHAIN RULE

1. Unitary Method.

§1. If we know the price, weight, length of any number of units, we can, by division ; find the price of one unit of the same substance and when that is found we can, by muliplication, find the price, weight, length, etc., of any number of units of the same substance. The process by which we combine the two processes described above, is called the **unitary method.**

Example 1. If 2 books cost Rs. 6, what will one book cost ?

Sol. The cost of 2 books = Rs. 6

$\therefore$,, ,, 1 book = Rs $6 \div 2$ Rs. 3 **Ans.**

Example 2. 1 book costs Rs. 3, what will 7 books cost ?

Sol. The cost of 1 book = Rs. 3

$\therefore$,, ,, 7 books = Rs. 3×7 = Rs. 21 **Ans.**

Now if we combine the two examples given above we form the following example :—

Example. If 2 books cost Rs. 6, what will 7 books cost ?

Sol. The cost of 2 books = Rs. 6

$\therefore$,, ,, of 1 book = Rs. $6 \div 2$ = Rs. 3

$\therefore$,, ,, of 7 books = Rs. 3×7 = Rs. 21. **Ans.**

First the price of one article is found by division and then the price of the required number of articles is obtained by multiplication.

Observe also that in every unitary question two of the three terms are **like** terms and the 3rd is an **unlike** term, it is similar to the term we obtain as answer.

Thus in the above example, 2 books and 7 books are like terms and the 3rd term Rs. 6 is an unlike term.

In solving such questions the 3rd unlike term is placed last in the first line as shown in the examples solved above. The following solved examples will further illustrate the method clearly :—

Example 3. If 7 yards of cloth cost Rs. 15, 5*a.* ; what will 11 yds. of cloth cost ?

Sol. Cost of 7 yards = Rs. 15, 5*a.*
∴ ,, ,, 1 yard = Rs. 15, 5*a.* ÷ 7 = Rs. 2, 3*a.*
∴ ,, ,, 11 yards = Rs. 2, 3*a.* × 11 = Rs. 24, 1*a.* **Ans.**

Example 4. If $\frac{2}{15}$ of an estate be worth Rs. 85, 8*a.*, what is the value of $\frac{13}{15}$ of it ?

Sol. $\frac{2}{15}$ of an estate is worth = Rs. 85, 8*a.*
∵ $\frac{1}{15}$ of ,, ,, = Rs. 85, 8*a.* ÷ 2
= Rs. 42, 12*a.*
∴ $\frac{13}{15}$,, ,, ,, = Rs. 42, 12*a.* × 13
= Rs. 555, 12*a.* **Ans.**

Example 5. 6 men or 9 women earn Rs. 75 in a certain time : how much will 10 men and 6 women earn in the same time ?

Sol. ∵ 6 men earn as much as 9 women,
∴ 1 man earns as much as $\frac{9}{6}$ women,
∴ 10 men earn as much as $\frac{9}{6} \times 10 = 15$ women.
∴ 10 men and 6 women earn as much as 15 + 6 = 21 women.

Now 9 women earn Rs. 75,
∴ 1 woman earns Rs. $\frac{75}{9}$
∴ 21 women earn Rs. $\frac{75}{9} \times 21$ = Rs. 175. **Ans.**

Example 6. 10 horses and 15 cows eat grass of 5 acres in a certain time ; how many acres will feed 15 horses and 10 cows for the same time, supposing a horse eats as much as 2 cows ?

Sol. Since 1 horse eats as much as 2 cows,
∴ 10 horses will eat as much as 20 cows,
∴ 10 horses and 15 cows will eat as much as 20 + 15 = 35 cows.

Similarly 15 horses and 10 cows will eat as much as 40 cows.

Now 35 cows eat grass of 5 acres,
∴ 1 cow will eat grass of $\frac{5}{35}$ acre,
∴ 40 cows ,, ,, ,, ,, $\frac{5}{35} \times 40 = \frac{40}{7}$ acres,
$= 5\frac{5}{7}$ **Ans.**

EXERCISE 37.

1. If 9 articles cost Rs. 17, 7*a*., what will one article cost ?

2. Railway fare for 41 miles is 6*a*. 10*p*. ; what is the fare for 66 miles ?

3. If I run a mile in 7 min. 12 sec., how long will I take to run 440 yards ?

4. If one article costs Rs. 2, 3*a*., what will 12 articles cost ?

5. If 6 chairs cost Rs. 32, 12*a*., what will 3 dozen cost ?

6. If 21 yards of cloth cost Rs. 43, 5*a*., what will one yard of cloth cost ? And also what will 15 yards of cloth cost ?

7. If 16 seers of sugar cost Rs. 6, 8*a*., what is the cost of 9 seers ?

8. 52 yards of cloth cost Rs. 39 ; what will 8 yards of cloth cost ?

9. A man earns Rs. 33, 12*a*. in 15 days ; what will he earn in 20 days ?

10. A man walks $4\frac{1}{2}$ miles in 72 min. ; find his rate per hour in metres.

11. Find the cost of 37 yards of silk if 25 yards cost Rs. 65, 10*a*.

12. 16 men can reap 85 bighas of field, how many bighas will 24 men reap ?

13. If the railway fare for 75 miles be Rs. 4, 11*a*, what is the fare for 49 miles. ?

14. The price of 10 bullocks is equal to the price of 15 sheep ; how many bullocks can be purchased for the price of 39 sheep ?

15. If 37 chairs cost £8, 11s. $1\frac{1}{2}$*d*., how many can be had for £70, 6*s*?

16. If 15 lbs. of tea cost 18s. 9*d*., how much would 1 qr. of tea cost ?

17. What would be the simple interest on Rs. 625 if the interest on Rs. 100 is Rs. $3\frac{1}{2}$?

18. If $2\frac{1}{2}$ cwt. cost £1, 9*s*. 2*d*., what will 21 lbs. cost ?

19. If 15 seers of sugar cost Rs. 3, 12*a*., find the price of $1\frac{3}{4}$ maunds.

20. A man runs a mile in 5 minutes 30 seconds, how long will he take to run 440 yards ?

21. If 3 md. 12 sr. 8 chk. of sugar cost Rs. 16. 9*a*, what will 1 md. 7 sr. 5 chk. cost ?

22. I travel 3 miles 7 fur. 20 poles in 1 hour ; how far can I go in 6 hrs. 14 min. 40 sec. ?

23. If 24 men can reap 16 acres in a cartain period, how many men will reap 40 acres in the same time ?

24. If $\frac{3}{17}$th of an estate be worth Rs. 110, 1*a*., what is the value of $\frac{11}{17}$th of the estate ?

25. If $\frac{1}{8}$th of a ship is worth £70, 19*s*· 6*d*., what part of her is worth £250, 10*s*. ?

26. If $\frac{7}{8}$th of a property be worth Rs. 141, 12*a*., find the value of $\frac{5}{8}$th of that property.

27. The value of $\frac{4}{13}$th of a cargo is Rs. 575, 12*a*. ; what is the value of $\frac{7}{13}$th of that cargo ?

28. If $\frac{7}{8}$th of a house be worth Rs. 856, 3*a*., find the value of $\frac{5}{12}$th of that house.

29. If $\frac{5}{16}$th of a cargo is worth Rs. 1055, 5*a*., what is the value of $\frac{7}{24}$th of that cargo ?

30. A man rides $46\frac{1}{2}$ miles in 5 hours 10 minutes, how much time will he take to ride $28\frac{1}{2}$ miles ?

31. If 6 men do as much work as 8 women, how many men will do as much work as 12 women ?

32. If 6 bullocks or 15 sheep cost Rs. 225, what will 9 bullocks and 12 sheep cost ?

33. A railway train runs at the rate of 22 miles an hour ; how far will it go between 6-15 A.M. and 2-35 P.M. ?

34. 6 cows and 5 horses graze 5 acres of grass in a certain time ; how many acres of grass will 8 cows and 9 horses graze for the same time, supposing 2 cows to eat as much as 3 horses ?

35. If 6 bullocks and 16 sheep eat the grass of 2 acres in a certain time, how many acres will feed 15 bullocks and 6 sheep for the same time, supposing a bullock eats as much grass as 3 sheep ?

36. A man employed a servant on the condition that he would get Rs. 45 and a horse after a month ; but after 12 days he dismissed the servant, giving him only a horse ; find the value of the horse. (*Hint.* Salary for 18 days=Rs. 45.)

37. A man employed a servant on the condition that he would get a horse and Rs. 40 after a month; but after 18 days he dismissed him giving him only a horse; find the value of the horse.

§2. **An increase in the number of workmen corresponds to diminution in the number of days and** *vice versa.*

Example 1. If 15 men can do a piece of work in 10 days, how long will it take one man to do it ?

Sol. 15 men can do the work in 10 days

$\therefore$ 1 man ,, ,, ,, ,, 10×15 days

i.e., 150 days. **Ans.**

Example 2. If one man can do a piece of work in 150 days ; how long will 25 men take to do it ?

Sol. 1 man can do the work in 150 days.

$\therefore$ 25 men ,, ,, in $150 \div 25$ days.

i.e., 6 days. **Ans.**

Example 3. If 15 men can do a piece of work in 10 days, how long will it take 25 men to do it ?

Sol. 15 men can do a piece of work in 10 days

$\therefore$ 1 man can do the same work in 10×15 days

$\therefore$ 25 men ,, ,, in $(10 \times 15) \div 25$ days.

i. e., 6 days **Ans.**

Example 4. If 10 men can mow a field in 5 days, how many men will be required to mow the same field in $12\frac{1}{2}$ days ?

Sol. In 5 days the field is mowed by 10 men

$\therefore$ in 1 day ,, ,, by 10×5 men

$\therefore$ in $\frac{25}{2}$ days,; ,, by $\frac{10 \times 5 \times 2}{25}$ men

i. e., 4 men. **Ans.**

Example 5. A garrison of 1500 men is provisioned for 60 days. After 25 days the garrison is reinforced by 500 men ; how long will the remaining provisions last ?

Sol. Since the garrison is reinforced by 500 men therefore there are $(1500+500)$ or 2000 men now ; $60-25=35$ days.

The provisions left would last 1500 men 35 days

$\therefore$,, ,, ,, 1 man 35×1500 days

$\therefore$,, ,, ,, 2000 men $\frac{35 \times 1500}{2000}$ days.

$= \frac{105}{4}$ days, *i.e.*, $26\frac{1}{4}$ days. **Ans.**

Note. Sometimes it so happens, that in a question, at first sight it would appear that more than 3 terms are given but such an example in some cases, comes under the same rule, as in the following example:-

☞ Note carefully the solutions of Ex. 6, 7.

Example 6. A contractor undertook to do a certain work in 55 days and employed 18 men to do it. In 11 days only $\frac{1}{5}$ of the work was done. How many extra men should he employ in order to complete the work in time ?

Sol. In 11 days only $\frac{1}{5}$th of the work was done. It is evident, therefore, that the remaining $\frac{4}{5}$th will be finished in 55 days ; but he wants to finish the rem. work in $55-11=44$ days.

In 55 days the rem. work can be finished by 48 men

$\therefore$ in 1 day " " " by 48×55 men

$\therefore$ in 44 days " " " by $\frac{48\times55}{44}$ men

i.e., 60 men.

$\therefore$ he should employ $60-48$ or 12 men more. **Ans.**

Example 7. 5 men and 6 boys finish a piece of work in 4 days : 4 men and 6 boys in 6 days. In how many days would 3 men and 6 boys finish the same work ?

Sol. 5 men and 6 boys finish the work in 4 days.

$\therefore$ 20 men and 24 boys will finish it in 1 day.....(1)

also 4 men and 3 boys will finish it in 6 days,

$\therefore$ 24 men and 18 boys will finish it in 1 day.....(2)

From (1) and (2) it follows, therefore, that the work of 20 men and 24 boys=the work of 24 men and 18 boys.

Or the work of 4 men=the work of 6 boys, [by subtraction]

" " 2 men= " " 3 boys.

$\therefore$ the work of 5 men and 6 boys=the work of 9 men.

also the work of 3 men and 6 boys=the work of 7 men.

Now the question is : " 9 men can finish a piece of work in 4 days. In how many days would 7 men finish it "?

EXERCISE 88.

Note. Some of the examples in this exercise are cases of direct method.

1. If 8 men can finish a piece of work in $17\frac{1}{2}$ days, how many men are required to do the same work in 7 days ?

2. If 15 men can reap a certain field in 17 days, in how many days could 9 men reap the field ?

3. If 28 men can finish a piece of work in 48 days, in how many days could 35 men do it ?

4. If 9 maunds of grass are eaten by 21 cows in 14 days how long will it take 7 cows to eat the same grass?

5. If 8 men or 12 women can do a piece of work in 15 days, how long will it take 6 men and 9 women to do the same work?

6. If 15 bullocks or 25 horses can mow a filed in 10 days, in what time could 9 bullocks and 15 horses mow the same field?

7. If 17 oxen or 11 horses eat the grass of a field in 37 days, how long will it take 5 oxen and 8 horses to eat it?

8. 9 men and 15 boys can reap a field in 15 days; in what time could 15 men and 16 boys reap the same field, supposing 3 men do as much work as 4 boys?

9. If 7 boys earn as much as 4 men and 48 men with the help of 14 boys earn Rs. 423. 8*a*., how many boys must help 20 men, in order to earn Rs. 272, 4*a*, in the same time?

10. A garrison of 2100 men has provisions for 36 days; how many men must be turned out, so that the provisions may last for 54 days?

11. A garrison of 2700 men has provisions for 51 days; how long would the provisions last if the garrison be increased by 900 men?

12. A garrison of 1400 men has provisions for 54 days; if after 14 days the garrison be increased by 600 men, how long will the remaining provisions last?

13. A ship leaves a port with food enough to last 14 weeks; 6 of the crew absconded and the voyage lasted 16 weeks when it was found that the food had just exhausted. Find the number of the full crew.

14. A besieged garrison have 4 months provisions at the rate of 18 chks. per man per day. How long would they be able to hold out, if each man were allowed only 12 chks. per day?

15. A train travelling 40 miles an hour reaches its destination in 3 hours 15 minutes; how long will it take to reach the same place, travelling at the rate of 25 miles per hour.

16. A contractor undertook to do a certain work in 75 days and employed 60 men to do it; after 25 days he found that only $\frac{1}{4}$th of the work was done. How many more men must be employed in order that the work may be finished in time?

17. A contractor undertook to do a certain work in 80 days and employed 72 men to do it; after 20 days he found that $\frac{1}{3}$rd of the work has been finished. How many men should

he dismiss in order that the work may be finished on the date agreed upon ?

18. A contractor takes up the work of making a road 189 miles long in 15 months. He engages 256 men, but after $\frac{1}{5}$th of the time finds that only $\frac{1}{6}$th of the work has been completed How many more men should he now employ to complete the contract in time ?

19. If the 6*d.* loaf weighs 3·4375 lbs. when wheat is 55·5*s.* a quarter; what is the price of wheat when the loaf weighs 2·8125 lbs. ?

20. If the 4*d.* loaf weighs 3 chataks when the price of wheat is Rs. 3 a maund, what should its weight be when wheat is at Rs. 3, 6*a.* a maund ?

21. A piece of gold at Rs 38, 15*a.* per oz. is worth Rs. 1500 ; what should be the price of a piece of silver of equal weight at Rs. 27, 4*a.* per lb. ?

22. If 17 men can do a piece of work in 89 days and if after working for 33 days, 3 men leave, in how many days *in all* will the work be done ?

23. I take 55 min. to walk to cantonment by the road and 58 min. 40 sec. to return by the fields, walking at the same rate. The distance by the road is $3\frac{3}{4}$ miles ; what is it by the field ?

24. 6 men and 7 boys can finish a piece of work in 21 days. In how many days could 12 men and 6 boys finish the same work, supposing 2 men work as much as 3 boys ?

25. The Bombay Express travels 27 miles an hour including stoppages and 30 miles an hour when it does not stop. In what distance will it lose 45 minutes by stopping ?

26. 25 men were employed to do a piece of work in 24 days. After 15 days, 10 more men were engaged and the work was finished a day too soon. In what time could they finish the work if extra men were not employed ?

27. A fort is provisioned for 75 days ; after 25 days a reinforcement of 400 men arrived and the food will now last only 40 days. How many men were there in the fort ?

28. 7 men and 5 boys can finish a piece of work in 12 days ; 9 men and 25 boys in 5 days. In how many days could 6 men and 5 boys do the same work ?

29. 6 men and 8 boys finish a piece of work in 10 days ; 12 men and 4 boys in 8 days. In how many days would 3 men and 2 boys finish it ?

§Double Unitary Method.

Example 1. If 7 men working 9 hours a day can finish a piece of work in 14 days, in how many days will 12 men working 7 hours a day, finish the same work ?

Note. The answer required is in number of days, therefore put 14 days at the end of the first line.

Sol. 7 men, 9 hrs. a day, will finish work in 14 days

$\therefore$ 1 man, 9 hrs. ,, ,, ,, ,, in 14×7 days

$\therefore$ 1 ,, 1 hr. ,, ,, ,, ,, in $14\times7\times9$ days

$\therefore$ 12 men, 1 hr. ,, ,, ,, ,, in $\frac{14\times7\times9}{12}$ days

$\therefore$ 12 ,, 7 hrs. ,, ,, ,, ,, in $\frac{14\times7\times9}{12\times7}$ days

$=\frac{21}{2}$, *i. e.*, in $10\frac{1}{2}$ days. **Ans.**

Example 2. If 5 men working 8 hours a day can compose a book of 240 pages in 21 days, how long will it take 3 men to compose a book of 360 pages working 10 hours a day ?

Sol. 5 men, 8 hrs. a day, comp. 240 pages in 21 days

$\therefore$ 1 man, 8 hrs. ,, ,, 240 ,, ,, 21×5 ,,

$\therefore$ 1 ,, 1 hr. ,, ,, 240 ,, ,, $21\times5\times8$,,

$\therefore$ 1 ,, 1 hr. ,, ,, 1 ,, ,, $\frac{21\times5\times8}{240}$,,

$\therefore$ 3 ,, 1 hr. ,, ,, 1 ,, ,, $\frac{21\times5\times8}{240\times3}$,,

$\therefore$ 3 men, 10 hrs. ,, ,, 1 ,, ,, $\frac{21\times5\times8}{240\times3\times10}$,,

$\therefore$ 3 ,, 10 hrs. ,, ,, 360 ,, $\frac{21\times5\times8\times360}{240\times3\times10}$

i. e., in 42 days. **Ans.**

EXERCISE 89.

1. If 5 men working 8 hours a day can finish a piece of work in 20 days, how long will it take 8 men, working 12 hours a day, to do the same work ?

2. If 9 men reap a field of 8 acres in 12 hours, how many men will reap a field of 28 acres in 18 hours ?

3. If 6 men dig a trench in 5 days, working 12 hours a day, how many men will dig the same trench in 4 days working 10 eours a day ?

4. If 13 men can do a piece of work in 12 days of 8 hours, how many hours a day must 9 men work to do the same work in 16 days ?

5 If the wages of 12 men for 25 days amount to Rs. 450, what will be the wages of 20 men for 16 days ?

6. What will be the wages of 15 men for 10 months when 9 when receive £261, 15s. for 8 months ?

7. If 40 maunds of rice be carried 150 miles for Rs. 12,8a., for what sum of money will 50 maunds of rice be carried 320 miles ?

8. If a man travels 51 miles in $1\frac{1}{2}$ days by walking 9 hours a days in how many days will he travel 170 miles by walking $7\frac{1}{2}$ hours a day ?

9. If 10 masons build a house in 25 days of 6 hours, in how many days of 8 hours will 15 masons build the house ?

10. If 144 men in 5 days of 11 hours each can dig a trench 132 yds. long, 5 ft. wide aud 2 ft. deep ; in how many days of 9 hours each, can 56 men dig a trench 210 yds. long, 8 ft. wide and 3 ft. deep ?

11. If Rs. 500 bear an interest of Rs. 40 in 2 years, what interest at the same rate will Rs. 625 bear in $1\frac{1}{2}$ years ?

12. If with a capital of Rs. 10000, a person gains Rs. 500 in 16 months, in how many months will he gain Rs. 1250 with a capital of Rs. 4000 ?

13. If 200 men can make an embankment 5 miles long in 25 days, how much over time must 60 men work to finish an embankment 2 miles long in 32 days, 12 hours being a day's work ?

14. If 5 men can reap a field 800 ft. long and 700 ft. broad in $3\frac{1}{2}$ days of 14 hours each, in how many days of 18 hours each can 7 men reap a field 1800 ft. long and 960 ft. broad ?

15. If 39 men can reap a field 65 yards lon and 55 yards broad in 2 days, how many men are required to reap a field 77 yard long and 75 yards broad in 14 days ?

16. If 12 men working 8 hours a day take 15 days to do a piece of work, how long will 16 boys working 10 hours a day take to do double the work, the work of one man being equal to that of two boys ?

17. If 12 men or 15 women or 18 boys can do a piece of work in 15 days of 8 hours ; find how many men assisted by 5 women and 6 boys will finish the same work in 16 days of 9 hours each.

18. If 7 men or 8 women or 10 boys can finish a piece of work in 24 days of 9 hours; find how many men with the help of 4 women and 5 boys can finish it in 18 days of 6 hours.

19. When wheat is sold at Rs. 6 a maund, Rs. 27, 8*a*. maintain a family of 10 men ; what sum of money will maintain a family of 15 men, when wheat is sold at Rs. 5 a maund ?

20. When rice is 10 seers a rupee, a sum of money maintains a family of 18 men for 15 days ; how long will the same amount of money maintain a family of 6 men when rice is 14 seers a rupee ?

21. If a penny loaf weighs 6 oz. when wheat is 5*s*. 6*d*. a bushel, what should be the weight of a shilling loaf when wheat is 8*s*. 3*d*. a bushel ?

22. If 12 pumps can raise 1218 tons of water in 11 days of 9 hours each, how many pumps will raise 2030 tons of water in 12 days of 11 hours each ?

23. If 5 pumps each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of mine, how many pumps with a length of stroke of $2\frac{1}{2}$ ft. working 10 hours a day for 12 days will be required to empty the same mine, the strokes of the former pumps being performed 4 times as fast as those of the other ?

24. A besieged garrison consists of 300 men, 120 women and 40 children, and has provisions enough for 200 men for 30 days. If a woman eats $\frac{2}{3}$rd as much as a man and a child half as much as a man and after 6 days 100 men with all the women and children escape, for how long will the remaining provisions last the garrison ?

25. If when wheat is at Rs. 3 per maund, the 4*a*. loaf weighs 8 chk., what should be the price of wheat per maund, when 3 sr. 2 chk. of bread costs 12*a*. 6*p*. ?

26. If Rs. 240 be paid for bread for 49 persons for 20 months, when wheat is at Rs 3 per maund, how long will Rs. 234 supply bread for 91 persons, when wheat is at Rs, 3. 8*a*. per maund ?

27. If 44 cannon firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 400 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day ?

28. If 60 guns firing 5 rounds in 6 min. kill 350 men in $1\frac{1}{4}$ hrs., how many guns firing 7 rounds in 9 min. will kill 980 men in 25 min. at the same rate ?

29. If 5 horses require as much corn as 8 ponies and 15 quarters of corn last for 12 ponies for 64 days, how long may 25 horses be kept for £41, 5*s*. when corn is 22*s*. a quarter ?

30. If 240 men working 10 hrs. a day can dig a trench 300 yds. long, $3\frac{1}{2}$ ft. deep and $2\frac{1}{2}$ ft. wide in 6 days, in how many days would 80 men. working 9 hrs. a day dig a trench 500 yds. long, 3 ft. wide and $2\frac{1}{4}$ ft. deep ?

31. If 10 compositors, in 16 days of 14 hours each can compose 20 sheets of 24 pages in each sheet, 50 lines in a page and 40 letters on the average in a line, in how many days of 7 hrs. each can 20 compositors compose a volume to be printed in the same letter, containing 40 sheets, 16 pages in a sheet, 60 lines in a page and 50 letters in a line ?

32. If 7 women earn as much as 4 men and 48 men assisted by 14 women earn 121 guineas in 17 days ; what number of women with the help of 20 men will earn £21, 3*s*. 6*d*. in one-third of the time ?

33. If the wages of 25 men amount to Rs. 766, 10*a*. 8*p*. in 16 days, how many men must work for 24 days to earn Rs. 1035, the daily wages of the latter set being half of the former ?

34. If 17 men working 8 hrs. a day made an excavation 121 ft. 6 in. long, 25 ft. 6 in broad and 24 ft. deep in 54 days, how many hours a day must 18 men work during 51 days in order that they may make an excavation whose length and breadth are 1 ft. 6 in. less and depth 1 ft. 6 in. greater than the preceding one, supposing 9 men of the latter group do as much work as 10 of the former ?

35. If 38 men working 6 hours a day can do a piece of work in 12 days, find in what time 57 men working 8 hrs. a day can do a piece of work twice as great, if 2 men of the first set can do as much work in 1 hour as 3 men of the second set can do in $1\frac{1}{2}$ hours ?

36. A contractor agrees to finish a piece of work in a certain time. He engages 55 men who work 9 hrs. a day. But after $\frac{3}{4}$th of the time he found that only $\frac{3}{7}$th of the work had been done. How many more men must he now employ in order that all the men working 11 hours daily may finish the work in the fixed time ?

37. A town is garrisoned with 10000 troops who have provisions enough for 6 months, giving a ration of $3\frac{1}{2}$ lbs. daily to each man. How many men must be sent away, so that by giving $\frac{1}{2}$ lb. less to each man, the provisions many last for 8 months ?

38. If a family of 9 men in Lahore spend Rs. 16380 in a year, what must be the expenses of 8 men in Calcutta who live in the same style for 7 months, assuming that the prices at Calcutta are $\frac{3}{5}$th of what they are in Lahore ?

39. A farmer engages 30 men and 45 women to cut his crop in 20 days of 12 hours each ; but after 12 days' work, he intends to finish the work in 4 days more of 10 hrs. each. How many more men must be employed assuming that 2 men do as much work as 3 women ?

40. 6 men or 9 women or 12 boys working 10 hours a day can finish a piece of work in 12 days. How many men with 3 women and 4 boys working 5 hours a day would finish the same work in 4 days ?

II. Chain Rule.

§4. If we wish to express one quantity A in terms of another R and have data of the following type, *viz.*,

$aA = mM$.

$bM = nN$,

$cN = pP$, etc., etc., then $A = \frac{mnp}{abc}P$, *i. e.*,

the quantity required is obtained by dividing the product of all numbers on the right-hand side by that of the numbers on the left hand side.

Example. 2 horses cost as much as 5 cows, 6 cows as much as 8 oxen, 10 oxen as much as 50 sheep, 14 sheep as much as 9 goats. If the cost of one goat be Rs. 7, how much will one horse cost ?

Sol. Reqd. No. of rupees = one horse,

2 horses = 5 cows,

6 cows = 8 oxen,

10 oxen = 50 sheep,

14 sheep = 9 goats

One goat = Rs. 7.

$\therefore$ req. No. of rupees $= \frac{1 \times 5 \times 8 \times 50 \times 9 \times 7}{2 \times 6 \times 10 \times 14 \times 1} =$ Rs. 75. **An**

This method is known as **Chain Rule.** The given relations are put in the form of an equation in such a manner that each denomination occurs on each side of the sign of equality, which means "are worth," and then the product of the numbers on the right-hand side is divided by the product of the numbers on the left-hand side. The quotient is the required value.

EXERCISE 90.

1. 2 horses cost as much as 6 cows, 8 cows as much as 9 oxen, 3 oxen as much as 8 sheep. If 5 sheep cost Rs. 40, what will one horse cost ?

2. If 12 rupees are worth 4 dollars, 3 dollars are worth 18 francs, 15 francs are worth 10 marks ; how many marks can be exchanged for Rs. 6 ?

3. If 6 rupees are worth 10 sillings, 15 shillings are worth 20 francs and 25 francs are worth 5 dollars, how many dollars can be exchanged for Rs. 81 ?

4. 5 ducks are worth 6 chickens, 3 chickens are worth 2 geese and 9 geese are worth 8 turkeys ; what is the price of 3 ducks when 2 turkeys cost Rs. 15 ?

5. If 8 lb. of tea be worth 6 lb. of coffee, 5 lb. of coffee be worth 7 lb. of chicory and 9 lb. of chicory be worth 12 lb. of sugar ; how many pounds of tea can be given in exchange for 14 lb. of sugar ?

6. If 15 maunds of wheat be worth 18 maunds of grams, 16 maunds of grams be worth 25 maunds of barley and 20 maunds of barley be worth 12 maunds of rice ; how many mds. of wheat can be given in exchange for 27 mds. of rice ?

7. If 2 horses cost as much as 11 cows, 3 cows as much as 7 sheep, 14 sheep as much as 15 goats ; what will one horse cost when a goat costs Rs. 5 ?

8. *A* can do as much work in 5 days as *B* can do in 8 days and *B* can do as much in 4 days as *C* in 9 days : in what time will *C* do a piece of work which *A* can do in $2\frac{1}{2}$ days ?

9. If *A* can do as much work in 3 days as *B* can do in $2\frac{1}{2}$ days and *B* can do as much in 4 days as *C* in $4\frac{1}{2}$ days ; in what time will *C* do a piece of work which *A* can do in 8 days ?

10. If 3 kilog. of tea be worth 5 kilog. of cocoa and 7 kilog. of cocoa be worth 11 kilog. of coffee ; what will 5 kilog. of tea be worth, if 1 kilog. of coffee is worth 6 fr. 30 c. ?

CHAPTER XVII

RATIO AND PROPORTION

1. RATIO.

§1. In ratio we compare two quantities of the *same kind* and consider what multiple, part or parts one is of the other. In comparing 8 with 4 we observe that it is 2 times 4. This comparison can be represented as $8 \div 4$ or $\frac{8}{4}$. Hence

Def. Ratio is that relation between two numbers which is expressed by the fraction, the numerator of which is the measure of the first quantity and denominator is the measure of the second quantity.

The expresion $\frac{8}{4}$ is otherwise read as the ratio of 8 to 4 and is further usually expressed by the notation 8 : 4.

§2. The two numbers which form the ratio are called its **terms**, the first number is called the **antecedent** and the second number the **consequent**.

§3. If the terms of ratio be multiplied or divided by the same quantity the value of ratio is not altered.

Thus 2 : 5 is the same as 8 : 20
and 12 : 16 is the same as 3 : 4.

§4. **Comparison of ratio.**

Example. Compare the ratios 3 : 5 and 7 : 8.

We can compare the ratios thus :—

Sol. $3 : 5 = \frac{3}{5}$; $7 : 8 = \frac{7}{8}$.

$\frac{3}{5}, \frac{7}{8} = \frac{24}{40}, \frac{35}{40}$.

Hence 7 : 8 is greater than 3 : 5.

(i) If the antecedent in a ratio is greater than the consequent, the ratio is called one of **greater inequality** as 5 : 3.

(ii) If the antecedent is equal to the consequent, the ratio is called one of **equality** as 3 : 3.

(iii) If the antecedent is less than the consequent the ratio is called one of the **less inequality** as 3 : 5.

§6. A ratio of greater inequality is diminished and of less inequality increased by adding any same quantity to both its terms.

If 1 be added to the terms of the ratio of greater inequality 7 : 4 it becomes the ratio 8 : 5 which is less than the former.

If 1 be added to the terms of the ratio of less inequality 4 : 7 it becomes 5 : 8 which is greater than the former.

Conversely. A ratio of greater inequality is increased and less inequality diminished by subtracting any same quantity from both its terms.

If 1 be subtracted from the terms of the ratio of greater inequality 5 : 3 it becomes 4 : 2 which is greater than the former.

If 1 be subtracted from the terms of the ratio of less inequality 3 : 5 it becomes 2 : 4 which is less than the former.

Ratios are compounded by multiplying together the antecedents for a new antecedent and the consequents for a new consequent.

Take the following ratios 2 : 3, 5 : 6, 7 : 8. Then the ratio $2\times5\times7 : 3\times6\times8$ or 70 : 144 is said to be the ratio compounded of the three given ratios.

When the ratio is compounded with itself once it is called the *duplicate ratio*. Thus $2^2 : 3^2$ *i.e.*, 4 : 9 is the duplicate ratio of 2 : 3 and **conversely** 2 : 3 is the *sub-duplicate ratio* of 4 : 9.

When the ratio is compounded with itself twice it is called the *triplicate ratio*. Thus 8 : 27 is the triplicate ratio of 2 : 3 and **conversely** 2 : 3 is the *sub-triplicate ratio* of 8 : 27.

§8. **Inverse or reciprocal ratio.**

If 4 : 5 be the given ratio then $\frac{1}{4} : \frac{1}{5}$ or 5 : 4 is the **Inverse** or **reciprocal ratio.**

§9. We shall now solve some examples :—

Example 1. Find the ratio of of 3*a*. 4*p*. to 5*a*.

Sol. 3*a*. 4*p*. = 40*p*. and 5*a*. = 60*p*.

∴ ratio $= \frac{40}{60} = \frac{2}{3} = 2 : 3$. **Ans.**

Example 2. Which of the two ratios is greater 5 : 6 or 7 : 8 ?

Sol. $5 : 6 = \frac{5}{6} = \frac{20}{24}$ and $7 : 8 = \frac{7}{8} = \frac{21}{24}$

$\therefore$ $7 : 8$ is the greater ratio. **Ans.**

Example 3. A, B, C are three quantities of the same kind, their ratios are $A : B = 3 : 4$, $B : C = 5 : 6$; find the ratio between A and C.

Sol. $\because \frac{A}{B} = \frac{3}{4}$ and $\frac{B}{C} = \frac{5}{6}$,

$\therefore \frac{A}{B} \times \frac{B}{C} = \frac{3}{4} \times \frac{5}{6}$

$\therefore \frac{A}{C} = \frac{5}{8}$

$\therefore A : C = 5 : 8$. **Ans.**

Example 4. A, B, C, D are four quantities of the same kind. The ratios are $A : B = 3 : 4$; $B : C = 5 : 6$ and $C : D = 8 : 9$. Compare them.

Sol. $A : B = 3 : 4$,

$B : C = 5 : 6 = 1 : \frac{6}{5}$

$= 4 : \frac{24}{5}$

Also $C : D = 8 : 9 = 1 : \frac{9}{8}$

$= \frac{24}{5} : \frac{9}{8} \times \frac{24}{5}$ or $\frac{24}{5} : \frac{27}{5}$

$\therefore A : B : C : D = 3 : 4 : \frac{24}{5} : \frac{27}{5}$

$= 15 : 20 : 24 : 27$. **Ans.**

Example 5. Three numbers are in the ratio of $3 : 4 : 5$ and the sum of their squares is 450, find the numbers.

Sol. Suppose that the numbers are $3x$, $4x$, $5x$.

$\therefore 9x^2 + 16x^2 + 25x^2 = 450$

or $50x^2 = 450$

or $x^2 = 9$ or $x = 3$

$\therefore$ reqd. numbers are $3 \times 3 = 9$, $3 \times 4 = 12$, $3 \times 5 = 15$ } **Ans.**

EXERCISE 91.

Simplify the following ratios :—

1. $9 : 3$.
2. $12 : 16$.
3. $20 : 25$.
4. $125 : 225$.
5. $144 : 156$.

Find the ratio of :—

6. 2 yards and 2 feet.
7. One acre and 1760 sq. yards.
8. One mile and 440 yards.
9. 4 centimetres and 6 decimetres.

Which of the ratio is greater :—

10. 3 : 4 or 8 : 9 11. 5 : 6 or 3 : 4.
12. 15 : 20 or 11 : 16.

Form the ratio compounded of the ratio :—

13. 5 : 6, 8 : 9, 12 : 15.
14. Find the triplicate ratio of 9 : 8
15. Find the triplicate ratio of 6 : 7
16. Find the sub-duplicate ratio of 81 : 144.
17. (*a*) What is the antecedent if 24 be the consequent and $\frac{1}{5}$ the ratio ?

(*b*) What is the consequent if 13·5 be the antecedent and 4 the ratio ?

18. If $A=3\frac{1}{3}$ of B and $C=5\frac{1}{5}$ of B, find the ratio of A to C.

19. Compare the rates of two trains, one of which goes $395\frac{5}{8}$ miles in $11\frac{2}{5}$ hours and the other goes $262\frac{4}{15}$ miles in $8\frac{4}{9}$ hours.

20. Three quantities A, B, C are of the same kind such that $A : B=7 : 8$ and $B : C=9 : 14$. Find the ratio between A and C.

21. Four quantities A, B, C, D are of the same kind such that $A : B=5 : 6$, $B : C=7 : 8$ and $C : D=9 : 10$, (*i*) find the ratio patween A and D ; (*ii*) compare A, B, C, D.

22. If B makes a profit of Rs. 3, when A makes Rs. 2 ; C makes a profit of Rs. 10, when B makes Rs. 9 and D makes a profit of Rs. 5 when C makes Rs. 4 ; compare the profits of A, B, C and D.

23. Two numbers are in the ratio of 4 : 5 and the difference of their sqaures is 225 ; find them.

24. Two numbers are in the ratio of 3 : 5 and the difference of their cubes is 784 ; find the numbers.

25. Three numbers are in the ratio of 3 : 6 : 5 and the sum of their squares is 3430 ; find the numberes.

26. The sum of two numbers is 36 and their difference is 4 ; find their ratio.

27. ·7 of one number is equal to ·075 of another number ; find the ratio.

28. Two numbers are in the ratio of 3 : 5 but when they are increased by 10 their ratio is 5 : 7, find the numbers.

29. A greyhound pursues a hare and takes 5 leaps for every 6 leaps of the hare, but 4 leaps of the hound are equal to 5 of the hare ; compare the rates of hound and hare.

30. Find the continued ratios between A, B, C, D if A and B are in the ratio 1 : 2 ; B and C are in the ratio 3 : 4 and C and D are in the ratio 5 : 6 ; also find the ratio between A and D.

II PROPORTION

10. Compare the following ratios 12 : 16 and 18 : 24.

$$12 : 16 = \tfrac{12}{16} = \tfrac{3}{4},$$

and $$18 : 24 = \tfrac{18}{24} = \tfrac{3}{4}.$$

We find that the two ratios are equal ; this equality of two ratios is named **proportion.**

Def. Four quantities are said to be in proportion when the first bears to the second the same ratio as the third bears to the fourth. Thus, 12, 16 and 18, 24 are said to be in proportion since the ratio 12 : 16 is equal to the ratio 18 : 24.

The proportion is expressed thus :—

$$12 : 16 :: 18 : 24 \text{ or } \tfrac{12}{16} = \tfrac{18}{24}.$$

The first and the fourth terms are called the **extremes** and the second and the third are called the **means.** Here 12, 24 are the extremes and 16, 18 the means.

Note. Here 24 is also called the fourth proportional to 12, 16, 18.

§11. When four quantities are in proportion, the product of the extremes is equal to the product of the means.

Let 12, 15, 16, 20 be four quantities in proportion.

$$\therefore \tfrac{12}{15} = \tfrac{16}{20} \text{ or } 12 \times 20 = 15 \times 16$$

Hence any three terms in proportion being given the fourth may be determined by the formula proved above.

Conversely. If the product of two quantities be equal to the product of the two others, the four are proportionals, making the factors of one product, the means and the factors of the other the extremes.

§12. Three quantities are said to be in continued proportion when the first bears to the second the same ratio as the second bears to the third. Thus 2, 4, 8 are in continued proportion since 2 : 4 : : 4 : 8.

Note. In this case 8 is called the third proportional to 2 and 4 and 4 is called the mean proportional between 2 and 8.

Example 1. Find a fourth proportional to 6, 10 and 9.

Sol. Let x be the fourth proportional, then by definition

$$6 : 10 : : 9 : x.$$

$\therefore$ product of the extremes = product of the means

$$\therefore \quad 6x = 90$$

or $$x = 15. \quad \textbf{Ans.}$$

Example 2. Find a third proportional to 8 and 12.

Sol. Third proportional to 8, 12 means fourth proportional to 8, 12, 12. Let it be equal to x.

We have therefore, $8 : 12 : : 12 : x$.

$$\therefore \quad 8x = 144 \text{ or } x = 18. \quad \textbf{Ans.}$$

Example 3. Find the mean proportional between 4 and 9

Sol. Let x be the required mean proportional.

We have therefore, $4 : x : : x : 9$.

$$\therefore \quad x^2 = 36$$

or $$x = 6. \quad \textbf{Ans.}$$

Note. It is clear that the mean proportional between two numbers is equal to the square root of their product.

EXERCISE 92.

Find the fourth proportional of :—

1. 5, 6, 10. 2. 8, 12, 6. 3. 6, 15, 12.

Find the fourth proportional of :—

4. 8, 12. 5. 9, 12. 6. 21, 42.

Find the mean proportional between :—

7. 12, 147. 8. 36, 81. 9. 64, 225.

Find the value of x in the following proportions :—

10. $6 : 7 : : 3 : x$ 11. $12 : 18 : : x : 24$.
12. $x : 19 : : 9 : 12$.

Are the following proportions true ?

13. $5 : 7 :: 8 : 9$. 14. $16 : 8 :: 10 : 5$.

15. One man adds 3 seers of water to 21 seers of milk and another adds 4 seers of water to 27 seers of milk ; compare the amount of milk in the two mixtures.

16. Find the number which has to 40, the ratio of 3·75 to 3 and (*b*) complete the proportion of which the first, second and fourth terms are $\frac{1}{20}$, 35 and $3\frac{3}{4}$.

17. If 3 men and 11 boys working together, can do 5 times as much work per hour as a man and a boy together, campare the work of a boy with that of a man.

18. 270 sheep and 14 horses eat 101 acres of grass in 30 days ; 155 sheep and 21 horses eat 185 acres of grass in 75 days. Compare the amount eaten by a sheep and a horse in the same time.

19. Find x in 12 yds. : 4 yds. : : £9 : £x.

20. Find x in 1·5 : 7·5 : : x lbs. : 3 lbs.

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§ 13. Simple Proportion or Rule of Three.

When any three terms of a proportion are given, we can find the fourth (Art. 10). This method of finding the fourth term is called the method of **simple proportion** or the **Rule of Three**. The modern tendency is to discourage the method of "Rule of Three" and to encourage the "unitary method." But problems, which we have solved by the unitary method can very *easily* be solved by this method. A great mathematician thus writes :

"The unitary method was originally introduced as a protest against the too mechanical method of the old 'Rule of Three.' It was contended that while by the old method the pupil was merely taught to write down his three terms in certain places and then multiply or divide them by a fixed rule learnt by heart, by a new method he would be compelled to think out the step for himself, but as a matter of fact, the unitary method has tended to become even more mechanical than the old 'rule of three' and there is at the present time a reversion to the method of proportion." **(J. T. Best.)**

§ 14. Consider the following examples :—

Example 1. The price of 8 chairs is Rs. 40, find the price of 15 chairs.

Note. If the number of articles is increased the price will also be increased in the same ratio, therefore the ratio of the two articles is equal to the ratio of the two corresponding prices.

chairs chairs Rs. Rs.
Hence 8 : 15 :: 40 : x,

when x stands for the required answer.

$\therefore$ $8x$=Rs. 15×40, *i.e*, 600.

or x=Rs. 75. **Ans.**

This is an example of what is called the **Direct Rule of Three.** In this method the greater quantity requires the greater answer and the less quantity requires the less answer.

Example 2. 20 men can do a piece of work in 10 days : in how many days could 25 men do the same work ?

Note. Here we observe that an increase in the number of workmen corresponds to a decrease in the number of days and vice versa. Therefore the inverse ratio of the corresponding number of men is equal to the ratio of the corresponding number of days.

Hence 25 men : 20 men : : 10 days : x, when x stands for the required number of days.

$\therefore$ $25x=20 \times 10$, *i.e.*, 200 days.

$\therefore$ x=8 days. **Ans.**

This is an example of what is called the **Inverse Rule of Three.** In this method a greater number requires a less answer and less number requires a greater answer.

§ 15. In every question of simple proportion or **Rule of Three.** two of the terms are of the same kind and the third term is of the kind as the required answer. And the second term is evidently greater or less than the first term according as the fourth term, *i.e.*, the required answer, is greater or less than the third.

Hence to arrange the terms in a question of *Rule of Three* we give the following rule :—

Rule. *Denote the answer by the letter x and put it down as the 4th term. Of the three given quantities, put down in the third place that which is of the same kind as the answer.*

Next from the nature of the question determine whether the answer will be greater or less than the third term. If the answer is greater, then put down in the 2nd place the greater of the two remaining quantities; but if the answer is less, then put down in the 2nd place the smaller of the two remaining quantities. Having thus arranged the terms, x i. e., the required answer is obtained by dividing the product of the 2nd and 3rd terms by the first term.

Note. The first two terms in the proportion must be expressed by numbers which measure them by the same unit.

Example 3. If the third class railway fare for 110 miles is Re. 1, 11*a*. 6*p*., what is the fare for 350 miles ?

Note. Here x is the required fare and hence Rs. 1. 11a. 6p. is put in the 3rd place. Now ∵ the fare for 350 miles is greater than that for 110 miles, ∴ we put in the second place 350 which is the greater of the two remaining quantities. 110 and 350. and hence in the 1st place we put the remaining quantity, 110. This is evidently a Direct Rule of Three and should be written thus :-

Sol. miles miles Re. *a*. *p*.

$$110 : 350 :: 1\ 11\ 6 : x$$

or $$11 : 35 :: 1\ 11\ 6 : x$$

$$\therefore\ x = \frac{\text{Re. } 1.\ 11a.\ 6p. \times 35}{11} = \frac{\text{Rs.}60,\ 2a.\ 6p.}{11}$$

$$= \text{Rs. } 5.\ 7a.\ 6p. \quad \textbf{Ans.}$$

Example 4. 400 men have food enough to last 23 weeks if they are joined by 60 men, how long will the provisions last ?

Note. The number of men is increased from 400 to 460. Putting x in the 4th place, evidently 23 must be put in the 3rd place. The required answer will be clearly less than 23, since when the number, increases, the food must last for a shorter period and therefore we put 400 in the 2nd place and 460 in the first place. This is, therefore, the inverse Rule of Three and should be written thus:—

Sol. Men men wks. wks.

$$460 : 400 :: 23 : x$$

$$\therefore\ x = \frac{400 \times 23}{460} = 20 \text{ weeks.} \quad \textbf{Ans.}$$

Note. The above Examples show how very easily we can solve the questions of Unitary Method by Rule of Three.

EXERCISE (Repetition)

Students should do the question in Exercise 87 and 88 by the Rule of Three.

§16. **Compound Proportion or Double Rule of Three.** Complex problems which require two or more applications of the Rule of Three can be solved by a short method known as the **Double Rule of Three.**

Example 1. If 10 masons can build a wall 50 ft. long in 25 days of 8 hours each, in how many days of 6 hours each will 15 masons build a wall 36 ft. long ?

Now consider the following points :—

1. First wall, 10 masons are working. Second wall, 15 masons are working.

∴ less days are required. (Inverse proportion)

2. First wall is 50 ft. long. Second wall is 36 ft. long.

∴ less days are required. (Direct proportion)

3. First wall, work 8 hrs. a day. Second wall, work 6 hrs. a day. ∴ more days are required. (Inverse proportion)

The solution will be arranged as follows :—

Mason 15 : 10
Length 50 : 36 } : : 25 days : No. of days required.
Hours 6 : 8

$$\therefore \text{ days required} = \frac{10 \times 36 \times 8 \times 25}{15 \times 50 \times 6} = 16. \quad \textbf{Ans.}$$

Such statement of the question in which two more applications of the Rule of Three are involved is called **Compound Proportion** or **Double Rule of Three.**

Example 2. If 15 men can dig a trench 15 yards long, 9 ft. broad and 5 ft. deep in 12 days of 8 hours each, in how many days will 24 men dig a trench 25 yards long, 6 ft. broad and 9 ft. deep working 6 hours a day ?

Note that :—

1. First trench, 15 men are working ; second trench 24 men are working

∴ less days are required (Inverse proportion).

2. Firt trench 15 yds. long ; second trench, 25 yds. long.
∴ more days are required (Direct proportion).

3. First trench, 9 ft. wide ; second trench, 6 ft. wide
∴ less days are required (Direct proportion).

4. First trench, 5 ft. deep ; second trench 9 ft. deep
∴ more days are required (Direct proportion).

5. First trench, work 8 hours a day ; second trench, work 6 hours a day

∴ more days are required (Inverse proportion). Hence the following

Sol.

Men	24 : 15	
Length	15 : 25	
Breadth	9 : 6	: : 12 days : No. of days required
Depth	5 : 9	
Hours	6 : 8	

$$\therefore \text{No of days} = \frac{15 \times 25 \times 6 \times 9 \times 8 \times 12}{24 \times 15 \times 9 \times 5 \times 6} = 20. \quad \textbf{Ans.}$$

The examples solved above give us the following rule .—

Rule. *Denote the required answer by x and put it in the 4th place. The quantity which is of the same kind as the requirea answer should then be put in the 3rd place. Then take any pair of the quantities of the same kind and arrange these terms, comparing with the 3rd term, as in a separate question on the Rule of Three. Similarly all pairs ef quantities of the same kind are to be arranged. Then the required answer is abtained by dividing the product of all the terms in the 2nd and 3rd places by the product of all the terms in the first place.*

☞ § **17.** There is another method of arranging the several terms in a question of Rule of Three or Double Rule of three and students will find that in most cases, this method is not only simpler and more convenient but it saves much time also, though for some reason or other, modern writers generally neglect to explain the method. We however, take the liberty of clearly explaining the method with illustrations.

In every question, we should find out the **causes** and the **effects** produced by these causes. Assuming as an axiom that

effects have the same relation or ratio to each other as the *causes* which produce them, it is evident that in any two causes of the same kind we shall have the following proportion :—

First cause : second cause : : first effect : second effect

and then as explained in Art. 10, we can find any one term if the three others are given.

§18. We may denote the required term or quantity by x which must be considered as and treated in the same way as any other number. This x may occupy any place in the proportion either by itself or as one of the factors with other given numbers as shown in the following examples :—

Example 1. If 15 men can reap a field of 12 acres in certain time, how many acres will 10 men reap in the same time ?

Sol. Here $\left.\begin{matrix}15 \text{ men} \\ 10 \text{ men}\end{matrix}\right\}$ are the first and second causes.

and $\left.\begin{matrix}12 \text{ acres} \\ x \text{ acres}\end{matrix}\right\}$ „ „ „ „ „ effects.

$\therefore$ 15 men : 10 men : : 12 acres : x acres.

and $\therefore$ $15 \times x = 10 \times 12$,

whence $x = \dfrac{10 \times 12}{15} = 8$ acres. **Ans.**

Example 2. If the three penny loaf weighs 4 oz., when wheat is 10s. a bushel, what should be the price of wheat when the two-penny loaf weighs 6 oz. ?

Sol. In the former case, the price of 1 oz. is $\frac{3}{4}d.$,
In the latter „ „ „ „ „ „ is $\frac{1}{3}d.$

$\therefore$ we have the following proportion :—

$\frac{3}{4}d. : \frac{1}{3}d. : : 10s. : x\ s.$,

$\therefore \frac{3}{4} \times x = \frac{1}{3} \times 10.$

whence $x = \frac{1}{3} \times 10 \div \frac{3}{4} = \frac{40}{9} = 4\frac{4}{9}s.$ **Ans.**

In these two examples, the causes are simple terms being dependent upon only one magnitude.

Example 3. If 8 men can do a piece of work in 15 days, how many days will 20 men take to do a piece of work twice as much as the former.

Sol. Here the causes are to each other as 8×15 is to $20\times x$ and the effects may be taken as 1 and 2,

$$\therefore\quad 8\times15 : 20\times x :: 1 : 2,$$

whence $20\times x = 8\times15\times2$,

$$\therefore\quad x = \frac{8\times15\times2}{20} = 12 \text{ days.}\quad \textbf{Ans.}$$

Example 4. If 10 cannon which fire 3 rounds in 5 minutes, kill 270 men in $1\frac{1}{2}$ hours, how many cannon which fire 5 rounds in 6 minutes will kill 500 men in 1 hour, at the same rate ?

Sol. In the former case $\frac{3}{5}$ round is fired in 1 min, and in the latter case $\frac{5}{6}$ in I min. and hence in this case,

the causes are $10\times\frac{3}{5}\times\frac{3}{2}$ and $x\times\frac{5}{6}\times1$

and the effects are 270 and 500 men, therefore,

$10\times\frac{3}{5}\times\frac{3}{2} : x\times\frac{5}{6}\times1 :: 270 : 500,$

whence $x = \dfrac{10\times3\times3\times500\times6}{5\times2\times5\times270}$

$=20$ cannon. **Ans.**

In the exercise given below, students will be well advised if they solve many of them if not all, by the method explained above.

EXERCISE (Repetition).

Students should do the question in Exercise 89 by the method of the Double Rule of Three.

CHAPTER XVIII

MISCELLANEOUS PROPOSITIONS

1. TIME AND WORK.

§1. Suppose a field of grass is divided into 5 equal parts. If a man can reap it in 5 days he will reap $\frac{1}{5}$ of it in one day. *Conversely*, if he can reap $\frac{1}{5}$ of it in one day he can reap the whole in 5 days. In solving the problems of time and work, these two fundamental laws always regulate our steps.



EXERCISE (Oral)

1. If A can do a piece of work in 10 days—
   (*a*) what part of it will he do in 1 day ? and
   (*b*) what part of it in 7 days ?

2. If B can finish $\frac{1}{8}$ part of work in one day, how long will he take to finish it ?

3. If A can do a piece of work in 6 days aud B can do in 5 days—
   (*a*) what part of it will A do in one day ?
   (*b*) what part of it will A and B together do in one day ?
   (*d*) how long will they both take to do it ?

4. A can build a wall in 10 days and B in 15 days ?
   (*a*) what part of it will A build in one day ?
   (*b*) what part of it will B build in one day ?
   (*c*) what part of it will both build in one day ?
   (*d*) how long will they both take to build it ?

---

§2. **Some, solved examples.**

Example 1. A can do a piece of work in 8 days and B can do it in 12 days ; how long will both working together take to do it ?

Sol. ∵ A can do the work in 8 days,
∴ A can do $\frac{1}{8}$ part of it in 1 day ;
∵ B can do the work in 12 days,
∴ B can do $\frac{1}{12}$ part of it in one day.
∴ A and B together can do $(\frac{1}{8}+\frac{1}{12})$ part of it in one day
or " " $\frac{5}{24}$ " " " in one day
∴ A and B together can do the whole work in $\frac{24}{5}$ days,
i.e., $4\frac{4}{5}$ days. **Ans.**

Example 2. A can finish $\frac{3}{8}$ of a work in 15 days and B can finish the remaining work in 10 days. In what time can A and B together do the same work ?

Sol. (*i*) ∵ A can do $\frac{3}{8}$ of the work in 15 days,
∴ A can do $\frac{3}{8}\times\frac{1}{15}=\frac{1}{40}$ of the work in 1 day.
Remaining work $=1-\frac{3}{8}=\frac{5}{8}$.
(*ii*) ∵ B can do $\frac{5}{8}$ of the work in 10 days
∴ B can do $\frac{5}{8}\times\frac{1}{10}=\frac{1}{16}$ of the work in one day.
∴ A and B together can do $\frac{1}{40}+\frac{1}{16}$.
i.e., $\frac{7}{80}$ of the work in one day.
∴ they together can do the work in $\frac{80}{7}$ or $11\frac{3}{7}$ days. **Ans.**

Example 3. A and B together can finish a piece of work in $9\frac{3}{5}$ days which A alone can finish in 24 days ; how long will it take B alone to do it ?

Sol. ∵ A and B can do the work in $9\frac{3}{5}$ or $\frac{48}{5}$ days
∴ they can do $\frac{5}{48}$ of the work in one day.
But A alone can do $\frac{1}{24}$ of it in one day,
∴ B alone can do $\frac{5}{48}-\frac{1}{24}$ or $\frac{1}{16}$ of it in one day.
∴ B can do the whole work in 16 days. **Ans.**

Example 4. A and B can do a certain piece of work in 18 days, B and C can do it in 12 days and C and A can do it in 24 days. How long would each take separately to do it ?

Sol. A and B can do $\frac{1}{18}$ of the work in one day,
B and C " " $\frac{1}{12}$ " " in one day,
C and A " " $\frac{1}{24}$ " " in one day,
∴ A, B, C can do $\frac{1}{18}+\frac{1}{12}+\frac{1}{24}$ *i.e.*, $\frac{13}{72}$ of the work in 2 days,
∴ A, B, C can do $\frac{13}{72}\times\frac{1}{2}$ or $\frac{13}{144}$ of the work in 1 day.
But A and B can do $\frac{1}{18}$ of the work in one day,
∴ C can do $\frac{13}{144}-\frac{1}{18}$ or $\frac{5}{144}$ of the work in one day.
∴ C can do the whole work in $28\frac{4}{5}$ days.
Similarly A can do the whole work in 144 days.
and B can do the whole work in $20\frac{4}{7}$ days. } **Ans.**

Example 5. Mohan can do a piece of work in 20 days and Sohan can do it in 25 days ; after working at it together for 5 days, Mohan leaves off; how long will Sohan take to do the remaining work?

Sol. Mohan can do $\frac{1}{20}$ of the work in 1 day and Sohan can do $\frac{1}{25}$ of the work in 1 day.

$\therefore$ both can do $\frac{1}{20}+\frac{1}{25}$ or $\frac{9}{100}$ of the work in 1 day.

$\therefore$,, $\frac{9}{100}\times 5$ or $\frac{9}{20}$ of the work in 5 days.

Now at this stage Mohan leaves off. Therefore the remaining work $1-\frac{9}{20}$ *i.e.*, $\frac{11}{20}$ will be done by Sohan.

$\therefore$ Sohan will do $\frac{11}{20}$ of the work in $25\times\frac{11}{20}$ days.

i.e., $\frac{55}{4}$, or in $13\frac{3}{4}$ days. **Ans.**

Example 6. *A* can do a piece of work in $33\frac{1}{3}$ days and *B* in 20 days ; 4 days before the work was actually completed *A* is called off. In how many days will the work be completed ?

Sol. Since *A* is called off 4 days before the work is actually completed, therefore *B* worked alone for 4 days.

$\because$ *B*'s work for 4 days$=\frac{4}{20}$ or $\frac{1}{5}$.

$\therefore$ remaining work $\frac{4}{5}$ was done by *A* and *B* jointly.

A and *B* jointly can do $\frac{3}{100}+\frac{1}{20}$ or $\frac{2}{25}$ of the work in one day,

$\therefore$ *A* and *B* jointly can do the whole work in $\frac{25}{2}$ days,

$\therefore$,, ,, ,, $\frac{4}{5}$ of the work in $\frac{25}{2}\times\frac{4}{5}$ or 10 days.

$\therefore$ the work was completed in $10+4=14$ days. **Ans.**

Example 7. *A*, *B* and *C* can do a piece of work in 12, 15 20 days respectively. They work at it together, but *B* and *C* are called off 2 and 1 day respectively before the completion of the work. In what time was the work finished ?

Sol. $\because$ *A*, *B*, *C* can do $\frac{1}{12}+\frac{1}{15}+\frac{1}{20}=\frac{1}{5}$ of the work in one day

$\therefore$ *A*, *B*, *C* can finish it in 5 days.

$\because$ *B* is called off 2 days before the completion of the work and *C* is ,, 1 day ,, ,, ,, ,, ,,

$\therefore$ *A* worked for 2 days and *C* for 1 day after *B* was called off.

A's 2 days' work and *C*'s 1 day's work$=\frac{1}{6}+\frac{1}{20}=\frac{13}{60}$ *i.e.*, the rem. work $1-\frac{13}{60}$ or $\frac{47}{60}$ is completed by all the three men.

Since they can complete the work in 5 days

$\therefore$ $\frac{47}{60}$ of the work was done in $5\times\frac{47}{60}=\frac{47}{12}$ days.

$\therefore$ the work was completed in $2+\frac{47}{12}=5\frac{11}{12}$ days. **Ans.**

Example 8. A and B together can do a piece of work in 15 days, A worked for 8 days and the remaining work was completed by B in 29 days. In what time could each alone do the work ?

Sol. Since A worked for 8 days and the remaining work was completed by B in 29 days, it is evident that if they both together work for 8 days then the the remaining work can be finished by B in 21 days.

A and B together can do $\frac{8}{15}$ of the work in 8 days,
$\therefore$ the remaining work $\frac{7}{15}$ can be done by B in 21 days
$\because$ B can do $\frac{7}{15}\times\frac{1}{21}=\frac{1}{45}$ of the work in one day
$\therefore$ B can do the whole work in 45 days,
A can do $\frac{1}{15}-\frac{1}{45}=\frac{2}{45}$ of the work in one day.
$\therefore$ A can do the whole work in $22\frac{1}{2}$ days.

Hence $22\frac{1}{2}$ and 45 days. **Ans.**

Example 9. A and B can do a piece of work in 12 days, B and C together in 18 days. After A has been working at it for 4 days and B for 6 days, C finished it in 17 days. In how many days could each do the work by himself ?

Sol. Since A alone worked for 4 days, B for 6 days and C for 17 days, therefore let A and B together work for 4 days, B and C together for 2 days and C alone for 15 days. Both the statements mean the same thing.

The work of A and B for 4 days $=\frac{1}{3}$
The work of B and C for 2 days $=\frac{1}{9}$
$\therefore$ the work of C alone for 15 days $=1-(\frac{1}{3}+\frac{1}{9})=\frac{5}{9}$.
$\therefore$ the work of C for 1 day $=\frac{5}{9}\times\frac{1}{15}=\frac{1}{27}$
$\therefore$ C can do the work in 27 days.
Since B and C can do $\frac{1}{18}$ of the work in 1 day.
$\therefore$ B can do $\frac{1}{18}-\frac{1}{27}$, *i. e.*, $\frac{1}{54}$ of the work in 1 day.
$\therefore$ B can do the work in 54 days.
Similarly A can do the work in $15\frac{3}{7}$ days.
Hence $15\frac{3}{7}$, 54, 27 days. **Ans.**

EXERCISE 93.

1. A can do a piece of work in 10 days and B can do it in 15 days ; how long would they take to do it ?

2. A can do a piece of work in 8 days ; B can do it in 10 days and C can do it in 20 days. In what time will they do it, all working together ?

3. *A* and *B* together can finish a piece of work in 2(days which *A* alone can finish in 30 days ; what time will it take *B* alone to finish it ?

4. *A* can do a piece of work in 15 days and *B* can do $\frac{2}{5}$ of the same work in 15 days ; how long will they take to do it if they work together ?

5. *A* can do $\frac{3}{8}$ of the work in 9 days and *B* can do $\frac{5}{6}$ of the work in 15 days ; how long will they both working together take to do it ?

6. *A* and *B* can do a piece of work in 15 days, *B* and *C* in 20 days, *C* and *A* in 30 days ; in what time can they do it, all working together ?

7. *A* and *B* can do a piece of work in $4\frac{4}{9}$ days ; *B* and *C* in $5\frac{5}{11}$ days; *C* and *A* in $4\frac{4}{5}$ days ; in what time can each do it working separately ?

8. *A* and *B* together can do a piece of work in 21 days, they both worked at it for 4 days and the remaining work was completed by *C* alone in 34 days. In what time would *C* alone do the whole work ?

9. *A* and *B* can do a piece of work in 18 and 24 days respectively ; they worked at it together for 8 days and then *B* fell ill ; in what time will *A* alone finish the remaining work ?

10. *A* can do a piece of work in 20 days, he works at it for 5 days and then *B* finishes it in 9 days. In what time can *A* and *B* together do it ?

11. *A* can do a piece of work in 20 days and *B* in 30 days; *A* worked at it for 5 days and then *B* finished it with the assistance of *C* in 15 days. In what time can *C* alone finish the whole work ?

12. Paul finishes $\frac{3}{10}$ of a work in 6 days and then finishes the remaining with the assistance of Peter in 6 days. In what time can Peter alone finish the same work ?

13. *A* and *B* can do a piece of work in 25 and 30 days respectively. *A* and *B* work at it together for 10 days and then *C* joins them and the remaining work is finished in 2 days. How long will it take *C* alone to do the whole work ?

14. *A* can do a piece of work in 24 days and *B* in 30 days. *A* worked for 6 days and then *B* also joined him. In how many days will the whole work be completed ?

15. A, B and C can finish a piece of work in 10, 12 and 15 days respectively. If B stops after 2 days, how long will it take A and C to finish the remaining work ?

16. A can do $\frac{5}{8}$ of the work in 15 days and B $\frac{3}{8}$ of the work in 12 days ; A and B work at it together for 8 days and then C finishes it in 14 days. In what time would C alone do the whole work ?

17. A can finish a piece of work in 6 days and B in 9 days; they work at it together, but A is called off 4 days before the work is actually completed ; find in what time the work was finished.

18. A, B and C can complete a piece of work in 8, 12 and 16 days respectively ; they work at it together but B and C are called off 2 days and 3 days respectively before the work is actually completed. In what time was the work finished ?

19. A, B and C can do a piece of work in 12, 18 and 24 days respectively ; they work at it together ; A stops the work after 4 days and B is called off 2 days before the work is finished. In what time was the work finished ?

20. A, B and C can finish a piece of work in 20, 30 and 40 days respectively. They work at it together, but B and C are called off 4 and 5 days before the completion of the work. In what time was the work finished ?

21. A and B together can mow a field in 24 days. A works at it for 6 days and then B finishes it in 33 days. In what time could each mow it ?

22. A and B can finish a piece of work in 28 days, A works at it for $10\frac{1}{2}$ days and then B finishes it in $35\frac{1}{2}$ days. In what time could each do it ?

23. A and B can finish a piece of work in 18 days, and B and C in 24 days, A works at it for 6 days and then B for 8 days and then C finishes it in 23 days. In what time could each do it ?

24. A and B can finish a piece of work in $11\frac{1}{9}$ days, B and C in $13\frac{7}{11}$ days ; A works at it for 4 days and B for 5 days and then C finishes it in 18 days. In what time could each do it ?

25. A and B together can do a piece of work in 12 days ; A works as much in 10 days as B in 15 days ; in what time could each do it ?

26. A and B together can to a piece of work in 25 days; A does as much work in 15 days as B in 20 days; in what time can each do it?

27. A, B and C together can do a piece of work in 10 days; B and C together work thrice as much as A, and A and B together work 4 times as much as C. In what time can each alone do it? [*Hint.* $B+C=3A$.]

28. A, B and C together can finish a piece of work in 12 days; A and C together work twice as much as B; A and B together work thrice as much as C. In what time could each do it separately?

29. A works as much in 2 days as B in 3 days and B works as much in 4 days as C in 5 days; in what time will A, B and C working together finish the work if A alone can finish it in 11 days?

30. Parkash works as much in 3 days as Raj in 4 days and Raj works as much in 6 days as Paul in 5 days; in what time can Paul and Raj do it working together, if Parkash can finish it in 18 days?

31. 20 men can complete a piece of work in 10 days, but after every 4 days 5 men are called off, in what time will the work be finished?

32. 3 men and 4 boys finish a piece of work in 7 days and 2 men and 3 boys in 10 days; compare the rates of working of a man and a boy.

33. A can do a piece of work in 25 days, B in 20 days and C in 24 days. They three work together for 2 days and then A and B leave; but C continues, and after $8\frac{2}{5}$ days is rejoined by A, who brings D along with him and these three finish the remainder of the work in 3 days. In what time would D alone have done the whole work?

34. A can do a piece of work in 3 days; B can do three times the work in 8 days and C 5 times the work in 12 days. In what time will they do it together, supposing them to work at the rate of 9 hours a day?

35. A and B can do a piece of work in 10 days, B and C in 15 days, and C and A in 20 days. They all work at it for 6 days; then A leaves and B and C go on together for 4 days more. If B then leaves, how long will C take to complete the remaining work?

36. Four men working together all day, can finish a piece of work in 11 days; but two of them having other engagements can work only one half-time and quarter time respectively. How long will it take them to complete the work?

37. Six men and 5 boys can do a piece of work in 7 days; they work at it till they have completed $\frac{3}{4}$ of it; then two of the men leave and two more boys come in. How long will the work be in hand if a boy does half as much work as a man?

38. Some men and boys can do a piece of work in 20 days; if twice the number of boys more be added the work is finished in 12 days: in what time could the original number of boys do the work?

39. 12 men and 15 women can do a piece of work in 20 days; if 10 more women be added the work is finished in 5 days before the stipulated time. In what time could a man and a woman separately do the work?

40. If 8 men and 12 boys can finish a piece of work in 12 days; in what time will 40 men and 45 boys finish another piece of work 3 times as great, supposing that 16 men can do as much work in 8 hrs. as 12 boys in 24 hrs.?

41. Five men undertook to complete a piece of work in 48 days, working 8 hours a day. One stopped working at the end of 12 days and a second at the end of 15 days. The others then agreed to work 9 hours a day. By what percentage (to the nearest unit) must they increase their rate of working to finish within the specified time?

II. WORK AND WAGES

§3. Consider the following example :—

Example 1. *A* can do a piece of work in 15 days and *B* in 20 days. They finished the work with the assistance of *C* in 5 days and got Rs. 45 as their wages: find the share of each.

Sol. *A* did in 5 days $\frac{1}{3}$ of the work.

B did " " $\frac{1}{4}$ " "

C did " " $1-(\frac{1}{3}+\frac{1}{4})=\frac{5}{12}$ of the work

Since *A*, *B*, *C* did in 5 days $\frac{1}{3}$, $\frac{1}{4}$, $\frac{5}{12}$ of the work repectively.

$\therefore$ *A*'s share = Rs. $45\times\frac{1}{3}$ = Rs 15

B's share = Rs. $45\times\frac{1}{4}$ = Rs. 11, 4*a*.

C's share = Rs. $45\times\frac{5}{12}$ = Rs. 18, 12*a*.

} **Ans.**

Example 2. If 4 men and 5 men and 4 boys earn Rs. 39 in 6 days, 6 men and 3 boys earn Rs. 49 in 7 days, in what time will 6 men and 3 boys earn Rs. 60 ?

Sol. Since 4 men and 5 boys earn in 6 days, Rs. 39,

$\therefore$ 4 men and 5 boys " " 1 day Rs. $6\frac{1}{2}$.....(*i*)

also 5 men and 4 boys " " 7 days Rs. 49

$\therefore$ 5 men and 4 boys " " 1 day Rs. 7.....(*ii*)

Multiplying (*i*) by 5 and (*ii*) by 4 we get,

20 men and 25 boys earn in 1 day Rs. $32\frac{1}{2}$.

20 men and 16 boys " " " Rs 28.

$\therefore$ 9 boys earn in one day Rs $4\frac{1}{2}$ [By subtraction]

$\therefore$ 1 boy earns in one day $3a$

Again, since 4 men and 5 boys earn in one day Rs. $6\frac{1}{2}$

$\therefore$ 4 men earn in one day Rs. $6\frac{1}{2}$ − Rs. $2\frac{1}{2}$ = Rs. 4.

$\therefore$ 1 man earns in one day Re. 1.

$\therefore$ 6 men and 3 boys earn in one day Rs. 6 + Rs. $1\frac{1}{2}$ *i.e.*, Rs. $7\frac{1}{2}$

$\therefore$ No. of days required $= \frac{\text{Rs. } 60}{\text{Rs. } 7\frac{1}{2}} = 8$. **Ans.**

EXERCISE 94.

1. *A* can do a piece of work in 10 days and *B* in 12 days; they finished the work with the assistance of *C* in 4 days and got Rs. 22, $8a$. as their wages ; find the share of each.

2. *A* and *B* undertook to do a piece of work for Rs. 37 , $8a$. *A* alone could do it in 20 days and *B* in 30 days. With the assistance of *C* they finished it in 8 days. How should the money be divided ?

3. A field can be reaped by 10 women in 4 days, or by 6 boys in 10 days, or 2 men in 12 days. One man, three boys and three women are employed. What is the total expense, if the wages of a man, a woman and a boy are $8a$., $5a$., and $3a$. respectively ?

4. Three men can do as much work as five boys, the wages of three boys are equal to those of two men. A work on which 40 boys and 15 men are employed takes 8 weeks and costs £350 ; how long would it take if 20 boys and 20 men were employed and how much would it cost ?

5. If the work done by a man, a woman and a child b in the ratio of 3, 2 and 1 and there be in a factory 24 men, 2

women and 16 children, whose weekly wages amount to Rs. 204 ; what will be the yearly wages of 27 men, 40 women and 15 children ?

6. If 5 men with 7 boys can earn £ 5, 2*s*. in 6 days and 2 men with 3 boys can earn £ 1, 8*s*. in 4 days ; in what time will 7 men with 6 boys earn £ 30 ?

7. If 3 men with 4 boys earn Rs. 58 in 8 days, and 2 men with 3 boys earn Rs. 40 in the same time ; in what time will 6 men and 7 boys earn Rs. 210 ?

III. PIPES AND CISTERNS

§4. Consider the following examples :—

Example 1. A vessel can be filled by one pipe A in 10 minutes, by a second B in 15 minutes ; it can be emptied by a waste pipe C in 9 minutes ; in what time will the vessel be filled if all the three were turned on at once ?

Sol. A and B fill $(\frac{1}{10}+\frac{1}{15})$ or $\frac{1}{6}$ of the vessel in 1 min. and C can empty $\frac{1}{9}$ of the vessel in 1 min.

$\therefore$ when all the pipes are turned on $\frac{1}{6}-\frac{1}{9}$ or $\frac{1}{18}$ of the vessel will be filled by A, B, C in 1 minute.

$\therefore$ A and B will fill the vessel in 18 minutes. **Ans.**

Example 2. Three pipes A, B and C can fill a cistern in 15, 20 and 30 minutes respectively. They were all turned on at the same time after 5 minutes the first two pipes were turned off. In what time will the cistern be filled ?

Sol. A, B and C can fill $(\frac{1}{15}+\frac{1}{20}+\frac{1}{30})$ or $\frac{3}{20}$ of the cistern in 1 minute.

$\therefore$ A, B and C filled $\frac{3}{20}\times 5$ or $\frac{3}{4}$ of the cistern in 5 min.

Now A and B are turned off ;

$\therefore$ $1-\frac{3}{4}$ or $\frac{1}{4}$ of the cistern will be filled by C.

$\therefore$ C will fill $\frac{1}{4}$ of the cistern in $30\times\frac{1}{4}$ or $7\frac{1}{2}$ minutes.

$\therefore$ the cistern will be filled in $7\frac{1}{2}+5$ or $12\frac{1}{2}$ min. **Ans.**

Example 3. Two pipes A and B can fill a cistern in 6 and 8 minutes respectively. If they are turned on alternately for one minute each, how long will it take the cistern to fill ?

Sol. A can fill $\frac{1}{6}$ of the cistern in one minute ;

B can fill $\frac{1}{8}$ of the cistern in one minute.

Since A, B are turned on alternately for 1 min. each
$\therefore$ A and B can fill $\frac{1}{6}+\frac{1}{8}$ or $\frac{7}{24}$ of the cistern in 2 min.
$\therefore$,, the remaining portion $=1-\frac{7}{8}$ or $\frac{1}{8}$
$\therefore$ $\frac{7}{24}\times 3$ or $\frac{7}{8}$ of the cistern in 6 min.
$\therefore$ the cistern will be filled by A in $6\times\frac{1}{8}$ or $\frac{3}{4}$ min.
$\therefore$ the cistern will be filled in $6+\frac{3}{4}=6\frac{3}{4}$ minutes. **Ans.**

Example 4. A cistern can be filled by two pipes A and B in 30 and 40 minutes respectively and a third pipe C can empty it in 20 minutes. If all the three are kept open successively for 1 minute each, how soon will the cistern be filled?

Sol. When only $(\frac{1}{30}+\frac{1}{40}-\frac{1}{20})$ or $\frac{1}{120}$ of the cistern is filled in 3 min.
can fill this portion in 2 minutes in their turn
$\therefore$ the remaining portion $=1-\frac{7}{120}=\frac{113}{120}$
$\therefore$ $\frac{1}{120}$ of the cistern is filled in 3 minutes
$\therefore$ $\frac{113}{120}$ of the cistern is filled in $120\times 3\times\frac{113}{120}=339$ min.
$\therefore$ the required time $=339+2=341$ min. **Ans.**

EXERCISE 95.

1. Three taps A, B and C can fill a cistern in 12, 15 and 18 minutes respectively. They are all turned on, but after $4\frac{1}{2}$ minutes A and C are turned off. How many mitutes longer will B take to fill the cistern?

2. A cistern can be filled by two taps in 20 minutes and 30 minutes respectively and can be emptied by a third in 48 minutes. They are all turned on at once. When will the cistern be half full?

3. A cistern can be filled by two taps A and B in 12 minutes and 14 minutes respectively and can be emptied by a third in 8 minutes. If all the taps are turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 minutes?

4. A reservoir can be filled by two taps in 6 hours and 9 hours respectively. The first tap was opened at 7 A.M., and the second at 8 A.M., at what o'clock will the reservoir be full?

5. Two taps can fill a cistern in 6 hours and 8 hours respectively and a waste pipe can empty it in 3 hours. If the cistern be full and all the three taps are turned on, what time will it take to get it empty?

6. A cistern can be filled by two pipes in $5\frac{1}{4}$ hours and $4\frac{2}{3}$ hours respectively and emptied by an escape pipe in $2\frac{1}{3}$ hours. If all the three pipes are turned on and the cistern is half full, what time will elapse before the cistern is emptied ?

7. A cistern can be filled by 3 pipes in 30, 40 and 60 minutes respectively and emptied by an escape pipe in half an hour. The three pipes are turned on at noon but the escape pipe is at the same time accidentally left open and not closed for a quarter of an hour. At what time will the cistern be full ?

8. A cistern can be filled by two pipes in 8 and 12 hours respectively and emptied by a third in 24 hours. The first two were opened at 7 A. M., and the waste pipe was opened by a boy at 8 A. M. At what time will the cistern be full ?

9. A cistern can be filled by two pipes in 4 hour and 6 hours respectively and emptied by a waste pipe in 2 hours. If the cistern be half full and the pipes were all turned on at the same moment, when will the cistern be emptied ?

10. Two pipes can fill a cistern in 3 and 4 hours respectively and a waste pipe can empty it in one hour. If the first be opened at one, the second at two and the third at three o'clock, at what time will the cistern be emptied ?

11. Two pipes which can fill a cistern in 20 minutes and 30 minutes respectively were turned on at the same moment. After some time the first pipe was turned off and the cistern was then filled in 20 minutes more. When was the first pipe turned off ?

12. A cistern can be filled by two pipes in 30 and 40 minutes respectively. Both the pipes were turned on at the same moment, but after some time the first was turned off and the cistern was filled in 10 minutes more. How long after the pipes have been turned on the first one was turned off ?

13. Two pipes can fill a cistern in 10 and 12 minutes respectively. If they are kept open successively for one minute each, how soon will the cistern be filled ?

14. *A* can fill a cistern in 15 minutes and *B* in 20 minutes and *C* can empty it in 10 minutes. If they are kept open succes sively for one minute each, how soon will the cistern be filled ?

15. A cistern has three pipes, *A*, *B* and *C*. *A* and *B* can fill it in 12 and 15 minutes respectively and *C* can empty it in 10 minutes. If the three pipes are kept open alternately for one minute each, how soon will the cistern be filled ?

CHAPTER XIX

MISCELLANEOUS PROPOSITIONS (*Continued*)

1. TIME AND DISTANCE

§1. Consider the following simple question :—

A person walks at the rate of 5 miles an hour ; how far will he go in 3 hours ?

15 miles. **Ans.** [5×3=15]

Here 5 miles per hour is the **speed** or **velocity** of the person ; 3 hours is the **time** and 15 miles is the **distance.**

Clearly, therefore, **speed×time=distance.**

From this we can deduce the following conclusions :—

Time=distance÷speed and speed=distance÷time.

§2. The following observations are almost self-evident but are very useful in solving questions of this class :—

(*i*) When two bodies are moving in a straight line and approaching each other from opposite directions their *relative* speed, *i.e.*, the speed with which they approach each other is equal to the **sum** of their *absolute* speeds.

(*ii*) But if they are moving in the same direction, then their *relative* speed is equal to the **difference** of their *absolute* speeds.

(*iii*) If two bodies approaching each other from opposite directions meet, it means that they have together travelled the whole distance between them.

(*iv*) A moving train passes a fixed point when the train has gone over a distance equal to its own **length.**

(*v*) If two trains run on parallel lines either in opposite directions or in the same direction ; one of them shall pass the other only when the former has gained over the latter a distance equal to the sum of the lengths of the trains.

[See §2, page 316]

(*vi*) When a vessel goes down a stream, the rower is *helped* by the current but if it goes upstream, his progress is *opposed* by the current. Hence in the former case the total work done by the vessel is the **sum** of works done by the rower and the current and in the latter case 'it is their **difference.**'

[See Art. 3, page 322].

Now we shall solve some examples to illustrate this method :—

Example 1. Two persons who live in different towns 40 miles apart started to meet each other at 10 A. M. One walked at the rate of $3\frac{1}{2}$ miles an hour, and the other at $4\frac{1}{2}$ miles an hour. When will they meet and how many miles will each walk ?

Sol.

40 miles

$4\frac{1}{2}$ miles $3\frac{1}{2}$ miles

Since they are walking in opposite directions their relative speed would be $3\frac{1}{2}+4\frac{1}{2}$ *i. e.*, 8 miles per hour.

$\therefore$ time required to walk 40 miles $=40\div 8=5$ hours.

Hence they will meet at $10+5=15$ or 3 P. M. ;

One man had walked $3\frac{1}{2}\times 5=17\frac{1}{2}$ miles

and the other man had walked $4\frac{1}{2}\times 5=22\frac{1}{2}$ miles } **Ans.**

Example 2. A policeman goes after a thief who is 176 yards before him. When and where will the policeman catch the thief when they run at the rates of $6\frac{1}{2}$ and 6 miles per hour respectively ?

Sol. Policeman Thief

176 yards.

(*a*) Since they run in the same direction therefore relative speed of the policeman would be $6\frac{1}{2}-6$, *i.e.*, $\frac{1}{2}$ mile or 880 yds. per hour.

880 yds. he walks in 60 minutes.

$\therefore$ 1 yd. ,, ,, ,, $\frac{60}{880}$ minutes,

$\therefore$ 176 yd. ,, ,, ,, $\frac{60}{880}\times 176=12$ minutes.

$\therefore$ the time reqd. to overtake the thief $=12$ min. **Ans.**

(2) The distance from the starting point

$=6\frac{1}{2}\times\frac{12}{60}$ miles

$=\frac{13}{2}\times\frac{1}{5}=\frac{13}{10}$ mile $=1$ mile 528 yds. **Ans.**

Example 3. If I walk at the rate of 4 miles an hour I reach my destination 30 minutes too late ; If I walk at the rate of 5 miles an hour I reach 30 minutes too soon. How far is my destination ?

Sol. Suppose I have to go one mile.
∴ it will take me 15 minutes to reach the destination at the rate of 4 miles an hour, and 12 minutes at the rate of 5 miles an hour, *i.e.*, I will save 3 minutes to walk one mile.
But I save 30+30, *i.e.*, 60 minutes in all.
∴ distance required=60÷3=20 miles. **Ans.**

Example 4. A hare makes 4 leaps in the same time in which a dog makes 3 but 2 leaps of the dog cover as much distance as 3 leaps of the hare; how many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 60 leaps?

Sol. 2 leaps of the dog=3 leaps of the hare
∴ 1 leap of the dog=$\frac{3}{2}$ leaps of the hare.
But the dog takes as much time in one leap as the hare takes in $\frac{4}{3}$ leaps.
It is clear, therefore, that $\frac{3}{2}-\frac{4}{3}$ or $\frac{1}{6}$ leap of the hare can be made up by the dog in 1 leap.
∴ 60 leaps can be made up in $60\div\frac{1}{6}$ or 360 leaps. **Ans.**

Example 5. I have to go to a certain place 7 miles off in 1 hour 15 minutes. If I walk some way on foot at 4 miles an hour and ride some way on horse-back at 10 miles an hour I reach my destination in time. How far should I walk on foot?

Sol. If I walk the whole way on foot I shall take 1 hour 45 min. to reach the destination, *i.e.*, I shall be 30 min. too late.
Again if I go the whole way on horse back I shall take 42 min. only to reach the destination, *i.e.* I shall be 3 min. too soon.
∴ the ratio between distance travelled on foot and horse-back would be=33 : 30 } Inverse ratio
=11 : 10 }
∴ distance I should walk on foot=$\frac{11}{21}\times 7=3\frac{2}{3}$ miles. **Ans.**

Example 6. Two guns are fired from the same place after an interval of 12 minutes, but a person approaching the place on a motor cycle hears the report after an interval of 11 minutes 15 sec. Find his rate of speed, sound travelling at the rate of 1100 ft. per second.

Sol. The distance which the man travels in 11 min. 15 sec. is the same as that travelled by the report in 12 min.—11 min. 15 sec. *i.e.*, 45 seconds.

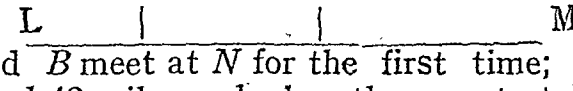
But in 45 sec. the sound travels 1100×45 ft.
$\therefore$ in $11\frac{1}{4}$ min. the man travels $=1100\times45$ ft.
$\therefore$ in 1 ,, ,, ,, $=1100\times45\times\frac{4}{45}$ ft.
$\therefore$ in 1 hour ,, ,, $=1100\times45\times\frac{4}{45}\times60$ ft.
$=\frac{1100\times45\times4\times60}{45\times3\times1760}$ miles
$=50$ miles **Ans.**

Example 7. The distance from P to Q is 24 miles, 6 miles of which is up-hill; 6 miles level and 12 miles down-hill; find the difference between the time in which a person would walk from P to Q and the time in which he will come back again, supposing his pace up-hill to be 3 miles, down hill 6 miles and on level 4 miles per hour.

Sol. From P to Q the distance is 6 miles up-hill, 6 miles level and 12 miles down hill
$\therefore$ whole time taken $=(\frac{6}{3}+\frac{6}{4}+\frac{12}{6})$ hrs. $=5\frac{1}{2}$ hrs.
Also from Q to P the distance is 12 miles up-hill, 6 miles level and 6 miles down-hill.
$\therefore$ whole time taken $=(\frac{12}{3}+\frac{6}{4}+\frac{6}{6})$ hrs. $=6\frac{1}{2}$ hrs.
Hence the diff. $=6\frac{1}{2}-5\frac{1}{2}=1$ hr. **Ans.**

Example 8. A and B start at the same time from L and M to go to M and L, a distance of 42 miles at the rates of 4 miles and 3 miles respectively. They meet at N, then go to M and L and return immediately and meet again at D. Find the distance DN.

Sol.



When A and B meet at N for the first time; they have together travelled 42 miles and when they meet at D for the second time they have together travelled 42×3 *i. e.*, 126 miles
The distance travelled by A when he meets at N for the first time $=LN$.

$\therefore LN=\frac{4}{4+3}\times42=\frac{4}{7}\times42=24$ miles.

Again, the distance travelled by B when he meets at D for the second time $=ML+LD$
$\therefore ML+LD=126\times\frac{3}{7}=54$ miles
$\therefore LD=54-LM$ miles
$=54-42=12$ miles
$\therefore DN=24-12=12$ miles **Ans.**

EXERCISE 96.

1. A man walks at the rate of $2\frac{3}{4}$ miles an hour; how long will he take to walk 605 yards?

2. The velocity of a train is 35 miles an hour; find velocity per second.

3. Two persons living in different towns 32 miles apart started at 1 P.M to see each other at the rate of $3\frac{1}{2}$ and $4\frac{1}{2}$ miles per hour respectively. When will they meet and how many miles will each then walk?

4. Two men started from a place P at the rate of $2\frac{1}{2}$ and $2\frac{3}{4}$ miles an hour. How many miles will they be apart after 5 hours (*i*) If they walk in opposite directions; (*ii*) if they walk in the same direction?

5. Two gentlemen start from Lahore at the same time at the rate of $3\frac{1}{2}$ and $4\frac{1}{2}$ miles an hour. When will they be 21 miles apart, supposing (*i*). they walk in the same direction (*ii*) in opposite directions?

6. Two persons A and B start from a place P. A starts at 7 A.M. and goes at the rate of $3\frac{3}{4}$ miles an hour and B at 8 A.M. at the rate of 4 miles an hour. At what time will B overtake him and how far will he then be from the starting place?

7. Two persons A and B start from Lahore and Amritsar respectively. A goes at the rate of $3\frac{1}{2}$ miles and B at the rate of $3\frac{3}{4}$ miles an hour. If they meet each other after 4 hours, find the distance between Lahore and Amritsar.

8. Two persons set out from Jullundur and Ludhiana respectively. One goes at the rate of $3\frac{1}{2}$ miles and the other at the rate of $3\frac{5}{6}$ miles an hour. They meet each other after $5\frac{1}{11}$ hours. Find the distance between the two cities.

9. A who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{2}$ hours before B who goes the same road at $4\frac{1}{2}$ miles an hour; where will B overtake A?

10. A policeman goes after a thief who has 100 yards start; if the policeman runs one mile in six minutes and the thief in 10 minutes, how far will the thief have gone before he is overtaken?

11. A thief who had escaped at $7\frac{1}{2}$ P.M. was followed by a policeman at 9 P.M. at the rate of 6 miles an hour. At what time will the policeman overtake him, supposing the thief runs at $4\frac{1}{2}$ miles an hour?

12. A policeman runs after a thief who has $3\frac{1}{2}$ miles' start; if the policeman runs at the rate of $3\frac{3}{4}$ miles and the thief at the rate of $3\frac{1}{2}$ miles an hour, when will the policeman overtake him?

13. A man rides at the rate of 11 miles an hour but stops 5 minutes to change horses at the end of every 7th mile; how long will he take to go a distance of 94 miles?

14. A student walks to school at the rate of $2\frac{1}{2}$ miles an hour and reaches 6 minutes too late. Next day he increases his speed by 2 miles an hour and then reaches there 10 minutes too soon. Find the distance of the school from his home.

15. I start to walk to a certain place at the rate of 4 miles an hour and reach there 15 minutes too late. When I increase my speed by 1 mile per hour, I reach there 30 minutes too soon. Find the distance of the place.

16. A man travelled 20 miles in 4 hours 10 minutes. He travelled some way on foot and some way on coach at 3 and 8 miles per hour respectively. How far did he travel on coach?

17. A person walks from A to B at the rate of 3 miles an hour and after transacting some business which occupies him an hour, returns to A by tramway at the rate of 5 miles an hour. He then finds that he has been absent for 2 hours 20 minutes. Find the distance from A to B.

18. A hare makes 5 leaps in the same time in which a dog makes 4, but 2 leaps of the dog cover as much distance as 3 leaps of the hare. How many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 100 leaps?

19. A hare makes 4 leaps in the same time in which a dog makes 3, but 2 leaps of the dog cover as much distance as 3 of the hare. How many leaps will the dog have to make before catching the hare, supposing the hare to have a start of 90 leaps?

20. A hare sees a dog 110 yards away from him and starts off in the opposite direction at the rate of 10 miles per hour. A minute later, the dog sees the hare too and pursues him at the rate of 14 miles an hour. When will the dog overtake the hare?

21. A person went from A to B, at the rate of 4 miles an hour and came back at the rate of 3 miles an hour. It took him 12 hours; find the distance from A to B.

22. A person goes from A to B at the rate of 5 miles an hour and comes back at the rate of 4 miles an hour. It took him 9 hours; find the distance from A to B.

23. A, B and C can walk at the rates of 3, 4, 5 miles an hour: they start from Poona at 1, 2, 3 o'clock respectively; when B catches up A, B sends him back with a message to C; when will C get the message?

24. A sets out from a place P to walk to a place Q. A quarter of an hour later a second person B sets out from P to walk to Q but after walking half a mile returns to P, where he is detained 10 minutes. Again setting out from P he reaches Q 5 minutes after A. If A walks throughout at the rate of 3 miles an hour and B at the rate of 4 miles an hour, find the distance from P to Q.

25. Two persons A and B set out together on a journey. They walked at the rate of 3 miles an hour; and when they had proceeded for three quarters of a mile, B returned, walking at the same rate to the place of starting. Here he was detained three quarters of an hour. Setting out again he overtook A, who had been walking all the time, at the end of $2\frac{1}{2}$ hours from the second time of starting. At what rate did he walk?

26. Sound travels at the rate of 1160 ft. per second. A shot is fired from a ship sailing at the rate of 10 miles an hour. How far will the ship have moved when the report is heard at the place $14\frac{1}{2}$ miles off?

27. A snail creeps 1 ft. 5 inches up a long pole during 12 hrs. in the night but slips down 10 inches during 12 hours in the day. If the pole is 12 ft. high, how long will it take to reach the top?

28. The distance P to Q is 30 miles, 8 miles of which is up-hill, 12 miles level and 10 miles down-hill; find the difference between the time in which a person would walk from P to Q and the time he will take to return from Q to P: supposing his pace up-hill to be 4 miles, down-hill 6 miles and on level 5 miles per hour.

29. A and B start from P and Q to go to Q and P respectively,—the distance PQ being 60 miles. A goes at the rate of 4 miles and B at the rate of 5 miles per hour. They meet at R,

reach their destinations, return immediately and again meet at S. Find the distance between R and S.

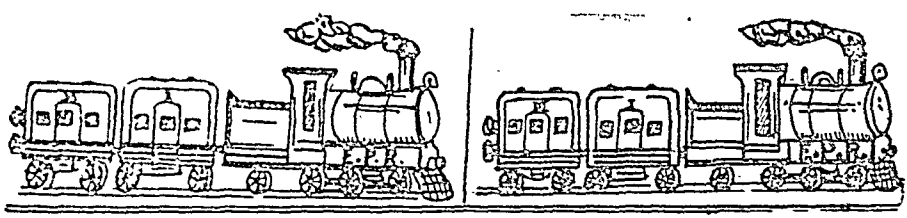
30. A starts from P and goes towards Q at the rate of 6 miles an hour. Two hours afterwards, B starts from P and going 10 miles per hour reaches Q 4 hours earlier than A. Find the distance between P and Q.

II. TRAINS.

§2. We shall solve some problems on Trains.

Example 1. A train 176 yards long moves at the rate of 40 miles an hour : how long will it take to pass a certain telegraphic post ?

Sol. *First position* *Second position.*



In the first position, distance between the engine and the telegraphic post is zero, and in the second position distance between the engine and the post is equal to the length of the train. It is clear, therefore, that *in passing a certain post or a standing man the train is to move its own length.*

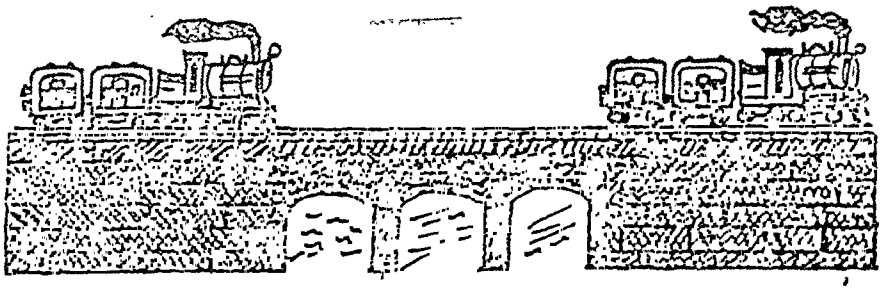
$$\therefore \text{ the required time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{176}{40 \times 1760} \text{ hrs.}$$

$$= \frac{176 \times 60 \times 60}{40 \times 1760} \text{ or } 9 \text{ sec.} \quad \textbf{Ans.}$$

Example 2. How long will a train 154 yards long take to cross a bridge 176 yards long at the rate of 40 miles an hour ?

Sol. *First position* *Second position.*



In the first position distance between the engine and the beginning of the bridge is zero. In the second position, the distance between the engine and the beginning of the bridge is equal to the **sum** of the lengths of the bridge and the train, *i.e.*, 176+154=330 yards. It is clear, therefore, that in *crossing a bridge or a station the train is to move its own length* **plus** *the length of the bridge or the station.*

$\therefore$ the required time=distance÷speed

$$=\frac{330}{40\times1760}\text{hrs.}=\frac{330\times60\times60}{40\times1760}\text{or } 16\tfrac{7}{8}\text{ sec. Ans.}$$

Example 3. Two trains 110 yards and 88 yards long respectively are running in the same direction. The first runs at the rate of 20 miles an hour and the second at the rate of 25 miles an hour. How long will they take to cross each other ?

Sol. It is clear that the trains will cross each other when they have travelled a distance equal to the sum of thier lengths *i. e.*, 110+88 or 198 yards. Since they are moving in the same direction, the relative speed=25—20 or 5 miles per hour.

$\therefore$ time reqd.=distance÷speed.

$$=\frac{198\times60}{5\times1760}\text{min.}=1\text{ min. }21\text{ sec. Ans.}$$

Note. Had they run in opposite directions, the relative speed would have been 25+20 or 45 miles per hour and then they would have passed each other in

$$\frac{198\times60\times60}{45\times1760}\text{ or } 9\text{ sec.}$$

Example 4. Two trains starting from the same station and travelling in the opposite directions are $227\frac{1}{2}$ miles apart in 3 hours 15 min. Had they been travelling in the same direction they would have been $32\frac{1}{2}$ miles apart in the same time ; find their speeds.

Sol. The sum of their rates per hr. $=\frac{455}{2}\div\frac{13}{4}$ or 70 miles.
and the diff ,, ,, ,, $=\frac{65}{2}\div\frac{13}{4}$ or 10 miles.
Hence the rate of the faster $=(70+10)\div 2$ or 40 miles
and ,, ,, ,, slower $=(70-10)\div 2$ or 30 miles } **Ans.**

Example 5. Two trains start at the same time, one from *A* to *B* and the other from *B* to *A*, at the rates of 30 and 40 miles per hour respectively. When they crossed each other it was found that the faster train had travelled 50 miles more than the slower one. Find the distance between *A* and *B*.

Sol. Ratio in the speeds of the trains $=40:30$
$=4:3.$
$=\frac{4}{7}:\frac{3}{7}.$
Diff. $=\frac{4}{7}-\frac{3}{7}=\frac{1}{7}$. But the real diff. is 50 miles.
$\therefore$ distance $=50\times 7$ or 350 miles. **Ans.**
Aliter. Difference in the speeds per hour $=10$ miles.
But the total diff. $=50$ miles.
$\because$ this difference is caused in $50\div 10$ or 5 hours
$\therefore$ the distance between *A* and *B* $=(40+30)\times 5$
$=350$ miles. **Ans.**

Example 6. Two trains travelling in the opposite directions pass each other in 8 seconds. But when they travel in the same direction at the same rates the man in the faster train passes the other in $31\frac{1}{2}$ sec. Find the lengths of the trains when their speeds are 25 and 20 miles respectively.

Sol. (*i*) When the trains travel in opposite directions, the distance travelled in one hour $=25+20=45$ miles.
$\therefore$,, ,, ,, one sec. $=\frac{45}{3600}$ mile
$\therefore$,, ,, ,, 8 ,, $=\frac{45}{3600}\times 8$ mile
$=\frac{45}{3600}\times 8\times 1760$ yds.
$=176$ yds.

It follows therefore, that the sum of the lengths of both the trains is 176 yds.

(*ii*) When the trains travel in the same direction, then the distance travelled in one hour $=25-20=5$ miles.
$\therefore$,, ,, ,, ,, one sec. $=\frac{5}{3600}$ mile
$\therefore$,, ,, ,, ,, $31\frac{1}{2}$ sec. $=\frac{5}{3600}\times\frac{63}{2}$ miles
$=\frac{5}{3600}\times\frac{63}{2}\times 1760=77$ yds.
i.e., the length of the slower train $=77$ yds.
$\therefore$,, ,, ,, faster ,, $=176-77=99$ yds. } **Ans.**

Example 7. A train starts from Lahore at 1-20 P. M. and reaches Ludhiana at 5-50 P. M. A second train starts from Ludhiana at 1-50 P.M. and reaches Lahore at 7-10 P. M. Find the ratio in the speeds of the two trains.

Sol. Time taken by the first train = 4 hrs. 30 min. $=\frac{9}{2}$ hrs.
and ,, ,, ,, second ,, = 5 hrs. 20 min. $=\frac{16}{3}$ hrs.

Ratio in their times $=\frac{9}{2} : \frac{16}{3}$
$\therefore$ ratio in their speeds $=\frac{16}{3} : \frac{9}{2} = 33 : 27$. **Ans.**

Example 8. A train after travelling 70 miles meets with an accident and then proceeds at $\frac{3}{4}$ of its former speed and arrives at its destination 35 min. too late. Had the accident happened 15 miles further on, it would have reached the destination only 23 min. late. Find (*a*) the distance and (*b*) the speed of the train.



Sol. (*a*) Let A be the starting place, B the destination; C is the place where the accident happened and D is the place 15 miles off from C.

Travelling at $\frac{3}{4}$ of its original speed means that the train would take $\frac{4}{3}$ of its usual time, *i.e.*, $\frac{1}{3}$ of the usual time more.

$\because$ the excess time taken to travel $CB = 35$ min.
and ,, ,, ,, ,, $DB = 23$ min.
$\therefore$,, ,, ,, ,, $CD = 12$ min.
12 min, is $\frac{1}{3}$ of the usual time
$\therefore$ usual time is 36 min.
$\because$ in 36 min. the distance travelled is CD., *i.e*,, 15 miles
$\therefore$ in 1 min. ,, ,, ,, $\frac{15}{36}$ miles
$\therefore$ in 60 min. ,, ,, ,, $\frac{15}{36} \times 60$ or 25 miles.
i.e., the speed is 25 miles per hour. **Ans.**

(*b*) $\because$ the excess time taken to travel $CD = 35$ min.
$\therefore$ usual time $= 35 \times 3 = 105$ min. $= \frac{7}{4}$ hrs.
$\therefore$ the distance $CB = 25 \times \frac{7}{4} = \frac{175}{4}$ miles.
$\therefore$ whole distance $= \frac{175}{4} + 70 = 113\frac{3}{4}$ miles.

EXERCISE 97.

1. A train runs at the rate of 15 miles an hour ; find its speed per minute.

2. A train moves 176 yards in 9 seconds ; find its speed per hour.

3. A train running at the rate of 20 miles an hour passes a certain telegraphic post in 18 seconds ; find the length of the train.

4. A train which is 110 yards long moves at the rate of 40 miles an hour ; what time will it take to pass a certain telegraphic post ?

5. A train 110 yds. long travels at the rate of 45 miles an hour ; what time will it take to pass a station 88 yards long ?

6. Two trains 195 ft. and 135 ft. long travel at the rates of 25 miles and 20 miles respectively. When will they pass each other when they are moving (*i*) in the same direction, (*ii*) in opposite directions ?

7. Two trains measuring 92 yards and 84 yards in length travel at the rates of 30 miles and 45 miles respectively in opposite directions. How long will it take them to cross each other ?

8. Two trains 88 yards and 110 yards in length respectively are running (*i*) in opposite directions, (*ii*) in the same direction. The first runs at the rate of 50 miles an hour and the second at the rate of 40 miles an hour. How long will they take to cross each other ?

9. A passenger train going 41 miles an hour and 431 ft. long overtakes a goods train on a parallel line of rails. The goods train is going 28 miles an hour and is 713 ft. long. How long does the passenger train take in passing the other ?

10. A train 99 yards long passes a man coming from the opposite direction at the rate of 4 miles an hour in 10 seconds. Find the speed of the train.

11. A train 99 yards long passes a man going in the same direction at the rate of 4 miles an hour, in 10 seconds. Find the speed of the train.

12. A train which runs at the rate of 50 miles an hour crosses a man going in the same direction at the rate of 5 miles an hour in 5 second. Find the length of the train.

13. A train which runs at the rate of 50 miles an hour crosses a man walking in the opposite direction at the rate of 5 miles an honr in 3 seconds. Find the length of the train.

14. A goods train moving at the rate of 20 miles an hour passes a man going in the same direction at the rate of 4 miles an hour in $7\frac{1}{2}$ seconds. Find the length of the train.

15. The whole time occupied by a train 126 yds. long, travelling at the rate of 45 miles an hour, in crossing a bridge is 12 seconds. Find the length of the bridge. *(Benares 1912)*

16. Two trains start at the same time from Lahore and Peshawar and proceed towards each other at the rates of 25 miles and 30 miles respectively. When they meet it is found that one train has travelled 35 miles more than the other. Find the distance between the two stations.

17. Two trains start at the same time from Mirzapur and Delhi and proceed towards each other at the rates of 16 and 21 miles per hour respectively. When they meet it is found that one train has travelled 60 miles more than the other. Find the distance between the two stations.

18. A lacal train which travels at the rate of 24 miles an hour, leaves Lahore at 20 minutes past 8 and reaches Amritsar at 5 minutes past 10 the same morning. It stops at Moghalpura for 10 minutes and at each of the three other stations for 5 minutes. Find the distance from Lahore to Amritsar.

19. Two trains 330 ft. and 264 ft.l ong respectively run on parallel lines of rails. When travelling in opposite directions they are observed to pass each other in 9 seconds, but when they are running in the same direction at the same rate as before the faster train passes the other in $27\frac{1}{2}$ seconds. Find the speed of the trains in miles per hour.

20. A train running at the rate of 40 miles an hour meets a person walking along the line in the opposite direction at the rate of 4 miles an hour and passes him in $5\frac{5}{8}$ seconds; find the length of the train. Had the person been going in the same direction as the train, in what time would it have passed him ?

21. A train 132 yards in length, travelling at a uniform speed, overtook a man walking along the line at the rate of 6 miles an hour and passed him in 12 second. Twenty minutes

later, the train overtook a second man and passed him in 11 seconds. How many hours after the train overtook the second man, would the first man also overtake him ?

22. A train moving at the rate of 45 miles an hour passes another train $1\frac{1}{2}$ times as long and moving at the rate of 27 miles an hour in the same direction in 25 seconds. How long would it take to pass a station 165 yards long?

23. Two trains running at the rate of 35 and 30 miles an hours respectively, on parallel rails in opposite directions are observed to pass each other in $4\frac{1}{2}$ seconds and when they are running in the same direction at the same rate as before a person sitting in the faster train observes that he passes the other in 36 sec. find the lengths of the trains.

24. A train starts from Lahore at 2-50 P.M. and reaches Ludhiana at 7-10P.M. and a second train starts from Ludhiana at-2-30 P. M. and reaches Lahore at 7-50 P. M. Find the ratio in the speeds of the trains.

25. A man after travelling 80 miles got his cycle punctured and then started at $\frac{4}{5}$ of his former speed. He reached his destination 30 min. too late. Had it punctured 10 miles further on, he would have reached the destination only 20 min. late. Find the distance and the speed of the cycle.

III. STREAMS.

§3. Suppose a man can swim in still water at the rate of 4 miles an hour. If he were to swim in a running stream the speed of which is one mile an hour then his speed would be 4+1, *i.e.* 5 miles an hour with the stream and 4−1, *i.e.*, 3 miles an hour against the stream.

From the above explanation we can deduce the following results :—

(i) Man's speed in still water = { half the sum of his speed with and against the stream. Thus $\frac{1}{2}(5+3)=4$.

(ii) Speed of the stream = { half the difference of his speeds with and against the stream. Thus $\frac{1}{2}(5-3)=1$.

Example 1. A boat is rowed down a river 20 miles in 4 hours and 44 yards up the river in 45 seconds. Find the velocity of the river.

Sol. Speed with the river $=\frac{20}{4}$, *i. e.*, 5 miles an hour.

Speed against the river $=\frac{44}{45}\times\frac{60\times60}{1760}$ miles

$=2$ miles an hour.

$\therefore$ velocity of the river $=\frac{1}{2}(5-2)$, *i. e.*, $1\frac{1}{2}$ miles an hour. **Ans.**

Example 2. A man can swim with the stream at the rate of 6 miles an hour and 352 yards against the stream in 4 minutes; find his speed in still water.

Sol. Speed with the stream $=6$ miles an hour.

Speed against the stream $=\frac{352}{4}\times\frac{60}{1760}$

$=3$ miles an hour.

$\therefore$ Speed in still water $=\frac{1}{2}(6+3)=4\frac{1}{2}$ miles. **Ans.**

Example 3. A man rows 18 miles down a river in 4 hours with the stream and returns in 12 hours; find his speed and also the velocity of the stream.

Sol. Speed with the stream $=\frac{18}{4}=4\frac{1}{2}$ miles an hour.

$\therefore$ Speed against the stream $=\frac{18}{12}=1\frac{1}{2}$ miles an hour.

$\therefore$ Speed of the stream $=\frac{1}{2}(4\frac{1}{2}-1\frac{1}{2})=1\frac{1}{2}$ miles an hour
and his speed $=4\frac{1}{2}-1\frac{1}{2}=3$ miles an hour } **Ans.**

EXERCISE 98.

1. A man can row in still water at the rate of 5 miles an hour; how long will he take to row 27 miles with the stream when the velocity of the stream is $1\frac{1}{4}$ miles an hour?

2. A man can row a mile with the stream in 15 min. and against the stream in 25 minutes. How long will it take him to row 2 miles in still water?

3. A man can swim with the stream at the rate of 3 miles an hour and against the stream at the rate of 2 miles an hour. How long will it take him to swim 7 miles in still water?

4. A man rows with the stream at the rate of $2\frac{1}{2}$ miles an hour and against the stream at the rate of $1\frac{1}{2}$ miles an hour. How long will it take him to row 12 miles in still water ?

5. A boat moves a mile down the stream in 15 minutes and up the stream in 25 minutes. How long will it take the boat to move 176 yards in still water ?

6. A man can swim 88 yards per minute down the stream and 55 yards per minute up the stream ; find the velocity of the river.

7. A person rows a mile down the stream in 20 minutes and up the stream in 25 minutes. Find the velocity of the stream.

8. A man can row in still water a distance of 4 miles in 20 minutes and 4 miles with the current in 16 min. How long will it take him to row the same distance against the current ?

9. A man rowe 15 miles down a river in 3 hours with the stream and returns in $7\frac{1}{2}$ hours, find the rate at which he rows and also the velocity of the stream.

10. A, B and C are three towns on a river which flows uniformly and B is equidistant from A and C. I can row from A to B and back in $5\frac{1}{4}$ hours and I can row from A to C in 7 hours ; compare the speed of my boat in still water with that of the river.

IV. RACES AND GAMES OF SKILL.

§4. The following terms used in questions of races and games of skill may be noted :—

(*i*) A **gives** B **a start of 20 yards** means that B stands 20 yards ahead of A.

(*ii*) **In a game of 100** A **can give** B **10 points** means that if A scores 100 points B scores only 90.

(*ii*) **Dead heat** means a race in which no one is the winner.

Example. *A* can beat *B* by 60 yards in a mile race ; *B* can beat *C* by 20 yards in a mile race. If *A* and *C* run a mile, by how much will *A* win ?

Sol. *A* can run 1760 yards while *B* runs 1700 yards.
and *B* can run 1760 yards while *C* runs 1740 yards.
Now if *B* runs 1760 yds. *C* runs 1740 yds.
∴ if *B* runs 1 yard *C* runs $\frac{1740}{1760}$ yards.
∴ if *B* runs 1700 yards *C* runs $\frac{1740}{1760} \times 1700$ yards.
i. e., $\frac{3692}{22}^{75}$ or $1680\frac{15}{22}$ yds.
if *A* runs 1760 yards *C* runs $1680\frac{15}{22}$ yards.
∴ *A* can win by $1760 - 1680\frac{15}{22}$ *i.e.*, $79\frac{7}{22}$ yards. **Ans.**

EXERCISE 99.

1. In a mile race *A* beats *B* by 40 yards and *C* by 60 yards. By how many yards will *B* beat *C* ?

2. In a mile race *A* gives *B* 44 yards, start and beats him by 44 yards. If *A* runs a mile in $9\frac{1}{2}$ minutes, how long will *B* take to do the same ?

3. *A* can give *B* 20 yards and *B* can give *C* 10 yards in a race of 200 yards. By how much could *A* beat *C* in the same race ?

4. In a mile race *A* beats *B* by 110 yards and *B* beats *C* by 120 yards. By how many yards could *A* beat *C* in the same race ?

5. *A* can give *B* 55 yards and *C* 80 yards in a race of 880 yards. By how much could *B* beat *C* in a mile race ?

6. In a mile race Madan Lal gives Tara Chand a start of 55 yards and beats him by 105 yards. If Madan Lal runs the mile in 5 minutes, how long will Tara Chand take ?

7. *A* can run 320 yards in 2 minutes and *B* 240 yards in one minute. By how much will *B* beat *A* in a mile race ?

8. In a mile race *A* can give *B* 40 yards and *B* can give *C* 40 yards ; how many yards start should *A* give *C* so that they may end in a dead heat ?

9. *A* runs 20 yards while *B* runs 21 yards ; *B* runs 31 yards while *C* runs 30 yds.; if *A* can run a mile in 5 min. 15 sec., what time will *C* take to do it ?

10. *A* can run 8 yards in the same time in which *B* can run 9 yds. They start together. When *B* has run 252 yards, how far behind is *A* ?

11. *A* works 15 sums in the same time in which *B* works 10 and *A* works 12 sums in the same time in which *C* works 7. Who is the quicker worker, *B* or *C* ?

12. In a half mile race *A* gives *B* 10 yards, start and beats him by 20 yds. *B* gives *C* 30 yards' start in half a mile and is beaten by 60 yards ; who runs faster, *A* or *C* ?

13. At a game of rackets, *A* can give *B* 8 points in 40 ; and *B* can give *C* 10 points in 50 ; how many points could *A* give *C* in 75 ?

14. *A* can give *B* 25 points, *A* can give *C* 40 points and *B* can give *C* 20 points. How many points make the game ?

15. In a game of skill, *A* can give *B* 8 points out of 38 and to *C* 10 points out of 95. Of *B* and *C* who is the better player and how many points can he give the other in 340 ?

16. *A* can give *B* 1 point in 5 at billiards. *B* can give *C* 3 in 20, *D* can give *C* 1 in 18. How many can *A* give *D* in 100 ?

V. TRAVELLING ROUND A CIRCLE.

§5. When two or more persons start from the same place at the same time and travel round a circle in the same direction or in opposite directions, then (*i*) they will be first together again after an interval of time which is the L. C. M. of the times in which one of the persons gains one complete round over each of the others, since each pair will be together after this time ; (*ii*) they will be first together at the starting point again after an interval of time which is the L. C. M. of the times in which each of them makes one complete round, since in that interval each of them shall make a complete number of rounds.

The following solved examples will illustrate the method of work :—

Example 1. Three cyclists *A*, *B* and *C* ride round a circular course 180 miles round at the rate of 24, 30, and 42 miles a day respectively. In how many days will they all come together again supposing they all ride in the same direction ?

Sol. B gains on A (30—24) or 6 miles a day,
$\therefore$ they will come together after every $\frac{180}{6}$, *i.e.*, 30 days,
C gains on B (42—30) or 12 miles a day,
$\therefore$ C and B come together after every $\frac{180}{12}$, *i.e.*, 15 days,
and C gains on A (42—24) or 18 miles a day,
$\therefore$ C and A come together after every $\frac{180}{18}$ or 10 days.

Since B and A meet after 30 days, C and B after 15 days and C and A after 10 days,

$\therefore$ A, B and C will meet at the end of any number of days which is a common multiple of 30, 15 and 10.

L. C. M. of 30, 15 and 10 being 30,
they will meet at the end of 30 days. **Ans.**

Example 2. Three cyclists A, B and C ride round a circular course 85 miles round at the rate of 8, 12 and 20 miles an hour ; A and B ride in the same direction and C in the opposite direction. In how many hrs. will they meet again ?

Sol. B gains 4 miles on A in 1 hour,
$\therefore$ they meet together after every $\frac{85}{4}$ hours,
and B and C together pass 12+20, *i.e.*, 32 miles an hour
$\therefore$ they will meet together after every $\frac{85}{32}$ hours.
Also A and C together pass 8+20, *i.e.*, 28 miles an hour,
$\therefore$ they will meet together after $\frac{85}{28}$ hours.

$\therefore$ A, B and C will meet together at the end of any No. of hours, which is a common multiple of $\frac{85}{4}$, $\frac{85}{32}$ and $\frac{85}{28}$
L. C. M. of $\frac{85}{4}$, $\frac{85}{32}$ and $\frac{85}{28}=\frac{85}{4}$,
$\therefore$ they will meet after $21\frac{1}{4}$ hours. **Ans.**

EXERCISE 100.

1. Two men A and B start together from the same place to walk round a circular course 20 miles round, A's speed is $2\frac{1}{2}$ miles and B's $3\frac{1}{2}$ miles an hour. When will they next meet supposing (*i*) they walk in the same direction ; (*ii*) in opposite directions ?

2. Two cyclists A and B ride round a circular course 60 miles round. A's speed is 10 miles and B's 15 miles per hour. When will they next come to the starting point, supposing (*i*) they ride in the same direction ; (*ii*) in opposite directions ?

3. Three cyclists A, B and C ride round a circluar course 40 miles round at the rate of 8, 10 and 12 miles per hour respectively. When will they meet together supposing (*i*) they all ride in the same direction and (*ii*) A and B ride in the same direction and C in the opposite direction ?

4. Three boys A, B and C run a race round a circular path 2 miles round at the rate of 2, 3 and 4 miles per hour respectively. When will they meet again, supposing (*i*) they all run in the same direction. (*ii*) A and C run in the same direction and B in the opposite direction ?

5. A, B and C start from the same point and travel in the same direction, round an island, A can go round it in 12 days, B in 20 days and C in 25 days. If they start simultaneously from the same point, A and B in the same direction and C in the opposite direction, when will they come together to the starting point again ?

VI. CLOCKS

§6. The dial of a clock or watch is divided into 60 minute spaces. Whilst the large hand (minute hand) moves 60 minutes, the small hand (hour hand) moves only 5 minute spaces. Clearly therefore the large hand gains 55 minute spaces on the small hand in 60 minutes. The fact is generally expressed in the words "M. H. gains 11 minute divisions in 12 minute divisions over H. H" Or

The large hand gains 1 minute space on the small hand in $\frac{12}{11}$ minute spaces. This is the fundamental principle which works throughout in solving,problems on clocks.

☞ The following points may also be remembered :—

(*i*) The hands coincide across and point in opposite directions once in an hour : but they are twice at right angles when the time is between 4 and 9 and only once at other times.

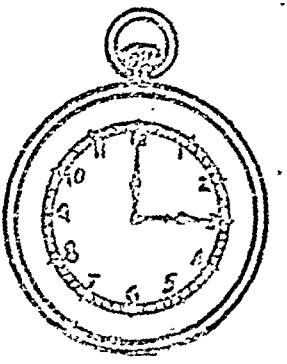
(*ii*) When at right angles they are 15 minute spaces apart and when they point in opposite directions, 30 minute spaces.

(*iii*) At 6 the two hands are opposite to each other. Hence they cannot be opposite again between 6 and 7.

(*i*) They are equidistant from a certain digit (*a*) when they coincide with (*b*) when they are on either side of the digit.

Example 1. At what time between 3 and 4 will the hands of a watch (*i*) coincide, (*ii*) be at right angles, (*iii*) point in opposite directions ?

Sol (*i*) Observe the figure of the watch. The hour hand is 15 minute spaces ahead at 3 o'clock ; hence the minute hand must gain 15 minute spaces to coincide with hour hand.



$\because$ the minute hand gains 1 minute space in $\frac{12}{11}$ minutes,

$\therefore$ the minute hand will gain 15 min. spaces in $\frac{12}{11} \times 15$ minutes.

i. e., in $16\frac{4}{11}$ minutes.

Hence the hands will coincide at $16\frac{4}{11}$ min. past 3. **Ans.**

(*ii*) In the figure, the hour hand is 15 minutes ahead. The hands are at right angles now. They will again be at right angles when the minute hand will gain 30 minutes.

$\because$ 1 minute is gained in $\frac{12}{11}$ minutes

$\therefore$ 30 min. will be gained in $30 \times \frac{12}{11}$ or $\frac{360}{11}$ or $32\frac{8}{11}$ min.

Hence they are at right angles at $32\frac{8}{11}$ minutes past 3. **Ans**

(*iii*) To be opposite, the large hand must gain 45 minutes.

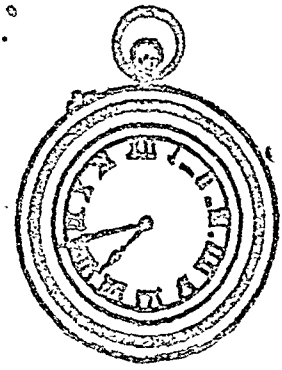
$\because$ 1 minute is gained in $\frac{12}{11}$ minutes

$\therefore$ 45 minutes will be gained in $45 \times \frac{12}{11}$ or $49\frac{1}{11}$ minutes.

Hence the hands will point in opposite directions at $49\frac{1}{11}$ min. past 3. **Ans.**

Example 2. At what time between 7 and 8 are the hands of a watch equidisonfrm tatthe figure VIII ?

Sol. The position of the hands when equidistant from the figure VIII will he as when they (*i*) coincide or (*ii*) as shown in the diagram. In the second case hour hand is at A and minute hand at B. Thus $A8=8B$.



Since distance from 7 to 8 is equal to the distance from 8 to 9, therefore $7A=9B$

It is obvious therefore that the two hands have moved 45 minute spaces between them.

$\therefore \quad 12B+7A=45$ minutes.

Since the minute hand moves 12 times as fast as the hour hand,

therefore, $12B=45\times\frac{12}{13}=\frac{540}{13}=41\frac{7}{13}$ minutes.

Hence the time is $41\frac{7}{13}$ minutes past 7. **Ans.**

Example 3. Two clocks were set right at noon on Sunday. One gains 2 minutes and the other loses 3 minutes in 24 hours. What will be the true time when (*i*) the first clock indicates 3 P. M. on Wednesday, (*ii*) the second clock indicates 8 P. M. on Thursday ?

Sol. (*i*) Time from noon on Sunday to 3 P. M. on Wednesday=75 hours.

24 hrs. 2 min. of the first clock= 24 hrs. of the correct one.

$\therefore$ 1 hr. of the first clock= $\frac{24\times30}{721}$ hrs. " "

$\therefore$ 75 hrs. of the first clock= $\frac{24\times30\times75}{721}$ hrs. " "

$=\frac{54000}{721}$ hrs.

$=74$ hrs. $53\frac{547}{721}$ min.

Hence the true time will be $53\frac{547}{721}$ min. past 2 P. M. **Ans.**

(*ii*) Time from noon on Sunday to 8 P. M. on Thursday $=104$ hours.

23 hrs. 57 min. of the 2nd clock=24 hrs. of the correct one

$\therefore$ 1 hour " " $=24\times\frac{60}{1437}$ " " "

104 hrs. " " $=\frac{24\times60\times104}{1437}$ " " "

$=104$ hrs. $13\frac{39}{1437}$ min.

Hence the true time will be $13\frac{39}{1437}$ min. past 8 P. M. **Ans**

Example 4. In the above question find, after what interval will the one hand gain 30 minutes on the other hand and what time will then each show ?

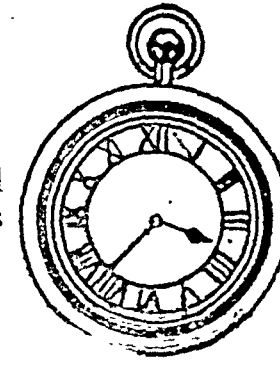
Sol. The first clock gains $2+3$, *i.e.* 5 min. in 24 hrs.

$\therefore$ it will gain 30 minutes in 144 hrs. or 6 days.

In 6 days the first clock will gain $6\times2=12$ minutes.
and in 6 days the second clock will lose $5\times3=18$ minutes.
Hence the first clock will indicate 12 min. past 12 P.M.
and the second clock will indicate 18 min. to 12 P.M. } **Ans.**

Example 5. A person went out between 3 and 4 and came back between 7 and 8. To his great surprise, he found that the hands had exactly changed their places. When did he go out ?

Sol. The position of the hands in the diagram indicates the time of his departure. As the hands now have exactly changed their places it is clear that the long hand has made 3 complete revolutions and one incomplete from B to A and the small hand has moved from A to B, *i.e.*, both the hands together have made full 4 revolutions.



4 complete revolutions $=4\times60$
or 240 minutes.

Now let us find out the minute spaces moved by the small hand in 240 minutes;

$\because$ the min. hand moves 12 times as fast as the hr. hand
$\therefore$ the hour hand has moved $\frac{240}{13}$ minutes only.

Now the question becomes:

When will the minute hand be $\frac{240}{13}$ minute spaces ahead of the hour hand between 3 and 4 ?

At 3 o'clock the minute hand is 15 minute spaces behind the hour hand, hence to be ahead $\frac{240}{13}$ minute spaces, it will have to gain $\frac{240}{13}+15$, *i.e.*, $\frac{435}{13}$ minute spaces on the hr. hand.

$\frac{435}{13}$ minute spaces can be gained in $\frac{435}{13}\times\frac{12}{11}$ minutes
i. e., in $36\frac{72}{143}$ minutes.

Hence he went out at $36\frac{72}{143}$ minutes past 3. **Ans.**

EXERCISE 101.

At what times are the hands of a clock (*i*) coincident, (*ii*) at right angles, (*iii*) pointing in opposite directions ? —

1. Between 4 and 5. 2. Between 5 and 6.
3. Between 2 and 3. 4. Between 8 and 9.
5. Between 9 and 10. 6. Between 6 and 7.
7. At what time between 4 and 5 is the minute hand 10 minute spaces ahead of the hour hand ?
8. At what time between 7 and 8 isthe minute hand 5 minute spaces ahead of the hour hand ?
9. At what time between 5 and 6 are the hands of a clock equidistant from the figure V ?
10. At what time between 3 and 4 are the hands of a clock equidistant from the figure IV ?
11. A watch which loses 3 seconds in one hour was set right at noon on Monday. What time will i indicate at 4 P. M. on Wednesday ?
12. A watch which loses 16 minutes in 24 hours was set right at noon on Sunday; find the correct time wnen teis watch induces 5 hours 40 min on Monday evening.
13. A clock which loses 7 seconds in one hour was set right at 8 A. M. on Sunday. What time will it indicate at 4 P. M. on Tuesday?
14. The school clock was set correctly at 4 P. M. on Saturday. It was fonnd 10 min. slow at 4 P. M. on Wednesday. What time will it indicate at 12 o'clock at noon Thursday?
15. One clock loses 10 sec. and the other goins 8 sec. in 24 hours. They were set riget at noonfion Tuesday? After weat interval will one be 2 min. in advance of the other and what o'clock will each of the clocks then indicate.?
16. Two watches were set correctly at 8 A. M. on Monday. One gains 4 min. and the other loses 5 min. in 24 hrs. After what idterval will one be half an hour in advadce of the other and what time will each of the watches then indicate ?
17. A watch loses 4 minutes daily. It was set right on September 17, 1640, at 9 A. M. When will it show correct time again ?
18. A clock loses 15 minutes in 24 hou s, It was set right at noon on July 1: when will st show correct time again ?
19. Two clocks are set right at 10 A. M. on Wednesday. One gains $2\frac{1}{2}$ minutes and the other loses 4 minutes daily

What difference will they indicate at 2 P. M. on Monday and after how many days will they indicate the same time ?

20. A watch loses 3 min. and the other gains 4 min. daily. They were set right at 3 P. M. What time will the slower watch indicate the next day when the faster shows 9 P. M.?

21. Two watches are set right at 1 P. M. on Wednesday. One loses $2\frac{1}{2}$ min. and the other gains $2\frac{1}{2}$ min. in 24 hours. What time will the fast watch indicate on Saturday when the slow watch shows 10 A. M.?

22. Two clocks point 2 o'clock at the same instant on the afternoon of April 25; one loses 7 seconds and the other gains 8 seconds in 24 hours. When will one be half an hour in advance of the other and what time will each clock then show?

23. Two watches, one of which gained at the rate of 1 min. 54·6 sec. and the other lost at the rate of 1 min. 55·8 sec. daily were set right at noon on the Ist January 1944. When did the watches next indicate the same time and what time did each then indicate ?

24. A clock which gains 3 min. 56 sec. in 24 hours was set right at noon on January 1, 1944. Find to the nearest minute the next date at which it indicated correct time.

25. Two clocks *A* and *B* whose rates are uniform at noon yesterday indicated 11 hours 55 minutes A. M. and 0 hour 2 minutes P. M. respectively. *A* indicated the correct time at 9 P. M. yesterday and *B* at 6 P. M. this morning. When did *A* and *B* last agree and what time did they then indicate ?

26. A man who went out between 5 and 6 and returned between 7 and 8 found that the hands of the watch had exactly changed places. When did he go out ?

27. A man who went out between 5 and 6 and returned between 6 and 7 found that the hands of the watch had exactly changed places. When did he go out ?

28. A man went out between 3 and 4 and coming back noticed that the hands of the clock had exactly changed places. Find the time of his daparture.

29. It is between 2 and 3 o'clock; but a person looking at the clock and mistaking the hour hand for the minute hand fancies that the time of the day is 57 minutes earlier than the reality. What is the true time ?

30. A clock which was 16 min. too slow 24 days ago, is 16 min. too fast to-day at the same hour. When did it last show correct time and when will it show correct time again ?

CHAPTER XX

PERCENTAGE

§1. **Percentage** means for every hundred.

When we say a man made a profit of 20 per cent. we mean to say that he gained Rs. 20 for every hundred rupees he invested in the business, *i.e.* $\frac{20}{100}$ for each rupee. Similarly 15 per cent. means $\frac{15}{100}$ for each rupee, and so on.

Percentage therefore is a fraction, the denominator of which is 100.

Abbreviation of per cent. is P. C., and it is generally denoted by the symbol %.

Example 1. What fraction is 40 per cent. ?

Sol. $40\% = \frac{40}{100} = \frac{4}{10} = \frac{2}{5}$. **Ans.**

Example 2. Write 80 per cent. as a decimal.

Sol. $80\% = \frac{80}{100} = \frac{8}{10} = \cdot 8$. **Ans.**

Example 3. What percentage is equivalent to $\frac{9}{20}$?

Sol. $\frac{9}{20} = \frac{\frac{9}{20} \times 100}{100} = \frac{45}{100} = 45$ P. C. **Ans.**

Example 4. Write $\frac{3}{7}$ as percentage.

Sol. $\frac{3}{7} = \frac{\frac{3}{7} \times 100}{100}$ $\frac{\frac{300}{7}}{100} = \frac{42\frac{6}{7}}{100} = 42\frac{6}{7}$ P. C. **Ans.**

Example 5. Express ·08 as percentage.

Sol. $\cdot 08 = \frac{8}{100} = 8$ p. c. **Ans.**

Example 6. Find 5 per cent. of Rs. 880.

Sol. 5% of Rs. $880 = \frac{5}{100} \times 880 =$ Rs. 44. **Ans.**

Example 7. In a school of 800 boys 45 per cent. are Hindus, 25 per cent. Mohammadans, 10 per cent. Sikhs and the remaining of other communities. Find the number of boys of each community.

Sol. Hindus $=\frac{45}{100}$ of $800=360$
Mohammadans $=\frac{25}{100}$ of $800=200$
Sikhs $=\frac{10}{100}$ of $800=80$
Others $=\frac{20}{100}$ of $800=160$
Ans.

Example 8. In mathematics a boy got 60 marks out of 150. What percentage of marks did he get ?

Sol. Marks he obtained $\frac{60}{150}=\frac{2}{5}$

$\therefore$ percen age $=\frac{\frac{2}{5}\times 100}{100}=\frac{40}{100}=40\%$. **Ans.**

Example 9. A man spends Rs. 513 which is 9% of his income ; find his income.

Sol. 9 per cent. of the sum = Rs. 513.
or $\frac{9}{100}$ of the sum = Rs. 513.
$\therefore$ the sum = Rs. $513\times\frac{100}{9}$
= Rs. 5700. **Ans.**

Example 10. A man gives 5 per cent. of his income in charity, spends 75 per cent. of the remainder for the maintenance of his family ; now he has Rs. 228 left. What had he at first ?

Sol. $100-75=25$.

Since 25 per cent. of the remainder = Rs. 228
$\therefore$ remainder = Rs. $228\times\frac{100}{25}$ = Rs. 912.
Now $100-5=95$;

$\because$ Rs. 912 is 95 per cent. of whole income
$\therefore$ income = Rs. $\frac{100}{95}\times 912$ = Rs. 960. **Ans.**

Example 11. The population of a village is 2500; $\frac{3}{5}$ of them are males and the rest females, 60 per cent. of the females are married; find the percentage of married males.

Sol. Population $=2500$.
$\therefore$ males $=2500\times\frac{3}{5}$ $=1500$
$\therefore$ females $=2500-1500$ $=1000$
$\because$ married females $=1000\times\frac{60}{100}$ $=600$;
$\therefore$ married males are also 600;
$\therefore$ percentage $=\frac{600}{1500}\times 100=40\%$ **Ans.**

EXERCISE 102.

Express as fractions in their lowest terms :—

1. 15 p. c. 2. 25. p. c. 3. 28 p.c. 4. $37\frac{1}{2}$ p.c.
5. $18\frac{3}{4}$ p.c.

Express as decimals .—

6. 16. p. c. 7. 45 p. c. 8. $7\frac{1}{2}$ p. c. 9. $17\frac{1}{2}$ p.c.

What rate per cent. do the following fractions denote ?

10 $\frac{7}{25}$ 11 $\frac{3}{8}$ 12 $\frac{9}{70}$ 13. $\frac{8}{125}$.

What rate per cent do the following decimals denote ?

14. ·09. 15. ·33. 16. ·0125. 17. ·075.

Find the value of :—

18. 8% of Rs. 1200. 19. 15% of Rs.560.
20. $37\frac{1}{2}$ of Rs.728 21. $12\frac{1}{2}$ % of £ 324.
22. $16\frac{2}{3}$ % of £ 127, 5 *s.* 6 *d.* 23. $3\frac{1}{8}$ of Rs. 515, 5 *a.*4 *p.*

Find the number of which

24. (*i*) 96 is 6%. (*ii*) $235\frac{1}{2}$ is $15\frac{7}{10}$ p.c.

25. What per cent. (*i*) of Rs. 35 is Rs. 7; (*ii*) of ·625 is $\frac{1}{5}$; (*iii*) of 4 hrs. 25 min. 12 sec. is 2 hours 12 min. 36 sec.?

26. In the school of 1200 students, 18% are absent; find the number of absentees.

27. In a school of 760 students, 15 per cent. are Christians, 25 per cent. are Mohammadans and the remaining are Hindus; find the number of each.

28. A man saves 12 per cent. of his income which amounts to Rs. 50 per month. Find his annual savings.

29. The population of a town increased from 4800 to 5000 in 2 years. What was the rate per cent. of the increase?

30. The population of a town decreased from 2115 to 1692 in 3 years. What was the rate per cent. of the decrease?

31. A person's annual income is £1250; he spends 55 per cent. of it. How much does he save ?

32. The population of a country is 7500000; out of them 1000000 are illiterate. Find the percentage of the illiterate people.

33. In a hospital there are 1225 sick people; out of them 450 are men, 530 women and the remaining are children. What is the percentage of the children ?

34. In a school of 2100 boys, 30 per cent. read in the High Department; 32 per cent. in the Middle Department and the remaining in the Primary Department. Find the number of students in the Primary Department

35. A man took from a bank Rs. 125 which was 20 per cent. of what he had deposited; find his balance in the bank.

36. A man gives Rs. 129, 8*a*. in charity which is $12\frac{1}{2}$ per cent. of his income, find his income.

37. 19 per cent. of the population of a town is 399, find the population.

38. What is the number, $66\frac{2}{3}$ per cent. of which is 128 ?

39. After spending 50 per cent. of his income and then 20 per cent. of the remainder a man has Rs. 600 left; find his income.

40 A man spends 5 per cent. of his income on the repairs of his house, 75 per cent. of the remainder on the other requirements of life. Now he has Rs. 1425 left, find his income.

41. *A* and *B* had equal sums of money, 33 per cent. of *A* s and 24 per cent. of *B*'s money amount to Rs. 543, 4*a*. 6*p*. Find how much money they had.

42 The population of a village is 4500, $\frac{5}{9}$ of them are males and the rest females, 40 per cent. of the males are married, find the percentage of the married females.

§2. Some Important Typical Examples

Example 1. The population of a town increases 5 per cent. every year. If the present population be 8000, find the population in 3 years time.

Sol. If the population is 100, the increase is 5
" " is 1, " " is $\frac{5}{100}$
" " is 8000 " " is $\frac{5}{100} \times 8000$
∴ the total population in one year will be

$8000 + (\frac{5}{100} \times 8000)$, *i.e.*, $8000(1 + \frac{5}{100})$ or $P\left(1 + \frac{r}{100}\right)$

where P stands for the original population and r for the rate%

$\because$ population in one year $= P\left(1+\frac{r}{100}\right)$

$\therefore$,, ,, two years $= P\left(1+\frac{r}{100}\right)^2$

and ,, ,, three years $= P\left(1+\frac{r}{100}\right)^3$

and so on. Hence the **formulae.**

1. Population (when increasing) in n years $= P\left(1+\frac{r}{100}\right)^n$

2. Population when decreasing in n years $= P\left(1-\frac{r}{100}\right)^n$

where n stands for number of years.

Now according to the formula the population in 3 years would be $P\left(1+\frac{r}{100}\right)^3$.

or $8000\ (1+\frac{5}{100})^3$ or $8000\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}=9261$. **Ans.**

Example 2. The population of a town was 31250 three years ago ; after a year there was 5 p. c. increase, after second year 4 p. c decrease and in the last year 2 p c. increase, find the present population.

Sol. Here the formula would be

final P=original $P\left(1+\frac{r_1}{100}\right)\left(1-\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$

$\therefore$ reqd. $P=31250\times\frac{105}{100}\times\frac{96}{100}\times\frac{102}{100}=32130$. **Ans.**

Example 3. The number of students in a school increases at a certain rate per cent. The number at present is 1323 and the number two years ago was 1200 ; find the rate per cent. of the increase.

Sol. By formula we have

$$1200\times\left(1+\frac{r}{100}\right)^2=1323$$

$$\therefore\quad\left(1+\frac{r}{100}\right)^2=\frac{1323}{1200}=\frac{441}{400}$$

or $$1+\frac{r}{100}=\frac{21}{20}\qquad\text{[by sq. root],}$$

or $$1+\frac{r}{100}=\frac{21\times5}{20\times5}=\frac{105}{100}$$

or $$1+\frac{r}{100}=1+\frac{5}{100}$$

or $$\frac{r}{100}=\frac{5}{100} \quad \therefore\ r=5.$$ **Ans.**

Example 4. The income of a person increases at a certain rate per cent. annually. The income at present is Rs. 625, four years ago it was Rs. 576. What will it be 2 years hence ?

Sol. According to the formula, we have

$$576\times\left(1+\frac{r}{100}\right)^4=625$$

$$\therefore\quad \left(1+\frac{r}{100}\right)^4=\frac{625}{576}$$

$$\therefore\quad \left(1+\frac{r}{100}\right)^2=\frac{25}{24} \qquad \text{[by sq. root]}$$

Now $$625\times\left(1+\frac{r}{100}\right)^2 = \text{income two years hence.}$$

or $625\times\frac{25}{24}$=required income.

i. e., Rs 651, 0*a*. 8*p*. **Ans.**

Example 5. A reduction of 20 per cent. in the price of ugar enables a man to buy 2 maunds of sugar more for Rs. 120. Find the original price of sugar.

Sol. Saving in Rs. 120 = Rs. $\frac{20}{100}\times120$

= Rs. 24.

For Rs. 24 he can buy 2 maunds of sugar.

$\therefore$ reduced price of sugar per maund = Rs. 12

$\therefore$ the original price must be Rs. $12\times\frac{100}{80}$

i. e., Rs. 15 per maund. **Ans.**

Example 6. A person had some eggs, 4% being worthless were thrown away, 80% of the remainder were sold and now there remained only 96. How many eggs had he at first ?

Sol. 100−80, *i. e.*, 20% of the remaining eggs = 96

$\therefore$ No. of eggs after being thrown away $=96\times\frac{100}{20}=480$

now 480 = 100−4, *i.e.*, 96% of the whole ;

$\therefore$ he had $480\times\frac{100}{96}=500$ eggs. **Ans.**

Example 7. If the price of sugar be raised 40%, find by how much per cent. a person must reduce his consumption of it so as not to increase his expenditure.

Sol. The raised price of sugar is $\frac{140}{100}$ of the former price.
$\therefore$ the person must now consume $\frac{100}{140}$ of the original amount.

$\therefore$ reduction $= 1 - \frac{100}{140}$ *i.e.*, $\frac{2}{7}$ of the original amount which is equal to $\frac{2}{7} \times 100 = 28\frac{4}{7}\%$. **Ans.**

Example 8. The price of coal being raised 40 p. c., a person reduced his consumption of it so much that his expenses in buying the coal only increased 10 p c. How much of coal does he now consume if he at first consumed $3\frac{1}{2}$ maunds.

Sol. Suppose the price of coal at first was Re. 1 per md.
$\therefore$ he consumed coal worth Rs.$3\frac{1}{2}$;

now he consumes ,, ,, Rs. $\frac{7}{2} \times \frac{110}{100} = \frac{77}{20}$
but now the price per md $=$ Rs $\frac{140}{100} = \frac{7}{5}$
$\therefore$ amount of consumption $= \frac{77}{20} \times \frac{5}{7} = 2\frac{3}{4}$ mds. **Ans.**

Example 9. In an examination 75% of the candidates passed in English ; 70% in Mathematics, 23% failed in both subjects and 102 passed in both subjects Find the number of candidates.

Sol. Let the number of candidates be 100;
$\therefore$ 75 candidates pass in English and 25 fail in it ;

70 candidates pass in Mathematics and 30 fail in it ;
and 23 candidates fail in both subjects.

$\because$ 25 failed in English and 23 failed in Mathematics also. it means 2 candidates failed in English alone and

$\because$ 30 failed in Mathematics and 23 failed in English also, it means 7 candidates failed in Mathematics alone.

$\therefore$ the total number of failures $= 2$ in English $+ 7$ in Mathematics $+ 23$ in both subjects $= 32$.

$\therefore$ No. of successful candidates $= 100 - 32 = 68$, *i. e.* 68%.
Now 68% of successful candidates $= 102$
$\therefore$ total No. of candidates $= \frac{100}{68} \times 102 = 150$. **Ans.**

EXERCISE 103.

1. The population of a town increases 5% every year, if the present population be 24000, what will it be in 3 years ?

2. The population of a town decreases 4 per cent. annually; if the present population be 390625, what will it be in 4 years ?

3. The present population of a town is 11979, if annual increase be 10 per cent. what was it 3 years ago ?

4. The population of a town was 25000 three years ago after one year there was 4% increase, next year 5% decrease and last year 2% increase. Find the present population.

5. The population of a town increases at a certain rate per cent. annually. Now itis 5408 ; if it was 5000 two years ago, find the rate per cent. of the increase.

6. The number of students in a school increases at a certain rate per cent. annually ; the present number is 1331 ; if it was 1000 three years ago, find the rate per cent. of the increase,

7. The income of a person decreases at a certain rate per cent. annually ; the present income is Rs 3610 and two years ago it was Rs. 4000. Find the rate per cent. of the decrease.

8. The population of a town increases at a certain rate per cent. Four years ago it was 6400; now it is 8100. What will it be 2 years hence ?

9. The income of a person decreases at a certain rate per cent. Four years ago it was 1600, now it is 900. What will it be 2 years hence ?

10. *A* sells his goods 10% cheaper than *B* and 10% dearer than *C* ; how much per cent. are *C*'s rates lower than *B*'s ?

11. The price of rice is raised 10 per cent., by how much per cent. should a man reduce his consumption of rice, so that his expenditure may be the same as before ?

12. A reduction of 5 per cent. in the price of tea enables a man to buy 2 lb. of tea more for Rs. 23, 12*a*. Find the original price per lb.

13. A person bought a quantity of tea for Rs. 11, 4*a*. On the next day the price fell down 20 per cent. Had he bought the tea on the reduced price he would have got 3 lbs. more. What price did he pay per lb. ?

14. A fruit vendor gives 4 per cent. of his oranges in charity and sells 90 per cent. of the remainder and then 96 oranges remain ; how many had he at first ?

15. Out of the eggs a man had 3 per cent. were found broken. By selling 80 per cent. of the remainder there remained 97 eggs only. Find the number of eggs he had at first?

16. If the price of coal be raised 20 P. C., find by how much per cent. a person must reduce his consumption of it, so as not to increase his expenditure.

17. If the price of rice be raised 30 P. C., find by how much per cent. a person must reduce his consumption of it so as not to increase his expenditure.

18. The price of sugar being raised 20 P. C. a family reduced its consumption so much that its expenses in buying sugar increased 5 P. C. only. Find how much sugar is consumed now if at first consumption was $4\frac{1}{2}$ mds.

19. In an examination 65·6 p. c. of the candidates took Science, 40·8 P. C. took Sanskrit. If 1280 candidates took both the subjects, find the number of candidates that appeared in the examination.

20 In an examination 75·8 P C. of the candidates took French and 49·4 p. c. took Latin. If the number of candidates that appeared in the examination be 2500, find how many took both the subjects.

21. 90 per cent. of the boys of a school pass in English, and 85 per cent. in Mathematics, 150 pass in both subjects and no boy fails in both. How many boys are there in the school?

22. In an examination 52 per cent. of the candidates failed in English and 42 per cent. in Mathematics. If 17 per cent. failed both in English and Mathematics, find the percentage of those who passed in both the subjects.

23. A man is able to save $12\frac{1}{2}$ P. C. of his weekly wages. His wages rise 2*s*. a week and expenses rise 10 P. C. in consequence. This diminishes his savings by 17*s*. 4*d*. a year. Find his weekly wages. [A year=52 weeks]

24. 68 P. C. of the persons employed in a business house are men getting an average wage of Rs. 235 a month each and the rest are women getting an average wage of Rs. 118, 8*a*. each. Find (to the nearest integer in each case) what percentage of the total wages is paid to (*a*) men, (*b*) women.

25. The population of Rangoon in 1921 was 342000 being an increase of 15 p. c. on the population of 1911; find the population of 1911. Assuming that it continues to increase at the same rate for each period of 10 years, find what the population will be in 1951. (Burma 1925)

CHAPTER XXI

AVERAGE

§ 1. Suppose *A* has Rs. 6, *B* Rs. 9 and *C* Rs. 15. If we collect their money and then divide it equally among them, each would get Rs. $(6+9+15)\div 3$ or Rs. 10. Ten is said to be an average number of 6, 9 and 15.

Definition. An average number is the intermediate of the given numbers of the same kind.

§ 2. **To find the average number.**

From the above illustration we deduce the following

Rule. *Add together the numbers and divide the sum by the number of the given numbers.*

Cor. The sum of the numbers of quantities = average × their number.

The following examples will illustrate the rule :—

Example 1. Find the average value of 536, 728, 832 and 620.

Sol. The sum of the numbers $= 536+728+832+620$
$= 2716$
$\therefore$ average value $= 2716\div 4 = 679$. **Ans.**

Example 2. In a cricket eleven, 5 boys scored 10 runs each, 3 boys scored 15 runs each, 2 boys scored 20 runs each and the eleventh boy scored 8 runs only. Find the average runs per boy.

Sol. Runs of 5 boys $= 10\times 5 = 50$
,, ,, 3 boys $= 15\times 3 = 45$
,, ,, 2 boys $= 20\times 2 = 40$
,, 1 boy, $= 8$

Total No. of runs $= 143$
$\therefore$ average runs $= 143\div 11 = 13$. **Ans.**

Example 3. The average age of a class of 20 boys is 12 years 4 months ; the average of the first 5 boys is 13 years 2 months and that of the next 13 is 11 years ; find the average age of the remaining two.

Sol. The total age of 20 boys = 12 years 4 months × 20
= 246 years 8 months.
The ,, ,, of 5 boys = 13 years 2 months × 5
= 65 years 10 months.
The ,, ,, of 13 boys = 11 years × 13
= 143 years.
∴ The ,, ,, of 18 boys = 65 yrs. 10 mths. + 143 yrs.
= 208 years 10 months.
The age of the rem. 2 boys = 246 years 8 mths. − 208 years 10 months.
= 37 years 10 months.
∴ average of the rem. 2 boys = 18 yrs. 11 months. **Ans.**

Example 4. The average temperature for Monday, Tuesday and Wednesday was 55°, the average for Tuesday, Wednesday and Thursday was 60°, that for Thursday being 56° ; what was the temperature on Monday ?

Sol. Sum of temp. for Mon., Tues., Wed. = 55° × 3 = 165°
Sum of temp. for Tues., Wed. Thurs., = 60° × 3 = 180°
∵ temperature on Thursday = 56°
∴ temperature for Tues., Wed. = 180° − 56° = 124°
∴ temperature on Monday = 165° − 124° = 41°. **Ans.**

EXERCISE 104.

Find the average of the numbers:—

1. 12, 16, 18. 20.
2. 9, 14, 21, 16.
3. $6\frac{1}{4}$, $7\frac{1}{2}$, $12\frac{1}{8}$, $6\frac{1}{8}$,
4. 5·25, 9·56, 8·19.
5. Four boys are 12, 15, 18 and 20 years old ; find their average age.
6. The population of three towns is 8972, 5632, 7524 respectively : what is the average population of the towns ?

7. The income of four men is Rs. 35, 8*a*., ; Rs. 45, 12*a*. ; Rs. 36, 10*a*. ; and Rs. 48, 4*a*. respectively ; find the average income of each.

8. In a class of 20 boys there are 5 boys of the age of 14 years. 9 boys of the age of 12 and 6 boys of the age of 10. Find the average age of the boys

9. Average age of 5 boys is 17·4 years and the ages of first four boys are 15, 17, 19 and 21 years respectively. Find the age of the fifth boy.

10. Average marks obtained by a boy in English, Mathematics, General Knowledge and Science are 85. He obtained 75 marks in English, 95 marks in Mathematics and 55 marks in General Knowledge ; how many marks did he obtain in Science?

11. A student obtained 300 marks in 4 subjects ; his average marks in 3 subjects are 81·5. How many marks did he obtain in the fourth subject ?

12. The average age of 4 boys is 17 years 2 months and of the 6 more boys 15 years 1 month; find the average age of the boys.

13. The average age of the boys of a class of 8 boys is 18·5 years ; it is decreased to 18 years by the admission of a new boy, how old is the new boy ?

14. The average age of the boys of a class of 15 boys is 16·5 years and the average of the same class including the teacher is 17·5 years ; how old is the teacher ?

15. The average cost of 15 horses is Rs 60·5 ; but the average cost of 10 horses is Rs. 50·5. Find the average cost of the rest of the horses.

16. The average age of the boys of a class of 40 boys is 15·5 years ; what will be the average age if 10 new boys are admitted whose average age is 13·8 years ?

17. average age of 5 boys is 21 years, the average of the first 2 boys is 18 years and that of the last 2 is 22 years; find the age of the third boy.

18. In a cricket eleven the average runs are 8, the average of the first 4 boys is 10 and that of the next 5 boys is 6 ; find the average runs of the remaining two boys.

19. The average attendance of a class for Monday, Tuesday and Wednesday was 38. The average for Tuesday, Wednesday and Thursday was 42. The attendance on Thursday being 40, find the attendance on Monday.

20. The average income of a man from 1918 to 1922 was Rs. 485 and from 1919 to 1923 was Rs. 487; if the income for the year 1923 be Rs. 478, find the income for the year 1918.

21. The average temperature for Monday, Tuesday and Wednesday was 99° and for Tuesday, Wednesday, Thurday 100°: if the temperature on Thurday be 98°, find the temperature on Monday.

22 A person bought 15 maunds of wheat at Rs. 5 a maund, and mixed 5 maunds of barley in it; the average cost of the mixture became Rs. 4, 12*a*. Find the cost of barley per maund.

23. A person bought 20 maunds of wheat at Rs. 5, 4*a*. a maund, 5 maunds of grains at Rs. 3, 8*a*. a maund and 2 maunds of barley at a certain price. He mixed them together and by selling them at Rs. $5\frac{1}{16}$ a maund got a profit of 8 per cent. Find the cost price of barley per maund.

24 There are equal number of men and women. Rs. 140 are to be divided among them. If a man be given Re. 1, 5*a*. 4*p*. and a woman Re. 1 only, find the total number of men and women.

25. The average expenditure of a man for 5 months is Rs. 120 and for the next 4 months Rs. 80. Find his monthly income if he had saved Rs. 25.

CHAPTER XXII

PROFIT AND LOSS

§1. The price we pay for an article is called its cost price (C. P.) and the price we get by the sale of an article is called its selling price (S. P.) If an article is sold for more than the cost price, it is said to be sold at a profit or gain; but if it is sold for less than the cost price, it is said to be sold at a loss.

Consider the following examples:—

(*i*) A man buys a watch for Rs. 20 and sells it for Rs. 25; find his gain or loss.

Here Rs. 20 is the cost price and Rs. 25 is the selling price. As it is sold for Rs. 5 more than the cost price, he is said to sell it at a profit of Rs. 5.

$\therefore$ profit=Selling price—Cost price

(*ii*) A man buys a watch for Rs. 20 and sells it for Rs. 15 find his gain or loss.

Here Rs. 20 is the cost price and Rs. 15 is the selling price. As it is sold for Rs. 5 less than the cost price, he is said to sell it at a loss of Rs. 5.

$\therefore$ Loss=Cost price—Selling price.

From the above examples we also conclude that

Selling price=Cost price+Profit or Cost price—Loss.

and Cost price=Selling price—Profit or Selling price+Loss

Note—Cost price is olso called the outlay or prime cost.

§2. **To find net loss or gain.**

The following solved examples will illustrate the theorem of profit or loss clearly:—

Example 1. A man bought 60 maunds of wheat for Rs. 315 and sold it for Rs. 5, 8*a*. per maund; find his profit.

Sol. The profit=selling price—cost price.

=Rs. 5, 8*a*. × 60—Rs. 315.

=Rs. 330—Rs. 315=Rs. 15. **Ans.**

Example 2. A merchant bought 180 bags of wheat containing 3 maunds each at Rs. 5, 4*a*. per maund and sold 100 bags at Rs. 5 per maund and 80 bags at Rs. 5, 8*a*. per maund; find his loss.

Sol. The loss = cost price − selling price
$$= (\text{Rs. } 5, 4a. \times 180 \times 3) - \{(\text{Rs. } 5 \times 100 \times 3) + (\text{Rs. } 5, 8a. \times 80 \times 3)\}$$
$$= \text{Rs. } 2835 - \text{Rs. } 2820 = \text{Rs. } 15. \quad \textbf{Ans}$$

Example 3. A man buys 25 gallons of wine for £37, 8*s*. and sells it at £1, 12*s*. per gallon ; find his gain or loss.

Sol. Cost price of 25 gallons = £37, 8*s*.

Selling price of 25 gallons = £1, 12*s*. × 25 = £40.

Since he sells it for more than it costs him, therefore he makes a profit.

The profit = selling price − cost price
$$= £40 - £37, 8s. = £2\ 12s. \quad \textbf{Ans}$$

Example 4. A merchant buys 2 cwt. of tea at Re. 1, 2*a*. per lb. and sells it at Rs. 27, 5*a*. per qr. ; find his gain or loss.

Sol. Cost price of 2 cwt. of tea = Re. 1. 2*a*. × 2 × 112.
$$= \text{Rs. } 252.$$
Selling price of 2 cwt. of tea = Rs. 27, 5*a*. × 2 × 4
$$= \text{Rs. } 218, 8a.$$

Since he sells it for less than what it costs him, therefore he sells it at a loss.

∴ the loss = cost price − selling price
$$= \text{Rs. } 252 - \text{Rs. } 218\ 8a. = \text{Rs. } 33, 8a. \quad \textbf{Ans.}$$

§3. When net loss or gain is given the cost or selling price is found as in the following examples :—

Examples 5. A man buys 30 chairs at Rs. 4, 12*a*. per chair and sells them at a profit of Rs. 9, 6*a*. ; find his selling price per chair.

Sol. Cost price of 30 chairs = Rs. 4, 12*a*. × 30
$$= \text{Rs. } 142, 8a.$$
Profit on 30 chairs = Rs. 9, 6*a*.
∴ selling price = cost price + profit.
$$= \text{Rs. } 142, 8a + \text{Rs. } 9, 6a$$
$$= \text{Rs. } 151, 14a.$$
∴ Selling price per chair = Rs. 151, 14*a*. ÷ 30,
$$= \text{Rs. } 5, 1a. \quad \textbf{Ans.}$$

Aliter ∵ cost price of one chair = Rs. 4, 12*a*.
and profit of one chair = Rs. 9, 6*a*. ÷ 30 = 5*a*.
∴ selling price = cost price + profit
= Rs. 4, 12*a*. + 5*a*. = Rs. 5, 1*a*. **Ans.**

Example 6. A man buys 100 sheep at Rs. 150 per score, keeps them for a month at a cost of Rs. 50 ; at what price per sheep does he sell them, if he loses Rs. 75 ?

Sol. Cost price of 100 sheep = Rs. $\frac{150}{20} \times 100$ = Rs. 750
and expenses he incurred for a month = Rs. 50,
∴ total cost price = Rs 750 + Rs. 50 = Rs. 800
∴ selling price = cost price − loss
= Rs. 800 − Rs. 75 = Rs. 725.
∴ selling price per sheep = Rs. 725 ÷ 100
= Rs. 7, 4*a*. **Ans.**

Example 7. A grocer sold 3 cwt. of tea at Re. 1, 3*a*. per lb. gaining thereby Rs. 84 ; find his total cost price and also cost price per lb.

Sol. Selling price of one lb. = Re. 1, 3*a*.

and profit on ,, ,, = $\frac{\text{Rs. } 84}{3 \times 112}$ = 4*a*.

∴ cost price = selling price − profit
= Rs. 1, 3*a*. − 4*a*. = 15*a*. per lb.
and total cost price = Rs. $\frac{15}{16} \times 3 \times 112$ = Rs. 315 } **Ans.**

Example 8. A merchant sells 40 bags of rice containing $2\frac{1}{2}$ maunds each at Rs. 8, 4*a*. per maund. What is his prime cost if he loses Rs. 22, 5*a*. in this transaction ?

Sol. Selling price = Rs. 8, 4*a*. × 40 × $\frac{5}{2}$ = Rs. 825.
Loss = Rs. 22, 5*a*.
∴ cost price = selling price + loss
= Rs. 825 + Rs. 22, 5*a*.
= Rs. 847, 5*a*. **Ans.**

EXERCISE 105.

1. A man buys a horse for Rs. 65, 10*a*. and keeps it for a week at the cost of Rs. 15. What does he lose if he sells it for Rs. 70, 10*a*. ?

2. A merchant bought a flock of sheep containing 15 score at Rs. 5, 4*a*, per head. Having kept them for 2 weeks at the cost of Re. 1, 8*a*. per head per week, he sold the whole flock at Rs. 9 per head. What did he gain or lose ?

3. A grocer buys 4 cwt. of tea at Rs. 175 per cwt. and sells it at Re. 1, 12*a*. per lb. ; find his profit.

4. A man bought 500 mangoes at 5*a*. per score and sold at 4*a*. a dozen. What did he gain or lose ?

5. A milk seller buys two maunds of milk at 3*a*. per seer and mixes 8 sears of water with it. He then sells the mixture at 2*a*. 9*p*. per seer. What did he gain or lose ?

6. A merchant bought 150 quarters of wheat at £ 2 a quarter, the expenses of carriage, etc., being 1*s*. 4*d*. a quarter. He sold the whole of it at £ 2, 10*s*. a quarter ; find his profit.

7. A merchant bought 1 cwt. of tea at 1*s*. 6*d*. per lb. and 2. cwt at 1*s*. 2*d*. per lb. He mixed them and sold the mixture at 1*s*. 8*d*. per lb. ; calculate his loss or gain.

8. A man bought goods for Rs. 250 and sold $\frac{1}{5}$ of it the same day losing $\frac{1}{10}$ of its cost ; at what price must he sell the remainder so as to gain Rs. 12 on the whole transaction ?

9. A milk seller buys $2\frac{1}{2}$ maunds of milk at 2*a*. per seer and mixes 20 seers of water with it ; at what price per seer does he sell it, if he makes a profit of Rs. 2, 8*a*. on the whole ?

10. A grocer bought 1 cwt. of tea at 1*s*. 2*d*. per lb. and 2 cwt. at 1*s*. 6*d*. per lb. ; at what price must he sell the mixture per lb. to gain £4, 13*s*. 4 *d*. ?

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§4. **To find loss or gain per cent.**

Loss or gain per cent. is always reckoned on the cost price The following examples will illustrate the rule :

Example 1. A man bought a horse for Rs. 250 and sold for Rs. 300 ; find his gain per cent.

Sol. ∵ cost. price=Rs 250.
and selling price =Rs. 300.
∴ total gain =Rs. 50.
This gain is on Rs. 250 (C. P.)
∴ gain on Rs. 250=Rs. 50.
∴ gain on Re. 1 =Rs. $\frac{50}{250}$=Re. $\frac{1}{5}$.
∴ gain on Rs. 100=Rs. $\frac{1}{5}\times 100$=Rs. 20,
∴ 20. p. c. is the gain. **Ans**

Example 2. A grocer bought 15 lbs. of tea at 10*a*. per lb. and 10 lbs. of tea at 12*a* per lb He mixed them together and sold the mixture at 11*a* per lb; find his gain or loss per cent.

Sol. C. P. of 15 lbs. of tea $=10a \times 15$ $=150a$.
and C. P. of 10 lbs of tea $=12a \times 10$ $=120a$.
$\therefore$ total cost price of 25 lbs. of tea $=150a. + 120a.$ $=270a$.
But selling price of mixture $=11a. \times 25$ $=275a$.
$\therefore$ gain on $270a. = 5a$.
$\therefore$ gain on $1a. = \frac{5}{270}a. = \frac{1}{54}a.$
$\therefore$ gain on $100a. = \frac{1}{54} \times 100 = \frac{50}{27} = 1\frac{23}{27}$
$\therefore$ gain per cent. $= 1\frac{23}{27}$. Ans.

From the above examples we learn the following rule :—

Rule *Profit or loss per cent.* $= \dfrac{\text{total profit or loss} \times 100}{\text{cost price}}$

Example 3 The cost price of 12 articles is equal to the selling price of 10 articles; find the gain per cent.

Sol. Let cost price of 12 articles = Rs. 100.
$\therefore$ selling price of 10 articles $=$ Rs. 100.
$\therefore$ selling price of 12 articles. $=$ Rs. 120.
$\therefore$ gain per cent. $=20$. Ans.

Note—For the sake of convenience we should always suppose the cost price in each question to be Rs. 100.

EXERCISE 106

1. A man bought a horse for Rs. 250 and sold it for Rs. 300 find his profit per cent.

2. A man buys a watch for Rs. 40 and sells it for Rs.50 ; what profit per cent. does he make ?

3. A table which costs Rs. 7, 8*a*. is sold for Rs. 10 ; find the profit per cent.

4. A horse which costs Rs. 80 is sold for Rs. 90 ; find the profit per cent.

5, An article which costs Rs. 15 is sold for Rs. 10 ; find the loss per cent.

6. A merchant bought some quantity of sugar at Rs.8, 5*a*. 4*p* a maund and sold it at Rs. 8, 10*a*. 8*p*, a maund. What profit per cent. did he make ?

7. A boy bought a number of pencils at 4*a*. a dozen and sold them at 5*a*. a score. What did he gain or lose per cent. ?

8. A shopkeeper bought a certain number of slates at Rs. 3. 6*a*. a dozen and sold them at 5*a*. 8*p*. per slate; find his gain or loss per cent.

9, A fruit-seller bought some mangoes at 10*a*. per hundred and sold them at 3*a*. a score; find his gain or loss per cent.

10. A fruit seller bought 100 oranges at 4 for 1*a*. and 200 oranges at 3 for 1*a*. He then sold the whole lot at 4*a*. 6*p*. per score; calculate his loss or gain per cent.

11. A man bought a horse for Rs. 250 and fed it for 2 months at the cost of Rs. 35 per month. He then sold it for Rs. 280, what did he lose per cent. ?

12. The cost price of 16 articles is equal to the selling price of 20 of the same kind; find the loss per cent.

13. If the cost price of 10 articles be equal to the selling price of 8, find the profit per cent.

14. A merchant buys a quantity of sugar at £2. 8*s*. per cwt. $\frac{1}{3}$ of it is wasted and the remainder he sells at 9*d*. per lb ; calculate his gain or loss per cent.

15. A merchant buys a quantity of sugar at £2, 16*s*. 3*d*. per cwt.; 4 lb. of sugar is wasted in every cwt.; what would be hisgain or loss per cent. if he sells the remainder at 6*d*. per lb?

16. By selling 66 yards of cloth I made a profit equal to the cost price of 22 yards; find my gain per cent.

17. What will be the loss p. c. if I sell 45 yds. of cloth and bear a loss equal to the cost price of 9 yards ?

18. A shopkeeper having purchased goods for Rs. 525, sold $\frac{1}{5}$ of them at a profit of 20 per cent. and the remainder at a profit of 5 per cent.; find his gain per cent. on the whole transaction.

19. A merchant bought teas worth 2*s*. 6*d*. and 2*s*. per lb respectively and mixed them together in the proportion of 2 lbs of the former to 3 lbs. of the latter and sold the mixture at 2*s* 9*d*. per lb. ; find his gain per cent.

20. A market woman bought eggs at 4 an anna and an equal number at 3 an anna; she sold them at 7 for 2 annas find her gain or loss per cent.

§5. **To find selling price.**

When the cost price and gain or loss per cent. are given and the selling price is required, we proceed as in the following examples .—

Example 1. A horse is bought for Rs. 350 ; find the selling price if it is sold at a gain of 20 per cent.

Sol. if the C. P. is Rs. 100, the S. P. $=$Rs. 120
" " Rs. 1 " " $=$Rs. $\frac{120}{100}$
" " Rs, 350 " " $=$Rs. $\frac{120}{100}\times 350$
$\therefore$ selling price $=$Rs. 420 **Ans.**

Example 2. I bought a house for Rs. 4200 and sold it at a loss of 15 per cent. ; find the selling price.

Sol. If the C. P. is Rs. 100, the S. P.$=$Rs. 85
" " Re. 1 " " $=$Rs. $\frac{85}{100}$
" " Rs. 4200 " " $=$Rs. $\frac{85}{100}\times 4200$
$\therefore$ selling price $=$Rs. 3570. **Ans.**

From the above examples we find that

$$\text{Selling price} = \begin{cases} 1. \ \dfrac{100+\text{gain p. c.}}{100} \times \text{cost price.} \\ 2. \ \dfrac{100-\text{loss p. c.}}{100} \times \text{cost price.} \end{cases}$$

EXERCISE 107.

1. I bought an article for Rs. 60 and sold it at a gain of 20 per cent . ; find the selling price.

2. At what price must a horse which costs Rs. 450 be sold so as to gain $7\frac{1}{2}$ per cent. ?

3. If by selling an article which costs me Rs. 66, 10*a*. 8*p*. lose 8 per cent., find the selling price.

4. A man bought a house for Rs. 2100 and spent Rs. 133, 5*a*. 4*p*. on its repairs ; at what price must he sell it so as to gain 20 per cent. ?

5. A grocer bought 4 lbs. of tea at 10*a*. a lb. and 8 lbs. of tea of another quality at 8*a*. a lb. ; at what price per lb. must he sell the mixture so as to gain 25 per cent. ?

6. 160 maunds of rice were bought at Rs. 8 per maund and sold at a loss of 10 per cent.; find the total selling price and the selling price per maund.

7. 180 maunds of sugar were bought at Rs. 20 per maund and sold at a profit of 20 per cent.; find the selling price per seer.

8. A merchant bought 80 maunds of wheat at Rs.5, 4*a*. per maund and paid Rs. 10 for expenses; at what price per maund must he sell it so as to gain 20 per cent. on his total outlay?

9. A shopkeeper bought oranges at the rate of 12 for a rupee and sold them at a profit of 20 per cent. How many did he sell for a rupee?

10. If the eggs are bought at the rate of 2 for a penny, how many must be sold for a shilling so as to gain 20 p.c.?

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§6. **To find cost price.**

When the selling price and gain or loss per cent. are given and the cost price is required, we proceed as in the following examples:—

Example 1. By selling goods for Rs. 650 a trademan gets a profit of 30 per cent. What did he give for them?

Sol. If the selling price is Rs.130, the cost = Rs. 100
" " " Re.1 " " = Rs. $\frac{100}{130}$
" " " Rs.650 " " = Rs. $\frac{100}{130} \times 650$
∴ cost price = Rs. 500. **Ans.**

Example 2. By selling a house for Rs. 2066, 10*a*. 8*p*. I lost $17\frac{1}{3}$ per cent.; find the cost price.

Sol. $100 - 17\frac{1}{3} = 82\frac{2}{3} = \frac{248}{3}$; Rs. 2066, 10*a*. 8*p*. = Rs. $\frac{6200}{3}$.
If the S. P. is Rs. $\frac{248}{3}$, the C. P. = Rs. 100
" " " Re. 1 " " = Rs. $100 \times \frac{3}{248}$
" " " Rs. $\frac{6200}{3}$ " " = Rs. $100 \times \frac{3}{248} \times \frac{6200}{3}$
∴ cost price " = Rs. 2500. Ans.

Note. From the above examples we find that the

$$\text{cost price} = \begin{cases} 1. & \dfrac{100}{100 + \text{gain p. c.}} \times \text{selling price} \\ 2. & \dfrac{100}{100 - \text{loss p. c.}} \times \text{selling price} \end{cases}$$

EXERCISE 108.

What is the cost price (1—5):—

1. When an article is sold for Rs. 24 at a loss of 4% ?
2. When an article is sold for Rs. 35 at a gain of 5% ?
3. When an article is sold for Rs. 108 at a gain of 20% ?
4. When an article is sold for Rs. 152 at a a gain of 14% ?
5. When an article is sold for Rs. 156 at a loss of 22% ?
6. If a yard of cloth sold for 5*a*. 6*p*. gives a profit of 20 per cent., find the prime cost.
7. By selling a horse for Rs. 175,2*a*. I gain 20 per cent., find the cost price.
8. By selling goods for Rs.77, 11*a*. a man gained 10 per cent. ; find the cost price.
9. Find the prime cost of a cycle which when sold for Rs. 91 gave a profit of 5 per cent.
10. A merchant by selling a horse for Rs.177, 3*a*. lost 16 per cent. ; find the cost price.
11. A shopkeeper by selling a sewing machine for £5,16*s*. gained 16 per cent.; find the prime cost.
12. A cow is sold for Rs.119,4*a*. at a gain of $12\frac{1}{2}$ per cent.; how much did it cost?
13. If apples are sold at the rate of 16 for a rupee and the gain is $12\frac{1}{2}$ per cent. , at what rate were they purchased?
14. If oranges are sold at a rate of 80 for a rupee and the gain is $22\frac{1}{2}$ per cent., at what rate were they purchased?
15. By selling a horse and a cow for Rs.112, 8*a*. I got a profit of 25 per cent. ; find the cost price of the horse if the cow cost Rs. 40.

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§7. Some Important Typical Examples.

☞ The following examples of different varieties are worthy of careful notice:—

(*a*) **Goods passing through successive hands.**

Example I. *A* sells an article to *B* at a profit of 25 per cent., *B* sells it to *C* at a profit of 20 per cent. If *C* pays Rs. 60 for it, what did it cost *A* ?

Sol. C's cost price=Rs. 60.
As C purchased the article from B
$\therefore$ B's selling price is also=Rs. 60.
and B's profit per cent. =Rs. 20.
$\therefore$ B's cost price =Rs. $\frac{100}{120}\times 60$=Rs. 50.
As B purchased the article from A,
$\therefore$ A's selling price is also =Rs. 50,
and A's profit per cent. =Rs. 25
$\therefore$ A's cost price =Rs. $\frac{100}{125}\times 50$=Rs. 40. **Ans.**

Aliter. Suppose A's cost price=Rs. 100.
$\therefore$ he sold it to B at Rs. 125;
and $\because$ B made a profit of 20 per cent. on Rs. 125
$\therefore$ his selling price=Rs. $\frac{100}{120}\times 125$=Rs. 150 which is evidently the price paid by C.
if when C pays Rs. 150, A's cost price=Rs. 100
$\therefore$ when C pays Re. 1, ,, cost price=Rs. $\frac{100}{150}$
$\therefore$ when C ,, Rs. 60 ,, ,, =Rs. $\frac{100}{150}\times 60$
=Rs. 40. **Ans.**

Note. The student should understand clearly this second solution which is much simpler than the first.

EXERCISE 109

1. A sells his horse to B at a profit of 20 % and B sells it to C at a profit of 5%. If C pays Rs. 252 for it, what did it cost A ?

2. A sold his watch to B at a profit of 5 per cent. B sold it to C for Rs. 49, 14*a*. at a loss of 5%, what did A pay for it?

3. A sold a piece of land to B at a profit of 25%, B sold it to C for £ 3668, 10*s*. at a loss of 13%; what did A pay for it ?

4. A sells an article to B at a gain of 20%, B sells to C at a gain of 5%. If C pays Rs. 70 for it what did A pay for it?

5. An article is sold by A to B at a profit of 12%. It is then sold for Rs. 2, 10*a*. by B who thereby gains 12½% on what he paid for it. What did A pay for it ?

6. A sells goods to B at a gain of 22½%. B sells them to C at a gain of 7½%. C gives Rs. 5267, 8*a*. for them. How much did A pay for them ?

7. An article passes successively through the hands of three dealers, each of whom in selling adds as his profit 10 per cent. of the price at which he bought it; if the third dealer sells the article for Rs. 332, 12*a*., what did the first dealer pay for it?

(*b*) **Different rates of gain or loss per cent. for di ent selling prices.**

Example 2. A watch was sold for Rs. 45 at a loss p. c.; for what should it have been sold to gain 30 p. c.?

Sol. ∵ selling price =Rs. 45
and loss per cent. =Rs. 20
∴ cost price =Rs. $\frac{100}{80} \times 45$=Rs. $\frac{225}{4}$
Now ∵ cost price =Rs. $\frac{225}{4}$
and gain per cent. =Rs. 30
∴ selling price =Rs. $\frac{130}{100} \times \frac{225}{4}$=Rs. 73, 2*a*. Ans.

Aliter. If the cost price is Rs 100, then a loss of 20 per cent. means it is sold at Rs. 80 ; and a gain of 30 per cent means it is sold at Rs.130.

In this question 45 corresponds to 80 and we are to find the number corresponding to 130 which is evidently

$$=\frac{45}{80} \times 130=\frac{585}{8}=73\frac{1}{8}.$$

∴ the required answer=Rs. 73. 2*a*. Ans.

Note. The student is very strongly recommended to understand the solution which is more intelligent than the previous one.

EXERCISE 110.

1. By selling an article for Rs. 48 a man gains 44%, how much must he reduce the price to gain 20%.

2. By selling an article for Rs. 66. 10*a*. 8*p*. a merchai loses 20%; at what price must it be sold to gain 20%.

3. By selling an article for Rs. 85 a merchant loses 15%; at what price must it be sold to gain $12\frac{1}{2}$%.

4. If a merchant, by selling an article for Rs. 172, 2*a*., gains $12\frac{1}{2}$%; what should he sell it for to gain 25%.

5. By selling goods for £ 130 per ton, I gain 30 per cent; what should I charge per cwt. to gain $7\frac{1}{2}$ per cent. ?

6. If I sell an article for £ 87, I gain $8\frac{3}{4}$ per cent., what must I sell it for to gain 20 per cent. ?

7. If by selling a table for Rs. 8, 8*a*. I lose $\frac{1}{18}$ of what it cost me, find at what price I ought to sell it to gain $16\frac{2}{3}$%.

8. By selling a cow for £ 9, 7*s*. I lose $6\frac{1}{2}$ p. c., what must I sell it for to gain 13 per cent. ?

9. By selling apples at 16 for 2*a*. 3*p*. a man loses 25 per cent. How many should he sell at 10 p.c. profit so that the total gain may be Rs. 3 ?

Example 3. A horse is sold for Rs, 510 at a loss of 15% l the gain or loss per cent. had it been sold for Rs. 575.

Sole. When the selling price $=$Rs. 510
the loss per cent. $=$Rs. 15,
$\therefore$ cost price $=$Rs. $\frac{100}{85}\times 510=$Rs. 600
Now $\because$ cost price $=$Rs. 600
and selling price $=$Rs. 575
$\therefore$ loss on Rs. 600 $=$Rs. 25
$\therefore$ loss per cent. $=$Rs. $\frac{25}{600}\times 100=4\frac{1}{6}$. **Ans.**

EXERCISE 111.

1. By selling a horse for Rs, 350, a merchant gains 25% , hat will he gain or lose per cent. by selling it for Rs. 250 ?

2 If I sell an article for Rs. 4, I lose $16\frac{2}{3}$% ; what would the gain or loss % if I sell the same article for Rs. 5, 3*a*. $2\frac{2}{5}p$.

3. By selling a cycle for Rs. 74 a person lost $7\frac{1}{2}$ per cent., nd the gain or loss per cent. if he sells it for Rs. 78.

4. By selling goods for Rs. 248, 1*a*. a peason lost 19 per nt. ;find the gain or loss per cent. if he sells it for Rs. 355, 4*a*.

5. By selling 7 oranges for 5*a*. a dealer gained 5 per cent.; hat per cent. wou'd he gain by selling it 9 for 7*a*. ?

6. Goods were sold for £9, 7*s*. at a loss of $6\frac{1}{2}$% ; what % ould have been gained or lost by selling them for £11,6*s*. ?

7. By selling a watch for Rs. 56, I gained $\frac{1}{7}$ of the cost rice; what would be the gain or loss % if I sell it for Rs. 63 ?

8 By selling 12 lb. of tea for £ 1, 11*s*. 6*d*., I gain 5% hat do I gain or lose per cent. by selling 50 lb. of the same ind of tea for £ 6, 9*s*. 2*d*. ?

(*c*) **To gain a certain p. c. on the whole transaction.**

Example 4. A shopkeeper having purchased 40 bags of ice at Rs. 5 a maund, sold $\frac{1}{2}$ of them at a loss of 20%; at what rice per maund should the remainder be sold so as to gain % on the whole transaction? A bag contians $2\frac{1}{2}$ maunds f rice.

Sol. Cost price of 40 bags $=$Rs. $40\times\frac{5}{2}\times 5=$Rs. 500
Total selling price he wants $=$Rs. $500\times\frac{105}{100}=$Rs. 525
$\because$ cost price of $\frac{1}{2}$ of rice $=$Rs. 250.
$\therefore$ selling price of first $\frac{1}{2}$ $=$Rs. $250\times\frac{80}{100}=$Rs. 200
$\therefore$ selling price of 2nd $\frac{1}{2}$ $=$Rs. $525-$Rs. $200=$Rs. 325
$\therefore$ selling price per maund $=$Rs. $325\div 50$
$=$Rs. 6, 8*a*. **Ans.**

1. A merchant bought good for £75 and sold $\frac{1}{2}$ of them at a loss of 20%. At what price should be sell the remainder so as to gain 20% on the whole transaction ?

2. A shopkeeper purchased goods for £60. He sold $\frac{1}{3}$ at a loss of 10%. At what price should be sell the remainder so as to clear a profit of 10% on the whole transaction ?

3. A merchant having purchased 300 maunds of rice at Rs. 8 a maund, sold $\frac{1}{3}$ of them at a loss of 10 per cent.; at what price per maund should he sell the remainder so as to gain 5 per cent. on the whole transaction ?

4. A shopkeeper having purchased 250 maunds of wheat at Rs. 8 a maund, sold $\frac{1}{2}$ at a profit of 10% and the remaining half at a loss of 15%. What did he gain or lose % on the whole?

5. A merchant having 100 mds.of grain sold 50 mds. at Rs. 9 per md. and thereby gained $12\frac{1}{2}$%. At what rate should he sell the remaining half so that he may gain $6\frac{1}{4}$% on the whole?

6. A merchant bought 150 maunds of wheat and sold 50 maunds of them at Rs. 9, 1*a*. $1\frac{1}{2}$*p*. a maund and thus cleared a profit of $7\frac{1}{2}$%. At what rate should he sell the remaining 100 maunds so as to gain 10% on the whole transaction ?

7. If goods be purchased for Rs.600 and $\frac{1}{3}$ be sold at a loss of 10 per cent. at what gain per cent. should the remainder be sold so as to gain 20 per cent. on the whole transaction?

8. A merchant having 150 mds. of grain sold 50 mds. at Rs. 5, 6*a*. per maund and thereby gained $7\frac{1}{2}$%. At what gain p.c. should he sell the remainder so that he may gain 10% on the whole ?

Example 5. A merchant purchased 111 maunds of wheat. He sold 26 maunds of them at a profit of 10% and the remainder at a profit of $6\frac{1}{4}$%. Had he sold the whole wheat at a profit of 8% he would have gained Rs. 4, 13*a*. $4\frac{4}{5}$*p*. more. **Find** the cost price per maund.

Sol- Suppose the cost price per md. $=$Re. 1

$\therefore$ gain on 26 mds. $=$Rs. $26\times\frac{10}{100}$ $=$Rs. 2·6

$\therefore$,, 85 mds $=85\times\frac{25}{4}\times\frac{1}{100}$ $=$Rs. 5·3125

Total gain ,, $=$Rs. 26$+$Rs. 5·3125 $=$Rs. 7·9124

$\because$ gain on 111 mds. at 8 p. c.$=$Rs. $111\times\frac{8}{100}=$Rs. 8·88

$\therefore$ difference in gains$=$Rs. 8·88$-$7·9125 $=$Rs. ·9675

But the actual difference$=$ Rs, 4, 13*a*. $4\frac{4}{5}$ *p*. $=$Rs. 4·8375

which is 5 times of Rs. ·9675

$\therefore$ cost price per maund$=$Rs. 5. **Ans.**

EXERCISE 113.

1. A merchant bought 124 maunds of corn. He sold 24 maunds at a gain of 25 per cent. and 100 maunds at a gain of 10 per cent. Had he sold the whole at a profit of $12\frac{1}{2}$ per cent., he would have got Rs. 2 less; find the cost per maund.

2. A man bought 91 sheep and sold $\frac{4}{7}$ at a profit of 15 per cent., $\frac{4}{13}$ at a profit of 10 per cent. and the remainder at a profit of 8 per cent. Had he sold all the sheep at a profit of 16 per cent. he would have got Rs. 46, 3*a*.2·4*p*.more. what did he pay for a sheep ?

3. A merchant having purchased 4000 maunds of rice, sold $\frac{1}{5}$ of them at a profit of 5 per cent., $\frac{1}{4}$ of them at a profit of 10 per cent., $\frac{1}{2}$ of them at a profit at 12 % and the remainder at a profit of 16 %. Had he sold the whole at a profit of 11% he would have got Rs. 728 more, find the cost price per maund.

4. I bought 50 horses and sold 15 of them at a gain of 20 per cent., 25 at a gain of 16 per cent. and the rest at cost price. Had I sold all at a gain of 18 per cent. I should have got Rs. 560 more. Find the cost price of each horse.

5. A merchant buys 5000 maunds of rice, one fifth of which he sells at a profit of 5 per cent., one fourth at a profit of 10 per cent. and the remainder at a profit of 16 %. If he had sold the whole at a profit of 15% he would have made Rs. 438, 12*a*. more. What was the cost of rice per maund ?

6. A merchant bought 630 maunds of ghee, sold $\frac{1}{5}$ at a gain of 5 per cent. and $\frac{1}{3}$ at a gain of 8 per cent. and the remainder at a gain of 12 per cent. Had he sold the whole at a gain of 10 per cent. he would have gained Rs. 115, 8*a*. more. Find the cost price per maund.

(*d*) **Mixture.**

Example 6. If 34 lbs. of tea at Re. 1, 2*a*. a lb. be mixed with 29 lbs. of tea at 12*a*. lb., at what per lb. must the mixture be sold to gain 5 per cent. on the whole outlay ?

Sol. The total cost $= 34 \times 18a. + 29 \times 12a.$

$= 612a. + 348a. = 960a.$

Add to it 5 per cent. gain. *i.e.*, $\frac{5}{100} \times 960 = 48a.$

$\therefore$ (34+29). *i.e.* 63 lbs. are sold at 960+48 or 1008*a*.

$\therefore$ 1 lb. is sold at $\frac{1008}{63}$ or 16*a*., *i.e.* Re. 1. **Ans.**

EXERCISE 114.

1. A man buys 100 lbs. of tea at 5*s*. per lb. and 40 lbs.at 3*s*.6*d*.per lb. ; he mixes them and sell the mixture at a loss of $12\frac{1}{2}$ per cent. ; at how mnch per lb. does he sell the tea?

2. A man buys 100 lbs. of tea at Rs.2 per lbs., 200 lbs. at Re. 1,8*a*. a lb. and 300 lbs. at Re. 1 per lb., he mixes them and sells the mixture at Re. 1,10*a*. per lb. How much per cent. is his profit?

3. Chicory is mixed with coffee bought at Re.1 4*a*. per lb. in proportion of 2 lbs. of chicory to 5 lbs. of coffee. The mixture is sold at Re. 1. 2*a*. per lb. which gives a loss of $6\frac{2}{3}$ per cent. Find the cost price per lb. of the chicory.

4. A gorcer bought 50 mds. of sugar at Rs.11,4*a*. a md. and mixed with it 70 mds. of sugar at a certain price By selling the mixture at 5*a*. a seer he gains $17\frac{3}{16}$ p.c.; find the price per md. of the latter kind of sugar.

5. A shopkeeper buys $\frac{1}{2}$ cwt. of tea, at 4*s*. 2*d*. per lb. and mixes it with tea at 2*s*. 11*d*. per lb. How much of the latter must he mix with the former that he might sell the mixture at 3*s*. 8*d*. per lb. and again 20 p.c. on his outlay ?

6. How much water must be added to a maund of milk so that by selling the mixture at cost price, there may be $12\frac{1}{2}$ per cent. profit ?

7. A man buys milk at $2\frac{1}{2}$ *d*. per quart, dilutes it with water and sells the mixture at 3*d*. per quart. How much water is added to each quart of milk if his profit is 60 p.c.

8. A man purchased milk at Rs.9, 8*a*. a maund and mixed some water in it; by selling the mixture at Rs. 10 a maund he gained 25 p.c. How much water did he mix with each maund of milk

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(*e*) **Different kinds of articles bought and sold at different rates.**

Example 7. A man buys eggs at 5*a*. a dozen and an equal number at 8*a*. a score; he sells them at Rs. 2, 8*a* a hundred and thus loses 4*a*. Find the number of egg she had bought

Note. We can suppose any number of eggs he bought but to avoid fractions we should suppose the number of eggs equal to the L. C. M. of 12 and 20 which is equal to 60.

Sol. Cost of 60 eggs at 5*a*. a dozen $=25a.$
,, 60 ,, at 8*a*. a score $=24a.$
∴ total cost of 120 eggs $=25a.+24a.$ $=49a.$
Selling price of 120 eggs at Rs. 2, 8*a*. a hundred $=48a.$
∴ loss on 120 eggs $=49a.-48a.$ $=1a.$
If the loss is 1*a*. the number of eggs $=120$
If the loss is 4*a*. the number of eggs $=120\times4$ $=480$
∴ the required number $=480$. **Ans.**

EXERCISE 115.

1. I bought a certain number of eggs at 4*a*. a dozen and sold at 7*a*. a score. How many eggs did I buy if I made a profit of Re. 1 ?

2. I bought some mangoes at 4*a*. a dozen and an equal number at 5*a*. a score. I then sold them at Re. 1, 14*a*. a hundred and thus cleared a profit of Re. 1; how many mangoes did I buy ?

3. A man buys apples at 5*a*. a dozen and an equal number at 8*a*. a score; he sells them at Rs. 2, 8*a*. a hundred and thus loses 2*a*.; how many apples did he buy ?

4. I buy a number of mangoes at 2 for a pice and half the same number at 4 for 3 pice; at what price must I sell them to gain 20 per cent. on the cost ? If my total profit is 14*a*., how many did I buy ?

(*f*) **Buying at a smaller price and selling at a greater price and** ***vice versa*****.**

Example 8. I sold a table at a profit of 10 per cent.; had I sold it for Rs. 2 more, 25 per cent. would have been gained. Find the cost price.

Sol. Let the cost price of the table be $=$Rs. 100
∴ selling price in the first case $=$Rs. 110
and ,, ,, ,, the second case $=$Rs. 125
∴ difference $=$Rs. 125$-$Rs. 110$=$Rs. 15.
If the diff. $=$Rs. 15 the cost is supposed to be Rs. 100
If the diff. $=$Re. 1 ,, ,, would be Rs. $\frac{100}{15}$
∴ If the diff. $=$Rs. 2 ,, ,, ,, Rs. $\frac{100}{15}\times2=$Rs. $\frac{40}{3}$
∴ cost price $=$Rs. 13, 5*a*. 4*p*. **Ans.**

Example 9. A man sold an article at a profit of 10 per cent. If he had bought it at 10 per cent. less and sold it for $\frac{1}{2}a$. more he would have got a profit of 25 per cent. Find the cost price.

Sol. Suppose the cost price $= 100a.$

$\therefore$ selling price in the first case $= 110a.$

Cost price in the second case $= 100 - 10 = 90a.$

$\therefore$ selling price in the second case $= \frac{125}{100} \times 90 = \frac{225}{2}a.$

Diff. between the two selling prices $= \frac{225}{2} - 110 = \frac{5}{2}a.$

But the actual difference $= \frac{1}{2}a.$

$\therefore$ actual cost price $= \frac{2}{5} \times 100 \times \frac{1}{2} = 20a. =$ Re. 1, 4*a*. **Ans.**

EXERCISE 116.

1. I sold an article at a profit of 25%; had I sold it for Rs. 2 more, $37\frac{1}{2}\%$ would have been gained. Find the cost price.

2. Alfred sold his watch at a profit of 20 per cent., had he sold it for Rs. 3 more, 32 per cent. would have been gained. Find his cost price.

3. Thomas sold his books at a profit of 5 per cent., had he sold them for Re. 1, 8*a*. less, he would have lost 4 per cent. Find his cost price.

4. Bashir Ahmad sold his chairs at a loss of 15 per cent; had he sold them for Rs. 8 more he would have cleared a profit of 35 per cent., find the cost price.

5. $12\frac{1}{2}$ per cent. more is gained by selling mangoes at 2*a*. a dozen than 3*a*. a score. Find the cost price of mangoes.

6. A person sold an article at a profit of 5%. If he had bought it at 5% less than he did and sold it for Re. 1 less he would have cleared a profit of 10%. Find the cost price.

7. A merchant sells a horse at a profit of 25 per cent. If he had bought it at 4 per cent. less than he did and sold it for Rs. 11, 8*a*. less he would have gained the same percentage of profit. What did it cost him?

8. Baldev sells his watch at a profit of 5 per cent. If he had bought it for 5 per cent. less than he did and sold it for 1s. less he would have gained 10 per cent. Find his cost price.

9. A watch was sold at a profit of $12\frac{1}{2}$ per cent., had it been bought at 5 per cent. more and sold it for 16s. more the seller would have gained $16\frac{1}{3}\%$. What was the cost price?

10. A man bought a watch and sold it at a loss of 10 per cent.; if he had received Rs. 15 more he would have gained 20 per cent. What did the watch cost?

(*e*) **Change in the total cost of articles owing to a change in their rates.**

Example 10. A lb. of tea and 4 lbs. of sugar cost Rs. 3 2*a*.; but if sugar rose 50 per cent. and tea 10 per cent., they would cost Rs. 3, 11*a*., find the prices per lb. of tea and sugar.

Sol. If both tea and sugar rose 50 per cent. the cost of 1 lb. of tea and 4 lbs. of sugar would be

$$\text{Rs. } 3.\ 2a. \times \tfrac{150}{100} = 75a.$$

But tea rose only 10 per cent

$\therefore$ 40% of the cost of 1 lb. of tea $= 75a. - \text{Rs. } 3,\ 11a. = \text{Re. } 1;$

$\therefore$ cost of 1 lb. of tea $= \text{Re. } 1 \times \tfrac{100}{40} = \text{Rs. } 2,\ 8a.$ **Ans.**

$\therefore$ cost of 4 lbs. of sugar $= \text{Rs. } 3,\ 2a. - \text{Rs. } 2,\ 8a. = 10a.$

$\therefore$ cost of 1 lb. of sugar $= 10 \div 4 = 2\tfrac{1}{2}a.$ **Ans.**

EXERCISE 117.

1. 2 lbs. of tea and 5 lbs. of sugar cost 7*s*. 6*d*.; but if sugar were to rise $33\tfrac{1}{3}$ per cent. and tea 20 per cent. they would cost 9*s*. 4*d*., find the cost of tea and sugar per lb.

2. 12 lbs. of tea and 25 lbs. of coffee together cost Rs. 43, 5*a*. 4*p*.; but if the tea were to rise $2\tfrac{1}{2}$ per cent. and the coffee to fall $4\tfrac{1}{2}$ per cent. the same quantities would cost Rs. 42, 15*a*. 4*p*.; find the price of tea and coffee per lb.

3. One lb. of tea and 3 lbs. of sugar cost Rs. 3; but if sugar were to rise 50 p. c. and tea 10 per cent. they would cost Rs. 3, 8*a*.; find the prices per lb. of tea and sugar.

4. 2 lbs. of tea and 5 lbs. of sugar cost Rs. 2, 13*a*. but if sugar were to rise 25 per cent. and tea 10 per cent. they would cost Rs. 3, 2*a*. 9*p*.; find the prices per lb. of tea and sugar.

5. 10 mds. of rice and 15 mds. of wheat cost Rs. 102, 8*a*.; but if rice were to rise 5 per cent. and wheat to fall 10 per cent. they would cost Rs. 99, 12*a*.; find the prices per md. of rice and wheat.

(*h*) **Discount.**

Example 11. A tradesman marks his goods at 25 per cent. above cost price and allows discount of $12\tfrac{1}{2}$ per cent. for cash payment. What profit per cent. does he make?

Sol. If the cost price is Rs. 100, his marked price = Rs. 125.

But discount to the cash purchaser $= 12\frac{1}{2}$ per cent. on Rs. 125 $\left\{ \frac{12\frac{1}{2}}{100} \times 125 = \text{Rs. } 15\frac{5}{8} \right.$

$\therefore$ the reduced price he gets from the cash purchaser

$= \text{Rs. } 125 - \text{Rs. } 15\frac{5}{8} = \text{Rs. } 109\frac{3}{8}$.

i.e., his gain per cent. $= 109\frac{3}{8} - 100 = 9\frac{3}{8}$. **Ans.**

Example 12. I buy goods for Rs. 39, 9*a*. 4*p*. What price must I charge in order to make $10\frac{1}{2}$ p. c. on my capital, after allowing 15 p. c. discount to the customers?

Sol. Suppose I fix the price = Rs. 100.

$\therefore$ I receive Rs. 100 − Rs. 15 = Rs. 85.

$\therefore$ cost price = Rs. $85 \times \frac{100}{110\frac{1}{2}} = \frac{85 \times 100 \times 2}{221} = \frac{1000}{13}$

Now, if the C. P. is Rs. $\frac{1000}{13}$ the fixed price is Rs. 100.

if the C. P. is Rs. 39, 9*a*. 4*p*. then the fixed price should be Rs. $\frac{13}{1000} \times 100$ of Rs. 39, 9*a*. 4*p*. = Rs. 51, 7*a*. 4*p*. **Ans.**

EXERCISE 118.

1. A publisher sells books to a retail dealer at Rs. 5 a copy, but allows 25 copies to account as 24. If the retailer sells each of the 25 copies at Rs. 6, 12*a*., what profit per cent. does he make?

2. A tradesman marks his goods 30 p. c. above the cost price, but he allows customers 10 p. c. off his bill and thus gains Rs. 3, 3*a*. on the goods; find the amount of the bill.

3. I buy goods for Rs. 5040 and incur 10 per cent. expenses; what must I charge in order to make 10 per cent. profit on my capital after allowing 10 per cent. discount?

4. A trader allows a discount of 5 per cent. to his customers. What price should he mark on an article, the cost price of which is Rs. 712, 8*a*. so as to make a clear profit of $3\frac{1}{3}$ per cent. on his outlay?

5. A manufacturer marks an article for sale at 50 per cent. over its cost price but gives the retail dealer 13 articles to the dozen and allows a discount of $12\frac{1}{2}$ per cent. for cash. What does he gain per cent. on the cost of the article, when he gets cash price?

(*i*) **To sell a number of articles at a loss or gain equal to the selling price of a part of them.**

Example 13. I bought 20 maund of rice for Rs. 62, 8*a*. and sold them at a loss equal to the selling price of 5 maunds. Find the selling price per maund.

Sol. Suppose S.P. of 20 maunds = Rs. 100
$\therefore$ loss = Rs. $\frac{100}{20} \times 5$ = Rs. 25.
and cost price = Rs. 100 + Rs. 25 = Rs. 125
but the actual cost price = Rs. 62, 8*a*.
If, C.P. is Rs. 125, the S.P. = Rs. 100
" " Re. 1 " " = Re. $\frac{100}{125}$
$\therefore$ " " Rs. $\frac{125}{2}$ " " = Rs. $\frac{100}{125} \times \frac{125}{2}$ = Rs. 50.
$\therefore$ selling price per maund = Rs. $50 \div 20$ = Rs. $2\frac{1}{2}$ **Ans.**

EXERCISE 119.

1. I bought 50 maunds of sugar for Rs. 662, 8*a*. and sold it at a loss equal to the selling price of 3 maunds. Find the selling price per maund.

2. I bought 80 chairs for Rs. 690 and sold them at a loss equal to the selling price of 12 chairs ; find the selling price per chair.

3. A merchant bought 56 cows for Rs. 2320, 8*a*. and sold them at a gain equal to the selling price of 5 cows. Find the selling price per cow.

4. I bought 50 chairs for Rs. 385 and sold them at a loss of as much money as I got for 5 chairs. Find the selling price per chair.

5. I bought 30 maunds of rice for Rs. 180, 7*a*. 6*p*. and sold them at a loss of as much money as I got for 3 maunds ; find the selling price per maund.

(*i*) **To get a percentage of profit equal to the cost price**

Example. 14. A person sold his watch for Rs. 24 and got a pescentage of profit equal to the cost price ; find the cost price.

Sol. $\because$ cost price $\times \frac{\text{cost price}+100}{100} = \text{selling price}$

$\therefore$ cost price $\times \frac{\text{cost price}+100}{100} = \text{Rs. } 24.$

or $(\text{cost price})^2 + 100 \text{ cost price} = \text{Rs. } 2400.$

Completing the square we have

$(\text{cost price})^2 + 100 \text{ cost price} + \text{Rs. } 2500 = \text{Rs. } 4900.$

Or cost price $+ \text{Rs. } 50 = \text{Rs. } 70$ [by square root.

$\therefore$ cost price $= \text{Rs. } 20.$ **Ans.**

It follows therefore that when a percentage of profit equal to the cost price is given we have the

Rule. $\sqrt{\text{S.P.} \times 100 + (50)^2} - 50 = \text{cost price.}$

EXERCISE 120.

1. An article when sold for Rs. 96 gives a certain profit per cent. which is equal to the cost price, find the cost price.

2. A person sold his watch for Rs. 85, 4*a*. It yielded a certain profit per cent. which is equal to the cost price ; find the cost price.

3. A horse when sold for Rs. 39 gives a certain gain p. c. which is equal to the cost price ; find the cost price.

4. An article when sold for Rs. 75 gives a percentage of profit equal to the cost price ; find the cost price.

5. A table when sold for Rs. 17, 4*a*. yields a percentage of profit equal to the cost price ; find its prime cost.

(*b*) Miscellaneous Types.

Example 15. A merchant sold a quantity of wheat at Rs. 5, 8*a*. $2\frac{2}{5}p$. a maund and cleared a profit of 2 per cent. ; what is the quantity of wheat sold if the total profit is Rs. 63 ?

Sol. Selling price of one md. $= \text{Rs. } 5, 8a. 2\frac{2}{5}p. = \text{Rs. } \frac{441}{80}$.

and gain $= 5$ per cent.

$\therefore$ cost rice ,, ,, $= \text{Rs. } \frac{100}{105} \times \frac{441}{80} = \text{Rs. } \frac{21}{4}$.

$\therefore$ gain on one maund $= \text{Rs. } \frac{441}{80} - \text{Rs. } \frac{21}{4} = \text{Rs. } \frac{21}{80}$

But the total gain $= \text{Rs. } 63$

$\therefore$ quantity sold $= 63 \div \frac{21}{80} = 240$ mds. **Ans.**

EXERCISE 121.

1. A merchant sold a quantity of wheat at Rs. 13, 4*a*. a maund and thus cleared a profit of 6 per cent. What is the quantity of sugar sold if the profit is Rs. 16, 8*a*. ?

2. A merchant by selling a quantity of rice at Rs. 7, 8*a*. a maund sustained a loss of $7\frac{9}{13}$ p.c., find the quantity sold if his total loss be Rs. 73, 2*a*.

3. A man sold a number of sheep at Rs. 15 each with a profit of 25 per cent. and a total gain of Rs. 180. Find how many sheep did he sell.

4. A merchant buys a certain number of yards of cloth at 5*s*. a yard ; he sells half of them at 6*s*. 4*d*. per yard, but the remainder, being damaged he can only dispose off at a loss of 10% under the invoiced price. His gain on the transaction is £14, 3*s*. 4*d*. Find how many yards were purchased.

Example 16. A person bought two horses for Rs. 550, sold one of them at a profit of 10 p. c. and the other at a loss of $8\frac{1}{3}$ p. c. He neither gained nor lost in this business; find the cost price of each horse.

Sol ∵ gain on one horse = loss on the other.

$$\therefore \frac{\text{cost price of one}}{\text{cost price of the other}} = \frac{8\frac{1}{3}}{10}$$

or C.P. of one : C.P. of the other $= 8\frac{1}{3} : 10$

$$= \frac{25}{3} : 10 = 5 : 6$$

∴ cost price of the first horse $= \frac{5}{11} \times 550 =$ Rs. 250 }
and cost price of the second horse $= \frac{6}{11} \times 550 =$ Rs. 300 }

Example 17. A person bought two articles for Rs. 270. He sold one of them at a gain of $12\frac{1}{2}$ p. c. and the other at a loss of 10 p.c. It was found that both the articles had fetched the same price. Find the cost price of each article.

Sol. Suppose Rs. 100 is the cost price of one article, then the S.P. of the article = Rs. $112\frac{1}{2}$.

∴ the S.P. of the second article is also = Rs. $112\frac{1}{2}$
and the loss on it = 10 p.c.
∴ C.P. of the second article = Rs. $\frac{100}{90} \times \frac{225}{2} =$ Rs. 125
∴ ratio between the prices = 100 : 125 = 4 : 5
∴ C.P. of the first article = Rs. $\frac{4}{9} \times 270 =$ Rs. 120 }
and C.P. „ „ second „ = Rs. $\frac{5}{9} \times 270 =$ Rs. 150 } **Ans.**

EXERCISE 122.

1. A person bought two horses for Rs. 560; sold one of them at a loss of 5 p. c. and the second at a gain of $6\frac{1}{5}$ p.c. He neither gained nor lost in this transaction. Find the cost price of each horse.

2. A person bought two horses for Rs. 3600. The bigger one he sold at a loss of 5 p.c. and the smaller one at a profit of 7 p.c. He neither gained nor lost in this transaction. Find the cost price of each horse.

3. The cost price of two clocks is Rs. 135. One was sold at a profit of $12\frac{1}{2}$ p.c. and the other at a loss of 10 p.c. but both the clocks fetched the same price; find the cost price of each clock.

4. A merchant has a horse and a cow together worth Rs. 225. He sells the horse at a loss of 20 p.c. and the cow at a gain of 60 p.c. It was found that both the animals had fetched the same price; find the price of each.

5. A person bought two articles for Rs. 325. He sold one at a gain of 5 p.c. and the other at a loss of 10 p.c. He found that both of them had fetched the same price. Find the price of each article.

6. A merchant bought two cows for Rs. 520. He sold one of them at a loss of 15 p.c. and the other at a gain of 36 p.c. It was found that both the cows had fetched the same price; find the price of each cow.

Example 18. A person had goods worth £300; he sells $\frac{1}{3}$ of them so as to lose 10 p. c. By how much per cent. should he raise the selling price in order to gain 10 p. c. on the whole?

Sol. S. P. of $\frac{1}{3}$ $= £\frac{90}{100} \times 100 = £90$

but total S. P. $= £\frac{110}{100} \times 300 = £330.$

$\therefore$ remaining S.P. $= £330 - £90 = £240.$

In case he sells the remaining goods also at a loss, the remaining S. P. $= £\frac{90}{100} \times 200 = £180.$

$\therefore$ diff. in the rem. selling prices $= £240 - £180 = £60,$

$\therefore$ excess per cent. $= \frac{60}{180} \times 100 = 33\frac{1}{3}.$ **Ans.**

EXERCISE 123,

1. A person has goods worth Rs. 75; he sells $\frac{1}{3}$ of them at a loss of 4 per cent. By how much per cent. should he raise that selling price in order to gain 4 p.c. on the whole?

2. A person has goods worth Rs. 450. He sells $\frac{2}{3}$ of them at a los of 4 p. c. By how much p. c. should he raise that selling price in order to gain 10 p. c. on the whole?

3. A person pays 10 p. c. expenses on the goods and allow 10 p. c. to his customers on cash payment. By how much should he raise the selling price from the cost price in order to get a profit of 8 p.c. on his outlay.

4. A man having bought a quantity of goods for Rs. 1500, sells $\frac{1}{3}$ at a loss of 4 p.c.; by how much p.c. must he raise that selling price in order that by selling the remainder at the increased rate, he may gain 4 p. c. on the whole transaction.

Example 19. A person bought some paper at 8*a*. per 5 quires and by selling it got on every 32 quires, a profit equal to the selling price of 8 quires; find the selling price per quire.

Sol. Since by selling 32 quires the profit=the selling price of 8 quires

$\therefore$ it is evident that the selling price of 24 quires=the cost price of 32 quires.

$\because$ the cost price of 32 quires$=\frac{8}{5}\times 32=2\frac{5}{5}\frac{6}{5}a$.

$\therefore$ the selling price of 24 ,, $=2\frac{5}{5}\frac{6}{5}a$.

$\therefore$ the selling price of 1 quire$=2\frac{5}{5}\frac{6}{5}\times\frac{1}{24}=\frac{32}{15}=2\frac{2}{15}a$. **Ans.**

EXERCISE 124.

1. A person bought some knives at the rate of 4 for 15*a*. and by selling them got on every 40 knives a profit equal to the selling price of 10 knives. Find the selling price of each knife.

2. A person bought some books at the rate of Rs. 5, 10*a*. per dozen and got a profit equal to the selling price of 5 books on the sale of every 30 books. Find the selling price of each book.

3. A person bought some chairs at Rs. $3\frac{1}{2}$ per chair and sold 20 of them at a loss equal to the selling price of 8 chairs. Find the selling price of each chair.

4. A person bought some bottles at 40*s.* per dozen. How should he dispose them off in order to get, on a capital of every £100 invested, a profit equal to the selling price of 5 dozen bottles?

Example 20. A person bought two horses at the same price. He sold one of them at a profit of 12 p. c. and for the other he received Rs. 15 more than he received for the first. By this bargain he got 15 p. c. profit on the whole. What did each horse cost him?

Sol. 15 p. c. − 12. p. c. = 3 p. c.

= Rs. $\frac{3}{100}$ of the real price

$\because$ $\frac{3}{100}$ of the real price = Rs. 15.

$\therefore$ real price = $\frac{100}{3} \times 15$ = Rs. 500

$\therefore$ cost of each horse is Rs. 250. **Ans.**

Example 21. A merchant bought two horses. The cost price of one of them is $\frac{4}{3}$ times that of the other. He sold the former at a gain of 15 p. c. and the latter at a loss of 5 per cent. He got Rs. 149 in all by this transaction. What did each horse cost him?

Sol. Suppose he bought the second horse for Re. 1.

$\therefore$ cost price of the first horse = Rs. $\frac{4}{3}$

$\therefore$ cost price of both the horses = $1 + \frac{4}{3}$ = Rs. $\frac{7}{3}$

Now the S. P. of the second horse = Re. $\frac{95}{100} \times 1$ = Re. $\frac{19}{20}$

and „ „ „ first horse = Rs. $\frac{115}{100} \times \frac{4}{3}$ = Rs. $\frac{23}{15}$

$\therefore$ selling price of both the horses = Rs. $\frac{19}{20} + \frac{23}{15}$ = Rs. $\frac{149}{60}$.

Now if the S.P. is Rs. $\frac{149}{60}$, the C.P. = Rs. $\frac{7}{3}$

if „ S.P. is Re. 1 the C. P. = Rs. $\frac{7}{3} \times \frac{60}{149}$

if „ S.P. is Rs. 149 the C. P. = Rs. $\frac{7}{3} \times \frac{60}{149} \times 149$.

= Rs. 140.

Now divide Rs. 140 in the ratio of 4 : 3,

$\therefore$ the cost of the horses = Rs. 80 and Rs. 60. **Ans.**

EXERCISE 125.

1. A person bought two motor cars, each for the same price. He sold one of them at a profit of 10 per cent. and the other for Rs. 480 more than the first. He thus made a profit of $17\frac{1}{2}$ per cent. ; find the price of the motor cars.

2. A person having bought two houses at the same price, sold one of them at a profit of 15 per cent. and for the other he received Rs. 320 more than what he received for the first. He thus got a profit of 18 per cent. on the whole. Find the profit of each house.

3. A merchant bought two horses. The price of one of them exceeds by $\frac{1}{4}$ the price of the other. He sold the horse of the higher price at a loss of $12\frac{1}{2}$ p.c. and the other at a gain of 5 per cent. Thus he received Rs. 428, 12*a*. in all ; find the price of each horse.

4. A person bought two horses. The price of one of them, was $\frac{2}{3}$ of that of the other. By selling the large house at a gain of 5 p. c. and the other at a gain of 12 p. c. he got Rs. 3773 in all. Find the price of each house.

5. I bought 200 books and sold 80 of them at a profit of Rs. 20 and the remaining for Rs. 1000 gaining there by 42 p. c. on the whole ; find the price of a book.

6. A person bought two clocks. The cost price of one of them exceeds by $\frac{1}{4}$ the cost of the other. He sold the better one at a gain of 10 p. c. and the other at a gain of $7\frac{1}{2}$ p. c. and thus got Rs. 98 in all ; find the cost price of each.

REVISION

§8. The examples given in the preceding exercises sum up the whole chapter and now we close it with questions of various types.

EXERCISE 126.

1. A trademan by selling an article for Rs. 35, 1*a*. gained 10 p.c. ; find the cost price.

2. A marchant buys sugar at 7*a*. 6*p*. per seer and sells it at 6*p*. per chhatak. Find the gain per cent.

3. By selling eggs at 3 for an anna I gain 5 per cent. on my outlay; what do I gain or lose per cent. by selling them at 7*a*. a score ?

4. A wholesale dealer sells an article at a gain of 20 per cent. to a reatail dearler, who selling it for Rs. 12. gains 20 per cent.; what did the wholesale dealer pay for it?

5. A tradesman gains 6 per cent. by selling an article for Rs. 795; how much per cent. does he lose by selling the same for Rs. 690?

6. A merchant buys some cloth at such a price that by selling it at Rs. 4, 6*a*. per yard he will gain 5 per cent. on his outlay; what per cent. will he gain or lose, if the cloth be sold at Rs. 3, 14*a*. per yard?

7. I bought 50 maunds of wheat for Rs. 262, 8*a*. and was obliged to sell it at a loss of as much as money as I received for $2\frac{1}{2}$ maunds of wheat. Find the sale price per maund.

8. A person sold his watch for Rs. 50 instead of Rs. 35. Thus he got 45 p.c. more profit; find the cost of a watch.

9. A person bought two watches for Rs. 106. He sold one of them at a profit of 25 p.c. and the other at a loss of $4\frac{1}{6}$ p.c. Both the watches fetched the same price; find the cost price of each watch.

10. Eggs were brought at 4*a*. a dozen and sold at 7*a*. a score; find the number of eggs bought if the gain is Re. 1.

11. By selling goods for Rs. 144, the profit p. c. is equal to the cost price; find the cost price.

12. A person has goods worth £275, he sells $\frac{1}{5}$ of them so as to lose 20 p.c. By how much p.c. should he raise the selling price in order to gain 20 p. c. on the whole?

13. A tradesman was making a profit of 26 p.c, on his outlay when he was selling an article at 1*s*. $3\frac{3}{4}d$. The cost price has since been reduced by a penny and he has taken a penny off the selling price. What percentage of profit is he now making?

14. A grocer sold a quantity of tea at Re. 1, 2*a*. a lb. with a profit of $12\frac{1}{2}$ p.c. and a total gain of Rs. 6. Find how many lbs. of tea did he sell.

15. How much should I have to pay for a watch which cost £3, 4*s*. to make, if the marker sold it to a dealer at a profit of 25 p.c. and the dealer sold it to me at a profit of $12\frac{1}{2}$ p.c.?

16, If a reduction of 20 p.c. were made in the price of eggs, it will enable a person to obtain 54 more for 21 shillings. Find the present price.

17. A trader allows a discount of 5 p. c. for cash payment; how much p.c. above the cost price must be mark his goods to make a profit of 10 p.c.

18. A coal merchant had 150 tons of coal, of which he sold 50 tons at Rs. 27 per ton and found that he was gaining $12\frac{1}{2}$ p. c. At what rate must he sell the remainder so that he may gain 10 p. c. on the whole ?

19. A man buys 27 sheep for Rs. 90 and sells 12 at a loss of 5 p. c.; at how much price must he sell the remainder, in order that he may gain $2\frac{1}{2}$ p. c. on the whole ?

20. Find the cost price of rice when an additional profit of $2\frac{1}{2}$ p. c. raises its price by 1*a*. 7*p*. per maund.

21. A man bought a number of oranges at 3 for an anna and an equal number at 2 for an anna. At what price per dozen should he sell them to make a profit of 20 p. c.?

22. By selling 5 yards of cloth for Re. 1, a man gains a profit of 15 p. c.; what will be the gain or loss p. c. if he sells the same cloth at 6 yards for Re. 1 ?

23. A quantity of tea is sold at Re. 1. 2*a*. per lb. at a profit of $12\frac{1}{2}$ p. c. and a total gain of Rs. 10. Find how many pounds of tea were sold.

24. An article when sold at a gain of 20 p. c. yields Rs. 7, 8*a*. more than when sold at a loss of 4 p. c. find its prime cost.

25. A piece of cloth is sold for Rs. 54 at a profit of $12\frac{1}{2}$ p. c. If it had been sold at Rs. 2, 6*a*. a yard, the profit would have been Rs. 9; find how many yards are there in the piece.

26. A merchant sells some goods to a retailer at 50 p. c. profit; but the latter failing he only receives 8*a*. a rupee; find the merchant's gain or loss per cent.

27. A tradesman by means of a false balance defrauds 10 p. c. in buying goods and 10 p. c. in selling. What p. c. does he gain on his outlay by his dishonesty ?

28. A merchant having purchased two horses for Rs. 1350, sold one at a gain of $7\frac{1}{2}$ p c. and the other at a loss of 6 p. c.; he found that he had neither gained nor lost in this transaction. Find the cost price of each horse.

29. A horse was sold at a loss of 10 p. c.; if it had been sold for Rs. 35 more, 4 p. c. would have been gained. Find the cost price.

30. A man sells two horses for Rs. 1955 each. On one he gains 15 p. c. and on the other he losses 15 p. c. Find his total gain or loss.

31. A merchant sells sugar to a customer, using false weight and thereby gains $11\frac{1}{9}\%$ on his outlay. What weight does he substitute for a seer ?

32. By selling a watch at a gain of 10 per cent. a man got Rs. 15 more than half its price. What is the price of the watch ?

33. By selling 4 dozen of mangoes for Rs. 13, it was found that $\frac{3}{10}$ths of the outlay was gained; what ought to have been the retail price per mango in order to have gained 60 per cent. ?

34. Two articles are sold for Rs. 396 each, one at a gain of 10 p. c. and the other at a loss of 10 per cent. Find the gain or loss per cent. on the whole.

35. A person bought some slates at 3*a*. per slate. Out of them 3 dozen were found broken and the remaining he sold at 4*a*. per slate and thus cleared a profit of Rs. 3. How many slates did he buy ?

36. I bought two horses for Rs. 385 and sold one of them at a gain of $12\frac{1}{2}$ p. c. and the other at a loss of $13\frac{1}{3}$ p. c. I neither gained nor lost in this transaction; find the cost price of each ?

37. A grocer bought 30 lbs. of tea at 12*a*. per lb. and 50 lbs. at Re. 1, 2*a*. per lb. At what rate per lb. must he sell the whole to gain at least 10 per cent. of the returns ?

38. By selling a horse and a cow for Rs. 310, a person gained 15 p.c. on the horse and 10% on the cow. Had he sold the cow at a gain of 15% and the horse at a gain of 10% he would have realized Rs. 308, 12*a*. Find the cost price of each.

39. Rs. 61, 4*a*. was spent in buying apples at Re. 1, 7*a*. 4*p*. a score. When they came to be sold, part of them was worthless; but the rest on being sold at a profit of 30 p. c. realized Rs. 68, 4*a*. ; how many scores were there of worthless ones ?

40. If I sell a horse for Rs. 132 and a cow for Rs. 88, I gain 10 p.c. on the whole. But if I sell the horse for Rs. 150, and the cow on the original price, I gian 15 p.c. Find the original cost price of each.

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CHAPTER XXIII

PROPORTIONAL PARTS AND PARTNERSHIP.

1. PROPORTIONAL PARTS

§ 1. To divide a given quantity into *proportional parts* is to divide into parts proportional to the given numbers.

Note. Again means a second time. As much again means as much once and as much a second time i.e., twice as much. Half as much again means as much once and half as much a second time, i.e., $1\frac{1}{2}$ times as much.

The following examples will illustrate the process :—

Example 1. *Divide Rs. 356, 4a. among A, B and C so that their share may be proportional to 3, 4 and 5.*

Sol. The sum of the parts $=3+4+5=12$.

Therefore the whole sum Rs. 356, 4*a*. is to be divided into 12 equal parts of which *A* shall have 3 parts, *B* 4 and *C* 5.

$\because$ 12 parts = Rs. 356, 4*a*.

$\therefore$ 1 part = Rs. 29, 11*a*.

$\therefore$ *A*'s share = Rs. 29, 11*a*. $\times 3$ = Rs. 89, 1*a*.
B's share = Rs. 29, 11*a*. $\times 4$ = Rs. 118, 12*a*. } **Ans.**
C's share = Rs. 29, 11*a*. $\times 5$ = Rs. 148, 7*a*.

Example 2. Divide Rs. 279 among *A*, *B*, *C* in the ratio of $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ respectively.

Sol. $A : B : C$
$= \frac{1}{2} : \frac{1}{3} : \frac{1}{5}$ } Multiply each term by 30, the L. C. M. of
or $15 : 10 : 6$ } the denominators 2, 3, 5.

$15+10+6$, *i.e.*, 31 parts = Rs. 279 or 1 part = Rs. 9

$\therefore$ *A*'s share = Rs. 9×15 = Rs. 135.
B's share = Rs. 9×10 = Rs. 90. } **Ans.**
C's share = Rs. 9×6 = Rs. 54.

Example 3. Divide Rs. 420 among *A*, *B* and *C* so that *A* may receive twice as much as *B* and *B* twice as much as *C*.

Sol. Let *C*'s share be = 1.

$\therefore$ *B*'s share = 2 [since *B* gets twice as much as *C*]
and *A*'s share = 4 [since *A* gets twice as much as *B*]

hence the sum of the parts $=1+2+4=7$.

$\because$ 7 parts = Rs 420

$\therefore$ 1 part = Rs. 60.

$\therefore$ A's share $=$Rs. $60\times4=$Rs. 240.
B's share $=$Rs. $60\times2=$Rs. 120.
C's share $=$Rs. $60\times1=$Rs. 60. } **Ans.**

Example 4. Divide Rs. 381, *3a.* among A, B and C so that A's share : B's share is as 5 : 6 and B's share : C's share as 3 : 4.

Sol. We can put the conditions given above as under :—

$A : B : C$
5 : 6
3 : 4

Now find out either C's share or A's share by the unitary method.

If B gets 3, C gets 4,
If B gets 1, C gets $\frac{4}{3}$,
If B gets 6, C gets $\frac{4}{3}\times6=8$.
Hence $A : B : C=5 : 6 : 8$.
Now proceed as in previous examples.

Example 5. Divide Rs. 375 among A, B and C so that if Rs. 4, Rs. 5, Rs. 6 be subtracted from their respective shares, the remainder may be in the ratio of 3 : 4 : 5.

Sol. $4+5+6=15$; Rs. $375-$Rs. $15=$Rs. 360.
Now divide Rs. 360 in the ratio 3 : 4 : 5 and then add Rs. 4, 5, 6, respectively to the shares.

Example 6 Divide Rs. 1680 among A, B and C so that A may receive $\frac{1}{5}$ as much as B and C together and B $\frac{2}{3}$ as much as A and C together.

Sol. *First condition*
If B and C get Rs. 1,
A gets Re. $\frac{1}{5}$.
Total in this case $=1\frac{1}{5}=\frac{6}{5}$

Second condition
If A and C get Re. 1,
B gets Re. $\frac{2}{3}$.
Total in this case $=1\frac{2}{3}=\frac{5}{3}$.

Now find the share of A in terms of the second condition.
If the total is $\frac{6}{5}$, A gets $\frac{1}{5}$
" " is 1, A gets $\frac{1}{5}\times\frac{5}{6}=\frac{1}{6}$
" " is $\frac{5}{3}$, A gets $\frac{1}{6}\times\frac{5}{3}=\frac{5}{18}$
$\therefore$ C will get $1-\frac{5}{18}=\frac{13}{18}$.
The proportional parts are $\frac{5}{18}$, $\frac{2}{3}$ and $\frac{13}{18}$,
or 5, 12 and 13.
Now the sum of the parts $=5+12+13=30$.
$\because$ 30 parts $=$Rs. 1680
$\therefore$ 1 part $=$Rs. 56.

$\therefore$ A's share = Rs. 56×5 = Rs. 280
B's hare = Rs. 56×12 = Rs. 672
C's share = Rs. 56×13 = Rs. 728 } **Ans.**

Aliter. A's share : ($B+C$)'s share = 1 : 5
B's share : ($A+C$)'s share = 2 ; 3
Now divide Rs. 1680 (*i*) in the ratio 1 : 5
(*ii*) in the ratio 2 ; 3
$\therefore$ A's share = $\frac{1}{6}$ of Rs. 1680 = Rs. 280.
B's share = $\frac{2}{5}$ of Rs. 1680 = Rs. 672. } **Ans.**
$\therefore$ C's share = 1680 − (Rs. 280 + Rs. 672) = Rs. 728

Example 7. Divide 184 into three parts such that 4 times the first = 5 times the second = 8 times the third.

Sol. Let 4 times the first = 5 times the second = 8 times the third = 1, then
first part = $\frac{1}{4}$, 2nd part $\frac{1}{5}$ and third part = $\frac{1}{8}$
$\therefore$ first part : 2nd part : 3rd part = $\frac{1}{4} : \frac{1}{5} : \frac{1}{8}$
= 10 : 8 : 5.
Now proceed as in previous examples.

Example 8. Divide Rs. 82 among 5 men, 8 women and 10 boys in such a way that a woman is to receive twice as much as a boy and a man as much as a woman and a boy together; what do the women receive ?

Sol, Since one woman receives as much as 2 boys.
$\therefore$ 8 women receive as much as 16 boys.
Again one man receives as much as one woman and one boy,
$\therefore$ 5 men receive as much as 5 women and 5 boys,
i.e., as much as 10 boys and 5 boys,
i.e., as much as 15 boys
$\because$ 5 men = 15 boys and 8 women = 16 boys
$\therefore$ men's share : women's share : boy's share = 15 : 16 : 10
Now proceed as in previous examples.

Example 9. 685 coins consist of guineas, half-sovereigns and half crowns : the *values* of guineas, half-sovereigns and half-crowns are in the ratio of 15 : 8 : 5 ; find the number of each coin.

Sol. Ratio in the values = 15 guineas : half-sovereigns worth 8 guineas : half-crowns worth 6 guineas.

$\therefore$ No of coins $= 15 : \frac{8\times21}{10} : \frac{6\times21}{2\frac{1}{2}}$

$= 15 : \frac{84}{5} : \frac{252}{5}$

$= 75 : 84 : 252$ or $25 : 28 : 84$.

Now $25+28+84=137$ and $685\div137=5$

$\therefore$ guineas $= 5\times25=125$
half-sovereigns $= 5\times28=140$ } **Ans.**
half-crowns $= 5\times84=420$

Example 10. How many eight anna pieces, four anna pieces and two anna pieces the *numbers* of which are in the ratio of 3 : 5 : 4 are together worth Rs. 156 ?

Sol. *Values* of three groups of coins are as
3 eight anna pieces : 5 four anna pieces : 4 two-anna pieces or as 12 two anna pieces : 10 two-anna pieces : 4 two-anna pieces or as 12 : 10 : 4 or 6 : 5 : 2, $\because$ $6+5+2=13$

$\therefore$ the amount in 8 anna bits $=$ Rs. $\frac{156}{13}\times6=$ Rs. 72
the amount in 4 anna bits $=$ Rs. $\frac{156}{13}\times5=$ Rs. 60
the amount in 2 anna bits $=$ Rs. $\frac{156}{13}\times2=$ Rs. 24

Therefore there are $72\times2=144$ eight-anna, $60\times4=240$ four-anna and $24\times8=192$ two-anna pieces. **Ans.**

Example 11. Total salary of A, B and C is Rs. 350 ; if they spend 75%, 80% and 56% of their salaries respectively, their savings are as 10 : 12 : 33 ; find their respective salaries.

Sol. A's saving $= 100-75=25\%$ of his salary
B's saving $= 100-80=20\%$,, ,, ,,
C's saving $= 100-56=44\%$,, ,, ,,

$\therefore \frac{25}{100}$ of A's salary : $\frac{20}{100}$ of B's : $\frac{44}{100}$ of C's $= 10 : 12 : 33$
i.e., (1) $\frac{1}{4}$ of A's salary : $\frac{1}{5}$ of B's salary $= 10 : 12$
and (2) $\frac{1}{5}$ of B's salary : $\frac{11}{25}$ of C's salary $= 12 : 33$.

From (1) we get $\frac{1}{4}\times12$ of A's salary $=\frac{1}{5}\times10$ of B's salary
or 3 times A's salary $=2$ times of B's salary
$\therefore$ A's salary : B's salary $= 2 : 3$.

From (2) we get $\frac{1}{5}\times33$ of B's salary $=\frac{11}{25}\times12$ of C's salary
or $\frac{33}{5}$ times of B's salary $=\frac{132}{25}$ times of C's salary
$\therefore$ B's salary : C's salary $=\frac{132}{25} : \frac{33}{5}$
or $132\times5 : 33\times25$ or $4 : 5$.

The ratios are therefore as follows :—

$A : B : C$
$2 : 3$
$4 : 5$

We can now proceed by the method already explained in Ex. 4.

Example 12. A's present age is to be B's as 8 : 5; and 20 years ago it was as 12 : 5. Find the present age of each.

Sol. A's present age to B's present age is as 8 : 5 ; and 8 is $2\frac{2}{3}$ times (8—5). Similarly.

A's former age was to B's former age as 12 : 5 ; here 12 is $1\frac{5}{7}$ times (12—5).

It follows therefore that
A's present age $=2\frac{2}{3}$ (A's—B's present ages)
and A's former age$=1\frac{5}{7}$ (A's—B's former ages)

Since the differance of the ages of two persons is always the same though the ratio of the ages is always varying.

$\therefore A$'s present age : A's former age$=2\frac{2}{3} : 1\frac{5}{7}$
or $2\frac{2}{3}$ times A's former age$=1\frac{5}{7}$ times A's present age
or A's former age $=(1\frac{5}{7} \div 2\frac{2}{3})$ or $\frac{9}{14}$ of his present age
but A's former age$=A$'s present age-20 years
$\therefore A$'s present age-20 years$=\frac{9}{14}$ of A's present age
$\therefore (1-\frac{9}{14})$ of A's present age$=20$ years
or $\frac{5}{14}$ of A's present age $=20$ years
$\therefore A$'s present age $=20\times\frac{14}{5}=56$ yrs.
$\because A$'s age : B's age$=8 : 5$
$\therefore B$'s present age $=\frac{5}{8}$ of $56=35$ yrs.
} **Ans.**

Example 13. Four years ago the ages of A and B were as 13 : 9 and eight years hence they will be as 4 : 3 ; find their present ages.

Sol. In $4+8$ *i.e.*, 12 years the ratio would be changed from 13 : 9 to 4 : 3. Note $13-9=4$ and $4-3=1$.

Now reduce the second ratio to its equivalent ratio so that the difference between the terms is the same as that between the terms of the first, *i.e.*, 4.

$\therefore 4 : 3 = 16 : 12.$

If A's age 4 years before had been 13 years it would have taken only 16—13, *i.e.* 3 years to become 16 years.

$\therefore$ 3 : 12 :: 13 : A's age 4 years ago.

or A's age 4 years ago $= \frac{12 \times 13}{3} = 52$ years.

$\therefore$ A's age $= 52 + 4 = 56$ years.
and B's age $= \frac{2}{13} \times 52 + 4 = 40$ years. } Ans.

EXERCISE 127.

1. Divide Rs. 526, 8*a*. into parts proportional to 2, 3 and 7.

2. Divide Rs. 487, 8*a*. into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

3. Divide Rs. 625, 9*a*. into 4 parts in the proportion of $\frac{1}{4}$, $1\frac{1}{2}$, ·75, ·625.

4. The magnitude of the three angles of a triangle are in the ratio of 3 : 5 : 1. Find the number of degrees in each angle.

5. A number is divided into 3 parts in the proportion of 3, 5 and 7, if the first part is equal to 24, find the number.

6. A sum of money was divided into parts proportional to 2·1, 2·5 and 3·2 ; the smallest part was Rs. 10, 8*a*. ; what was the sum divided ?

7. Divide Rs. 738 among A, B and C so that if their shares be diminished by Rs. 7, Rs. 5 and Rs. 6 respectively. the remainder may be in the ratio of 5 : 6 : 7.

8. Divide Rs. 586 among A, B and C so that if Rs. 15, Rs. 20 and Rs. 49 respectively be added to their shares the total may be in the ratio of 4 : 5 : 7.

9. Divide Rs. 517 among A, B and C so that A's share : B's share is equal to 4 : 5 and B's share : C's share is equal to 3 : 4.

10. Divide Rs. 126, 6*a*. among A, B and C so that A may have $1\frac{1}{2}$ of B's share and B double of C's share.

11. Divide Rs. 450 among three persons so that first man's share : second man's share = 4 : 5 and second man's share : third man's share = 5 : 6.

12. Divide Rs. 252, 8*a*. among *A*, *B* and *C* so that *A* may get twice as much as *B* and *B* thrice as much as *C*.

13. Divide £250 among *A*, *B* and *C* so that *A*'s share : *B*'s share $= 1 : \frac{3}{2}$ and *B*'s share : *C*'s share $= \frac{3}{4} : \frac{5}{6}$.

14. Divide Rs. 1125 among *A*, *B*, *C* and *D* so that *A* may receive twice as much as *B*, thrice as much as *C* and 4 times as much as *D*.

15. Rs. 350 is divided among *A*, *B* and *C*. *B*'s share is equal to *A*'s share and Rs. 40 more; *C*'s share is equal to *A*'s share and Rs. 70 more; find each one's share.

16. Rs. 625 is to be divided among *A*, *B* and *C*; if *A* receives Rs. 130 less than *C* and *C* receives Rs. 25 less than *B*, how much will each receive ?

17. Divide Rs. 395 among *A*, *B* and *C* so that *B* may get 25 per cent. more than *A* and 20 per cent. more than *C*.

18. Divide Rs. 1525 among *A*, *B* and *C* so that *B* may get 20 per cent. less than *A* and *C* may get 20 per cent. less than *B*.

19. The sum of the ages of three men is 108 years. Twelve years ago their ages were in the proportion 5 : 4 : 3; find their ages.

20. Divide a guinea between *A*, *B*, *C* and *D* so that *B*'s share is $\frac{1}{3}$ more than *A*'s, *C*'s $\frac{1}{3}$ more than *B*'s and *D*'s $\frac{1}{3}$ more than *C*'s.

21. Divide Rs. 551 in three parts so that 4 times the first, five times the second and twice the third part may all be equal.

22. A purse contains rupees, eight anna pieces and four-anna pieces; their *numbers* are in the proportion of 2, 3 and 4; find the numbers of each coin if the amount of money in the purse be Rs. 81.

23. How many sovereigns, crowns and florins whose *numbers* are proportional to 2·5, 3 and 4 are together worth £365 ?

24. 10880 coins consist of pounds, shillings and pence; the *values* of pounds, shillings and pence are as 4 : 3 : 2 ; find the number of each coin.

25. Divide Rs. 2440 among A, B, C and D so that B's share may be $\frac{5}{9}$ of A's share and C's share $\frac{7}{10}$ of B's share and D's share $\frac{1}{3}$ of B's and C's together.

26. Divide Rs. 6270 among A, B and C so that A shall receive $\frac{3}{7}$ of as much as B and C together and B shall receive $\frac{2}{5}$ of as much as A and C together.

27. Divide Rs. 3080 among A, B and C so that A shall get $\frac{2}{9}$ of as much as B and C together and B shall get $\frac{3}{11}$ of as much as A and C together.

28. Divide Rs. 12540 among A, B and C so that A may receive $\frac{3}{7}$ of as much as B and C together and B may receive $\frac{2}{5}$ of as much as A and C together.

29. The sum of £177 is to be divided among 15 men, 20 women and 30 children in such a manner that a man and a child may together receive as much as two women and all the women may together receive £60. What will they repectively receive ?

30. The sum of Rs. 2840, 1*a*. is to be divided between 7 men 11 women, 5 boys and 6 girls, so that for every Rs. 3, 12*a*. a man receives, a woman may get Rs. 2, 3*a*. and for every Rs.2, 10*a*. a woman receives, a boy may get Re. 1, 14*a*. and a girl Re. 1, 2. Find how much each person receives.

31. A five-storeyed building braught in a rent of Rs. 1250. The ratios of the rents yielded by the successive storeys, *i. e.*, the first to the second; the second to the third and so on are 3 : 2 ; 1 : 2 ; 16 : 9 ; 9 : 5. Find what rent each storey yielded.

32. The total of the salaries of A, B, C is Rs. 444 : if they spend 80 p. c., 85 p.c., 75 p. c., of their salaries respectively, their savings are as 7 : 6 : 9; find their respective salaries.

33. A, B, C's total income is Rs. 1440. If they spend 80%, 85 p. c.. 75 p. c. of their incomes, their savings are as 8 : 9: 20, find how much each earns.

34. Divide Rs. 950 among A, B, C so that Rs. 70 more than $\frac{4}{5}$ of A's share, Rs. 45 more than $\frac{3}{4}$ of B's share and Rs. 30 more than $\frac{2}{3}$ of C's share may be all equal.

35. A person died leaving Rs. 3800 with directions to his pragnant wife that she should keep Rs. 1520 for herself and

give Rs. 2280 to the child if a son is born and in case a daughter is born to give Rs. 1520 to her and keep Rs. 2280 for herself. But as God would have it, twins, *i.e.*, a daughter and a son, were born. How is the money to be divided now ?

36. *A*'s present age is to *B*'s as 10 : 9 and 12 years ago it was as 7 : 6. Find the present age of each.

37. *A*'s present age is to *B*'s as 13 : 10 and 20 years ago it was as 8 : 5. Find the present age of each.

38. Five years ago the age of *A* and *B* were as 4 : 3 and ten years hence they will be as 11 : 9. Find their present ages.

39. Six years ago the ages of *A* and *B* were as 10 : 7 and nine years hence they will be as 13 : 10. Find their present ages.

40. Divide 312 into three parts such that if they are divided by 5, 6, 7 respectively, the quotients shall be in the proportion of 4, 3, 2.

II. PARTNERSHIP.

§2. When some persons begin to trade together with a joint stock and the gain or loss is ascertained in proportion to the money contributed by each person, this system of business is called **partnership**.

partnership is of two kinds—*simple and compound.*

Simple partnership. When the capital contributed by each partner is supposed to remain for the same period of time, it is called simple partnership.

Compound partnership. When it remains for different times it is called compound partnership.

We give below examples of each kind :—

Example 1. *A*, *B* and *C* are partners in a business. *A* contributed Rs. 1200, *B* Rs. 900 and *C* Rs. 800 and they gain Rs. 580 ; how should the profit be divided. [Simple partnership.]

Sol. We shall proceed as in Art. 1 of this Chapter. *i.e.*, Rs. 580 will be divided in the ratio of 1200 : 900 : 800 or 12 : 9 : 8.

Example 2. Three men *A*, *B* and *C* enter into partnership. *A* puts in £300 for 4 months, *B* puts in £400 for 5 months and *C* puts in £200 for 8 months. They gain £240. what should each man receive as his share of gain ? [Compound partnership].

Sol. £300 for 4 months will produce the same gain as £300×4 or £1200 in one month. Likewise £400 for 5 months and £200 for 8 months will yield the same gain as £2000 and £1600 for one month, respectively.

Now proceed as in simple parrnership.

The profit will be divided in the ratios 1200 : 2000 : 1600
or 3 : 5 : 4

Now 3+5+4=12.

$\therefore$ *A*'s share=$\frac{3}{12}$ of £240=£ 60
B's share=$\frac{5}{12}$ of £240=£100 } **Ans.**
C's share=$\frac{4}{12}$ of £240=£ 80

Example 3. *A* and *B* rent a field for Rs. 60 ; *A* puts in 50 cows for 6 months and *B* puts in 40 cows for 5 months ; how much rhould each pay for the rent ?

Sol. 50 cows grazing for 6 months must eat as much as 50×6 or 300 cows in one month; also 40 cows for 5 months must eat as much as 40×5 or 200 cows in one month.

$\therefore$ the rent must be divided proportional to 300 and 200 or 3 and 2.

$\because$ 3+2=5 ;

$\therefore$ *A* must pay $\frac{3}{5}$ of Rs. 60=Rs. 36
B must pay $\frac{2}{5}$ of Rs. 60=Rs. 24 } **Ans.**

EXERCISE 128.

1. Two persons enter into patnership; one contributes Rs. 900 and the other Rs. 1200 and they gain Rs. 280; what is each person's share of the profit ?

2. *A*, *B* and *C* enter into partnership. *A* puts in Rs. 2,000; *B* Rs. 5000 and *C* Rs. 11000; what would be the share of each in Rs. 2179, 2*a*. profit.

3. *A*, *B* and *C* form a joint stock of Rs. 75000, of which Rs. 36000 are contributed by *A*, Rs. 30000 by *B* and the remainder by *C*. At the end of the year, the profit is found to be Rs. 16750; find the share of each, Rs 800 a month being allowed as salary to *C* as acting partner;

4. *A* and *B* are partners in a business: *A* puts in Rs, 833, 5*a*. 4*p*. and *B* Rs. 966, 10*a*. 8*p*. At the and of the year the profit is found to be Rs. 300 only; find the share of each, Rs. 2, 8*a*. a month being allowed as an allowance to *A* as acting partner.

5. *A* contributes a certain capital for 4 months and *B* contributes Rs. 400 for 5 months. If their profits be in the ratio of 3 : 4, how much did *A* contribute ?

6. Four merchants trading with a capital of Rs. 11900 find after a year their respective shares increased by Rs. 265, 13*a*., 4*p*., Rs. 372, 2*a*. 8*p*., Rs. 531, 10*a*. 8*p*. and Rs. 638. Find how much they subscribed to the original capital.

7. *A* starts business with a capital of Rs. 4,000; after 3 months he admits a partner *B* with a capital of Rs. 6000 and after further 4 months another partner *C* was admitted with a capit al of Rs. 9000. At the end of a year after *A* started the business the profit was found to be Rs. 2450. What is each person's share of the profit ?

8. *A*, *B* and *C* entered into a partnership, *A* advanced Rs. 1200, *B* Rs. 1500 and *C* Rs. 2000. After 2 months *C* withdrew Rs. 200. and after 5 months *B* put in Rs. 300 more. At the end of a year, the profits of the concern were Rs. 1130. Find each one's share of the profit?

9 *A* and *B* advanced Rs. 1500 and Rs. 1800 respectively in a business and took Rs. 2000 from the Punjab National Bank on the condition that they must pay Rs. 120 a year to the bank as interest ; at the end of a year the profit is found to be Rs. 1110; find the share of each in the profit.

10. *A* started a business with a capital of Rs. 2100, after 4 months he. admitted another partner *B*. What amount should he put in so that the profit may be divided equally at the end of the year ?

11. *A*,*B* and *C* are partners in a business and their shares are in the proportion of $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$. *A* withdraws half his capital at the end of 4 months and after 8 months more a profit of Rs. 2024 is declared. What is *A*'s share ?

12. *A* employs Rs. 2000 in a business for 8 months, *B* who joins later keeps his capital for 6 months and receives Rs. 150 out of a total profit of Rs. 400. How much does *B* subscribe to the business ?

13. *A* and *B* are partners in a business in which *A* has Rs. 4900 and *B* Rs. 1400; *B* is the working partner and hence receives 6% of the total profit. the rest being divided in proportion to the capital. If the profits are Rs. 450, what does each receive ?

14. *A* starts a business with a capital of Rs. 1500 and admits two partners *B* and *C* after 3 months and 6 months respectively. After a year the profits are divided in the ratio of 5 : 4 : 3 . What amount did *B* and *C* each contribute ?

15. *A* starts a business with a capital of Rs. 1700 and admits two partners *B* and *C* after 3 months and 6 months respectively. After a year the profits are divided in the ratio of 2 : 3 : 5. What amount of money did *B* and *C* each contribute ?

16. *A* is a working and *B* is a sleeping partner in a business. *A* possesses £ 1200 of the capital. *B* £ 2000. *A* receives 10% of all profits for managing and the rest is proportionally divided. The total profits being £ 800, find the share of each.

17. *A* is a working and *B* a sleeping partner in a business. The capital of *A* is £ 2400 and that of *B* £ 4000. At the end of one year, the profit amounted to £ 1600. Being the manager, *A* receives 10% of the profit. How should the remainder of the profit be divided ?

18. Kirpal Singh and Nihal Singh hire a meadow. Kirpal Singh puts in 120 cows and Nihal Singh 150. After 3 months Nihal Singh sold 100 of his cows. If the rent of the meadow be Rs. 585 for a year, how much of the rent will each pay ?

19. Two persons rented some fields for Rs. 2430 for 10 months. One of them put in 27 oxen for 3 months, and the other 270 sheep for 7 months. If 3 oxen eat as much as 11 sheep find how much of the rent each ought to pay.

20. *A* and *B* are partners in a business in which *A* subscribed Rs. 5000 and *B* Rs. 7500. The gross receipts for a year are Rs. 3000 ; $\frac{1}{8}$ of this is spent in salaries, Rs. 30 in insurance of the premises. *A* is to receive 8 p. c. on his capital and *B* 4% on his ; the remainder of the profits is to be divided proportionally to the capital. Find the net rent receipts of *A* and *B*.

21. *A* and *B* began business with capitals as 4 : 7 ; at the end of 7 months *A* withdraws; if they received profits in the ratio of 2 : 5, how long was *B*'s capital in the business ?

CHAPTER XXIV

MISCELLANEOUS PROPOSITIONS

1. ALLIGATION OR MIXTURE.

§1. **Alligation** is the method of mixing two or more things of the same kind but of different quantities. It consists of two kinds—*medial and alternate.*

§2. **Alligation Medial** is that in which the quantities and prices of several things which compose the mixture are given and we are to find the mean price of that mixture.

Example 1. A grocer mixes 3 lb. of tea at 10*a.* per lb., 4lb. of tea at 8*a.* per lb., 5 lb. of tea at 9*a.* per lb. and 6 lb. of tea at Re. 1, 2*a.* 2*p.* per lb., find the price of the mixture per lb.

Sol. Price of 3 lb. at 10*a.* per lb. $=30a.$
" 4 lb. at 8*a.* per lb. $=32a.$
" 5 lb. at 9*a.* per lb. $=45a.$
" 6 lb. at Re. 1, 2*a.* 2*p.* per lb. $=109a.$

$\therefore$ price of 18 lb.$=216a.$
$\therefore$ price of 1 lb.$=216\div 18=12a.$ **Ans.**

Rule. *Multiply the number of each quantity by the value of a unit of the same quantity and divide the sum of the products by the sum of the numbers.*

§3. **Alligation alternate** is that in which the prices of several things are given and we are to find in which ratio those things are to be mixed so that the mixture may be of a given price.

Two ingredients.

Example 2. How must a grocer mix teas at Re. 1, 2*a.* a lb. and Re. 1, 7*a.* a lb. so as to make a mixture worth Re. 1, 5*a.* a lb.?

Sol Rs. 1, 2*a.*$=18a.$, Re. 1, 7*a.*$=23a.$, Re. 1, 5*a.*$=21a.$

To make the mixture worth 21*a.* per lb. the grocer will gain $21-18=3a.$ per lb. in the first case and will lose $23-21=2a.$ per lb. in the second case. It is clear, therefore, that the

gain in using the cheaper tea must be equal to the loss in using the dearer one. Thus

No. of lbs. of cheaper tea $\times 3a.$ = No. of lbs. of dearer tea $\times 2a.$

$$\therefore \frac{\text{No. of lbs. of cheaper tea}}{\text{No. of lbs. of dearer tea}} = \frac{2}{3}$$

Hence the tea must be mixed in the ratio of 2 : 3, that is to say, *in the inverse ratio of the difference of the two prices and the mean price.*

We can now arrange the method of *B* and thus:—

$21-18=3a.$ gain
$23-21=2a.$ loss
Ratio in the gain and loss = 3 : 2
∴ tea must be mixed
in the inverse ratio 2 : 3. **Ans.**

	21	
18		23
23−21 :	21−18	
=	2 : 3 **Ans.**	

Rule. *Two ingredients are mixed in the inverse ratio of the differences of the two prices and the mean price.*

Three ingredients.

Example 3. In what proportion should teas at 9*a.*, 10*a.* and 13*a.* a lb. be mixed to make a mixture worth 12*a* a lb. ?

Note. When two ingredients are mixed the mean price of the mixture is always greater than the price of one ingredient and less than the price of the other. Assumiug this principle, let us find out which number out of 9, 10, 13 can be linked with another number when the mean price is supposed to be 12a. in all cases.

Sol. (*i*) First and third pairs can be linked because one is less than 12 and the other is greater than 12.

(*ii*) Second and 3rd, pairs can also be linked because the principle stated above holds good for this pair also.

In the (*i*) case the ratio of the profit and loss = 3 : 1
∴ the ratio in the quantities = 1 : 3
In the (*ii*) case the ratio of the profit and loss = 2 : 1
∴ the ratio in the quantities = 1 : 2.
Now put these ratios as under and then add:—

(1)	(2)	(3)
1		3
	1	2
1	1	5 **Ans.**

Test of correctness. Price of 1 lb. at 9*a*. = 9*a*.
„ 1 lb. at 10*a*. = 10*a*.
„ 5 lb. at 13*a*. = 65*a*.
∴ „ 7 lb. of mixture = 84*a*.
∴ „ 1 lb. „ „ = 12*a*.

Note. We may note that after having obtained one answer, we can obtain as many more as we please by multiplying or dividing each of the quantities found by 2, 3, 4, 5, etc. For if two ingredients when mixed together make loss and gain equal, their halves, third parts or doubles, triples, etc., when mixed together will make no difference.

Four ingredients.

Example 4. To mix water with spirits worth Rs. 7, Rs. 5, Rs. 2 per gallon respectively making a mixture worth Rs. 4 per gallon, how much quantity of each is to be taken ?

Note. Water having no price 0 must be put in its place and considered as the first quantity.

Sol. The following pairs can be linked:—
(*i*) first and 2nd, *i. e.*, 0 and 7
(*ii*) first and 3rd, *i. e.*, 0 and 5
(*iii*) second and fourth, *i. e.*, 7 and 2
(*iv*) third and fourth, *i. e.*, 5 and 2

In the (*i*) case the ratio of the profit and loss = 4 : 3
∴ the ratio of the quantities = 3 : 4.....(1)
In the (*ii*) case the ratio of the profit and loss = 4 : 1
∴ the ratio of the quantities = 1 : 4.....(2)
In the (*iii*) case the ratio of the loss and profit = 3 : 2
∴ the ratio of the quantities = 2 : 3.....(3)
In the (*iv*) case the ratio of the loss and profit = 1 : 2
∴ the ratio of the quantities = 2 : 1.....(4)
Now put these raios as under and then add:

(1)	(2)	(3)	(4)
3.....	4		
1.....		4	
	2.....		3
		2.....	1
4 :	6 :	6 :	4

Halves of the quantities found above can be taken. (See Note above)

$\therefore$ 2 gallons of water
2 gallons at Rs. 2 per gallon
3 gallons at Rs. 5 ,,
3 gallons at Rs. 7 : ,,
} Ans.

The student can verify this result as in **Ex. 4.**

§4. **An important result.**

If a vessel contains x gallons of wine and if y gallons are drawn and replaced by water, then if y gallons of the mixture are drawn and replaced by water and if this operation is repeated n times in all, then

$$\frac{\text{Wine left in the vessel after } n\text{th operation}}{\text{Whole quantity of wine in vessel}} = \left(\frac{x-y}{x}\right)^n.$$

Proof. After the first operation $x-y$ gallons of wine are left in the vessel, *i. e.*

$$\text{Quantity of wine after the Ist operation} = \frac{x-y}{x} \text{ of whole}$$

$$\text{,, ,, ,, 2nd ,,} = \left(\frac{x-y}{x}\right)^2 \text{ ,, ,,}$$

i. e., $\dfrac{\text{wine left in the vessel after 2nd operation}}{\text{whole quantity of wine}} = \left(\dfrac{x-y}{x}\right)^2$, ,,

and hence after n operations this ratio will be $= \left(\dfrac{x-y}{x}\right)^n$.

Example 5. Four gallons are drawn from a cask full of wine ; it is then filled with water. Four gallons of the mixture are drawn and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 36 : 13. How much does the cask hold ?

Sol. Here, $\dfrac{\text{wine left}}{\text{water}} = \dfrac{36}{13}$.

$$\therefore \frac{\text{wine}}{\text{wine left} + \text{water}} = \frac{36}{36+13} = \frac{36}{49}$$

$\therefore$ by the result proved above

$$\left(\frac{\text{total quantity of wine} - 4}{\text{total quantity of wine}}\right)^2 = \frac{36}{49} = \left(\frac{6}{7}\right)^2$$

$$\therefore \frac{\text{total quantity of wine} - 4}{\text{total quantity of wine}} = \frac{6}{7} = \frac{7-1}{7} = \frac{28-4}{28}$$

i. e., total quantity of wine $= 28$ gallons. Ans.

EXERCISE 129.

1. How must milk at 3*a*. per seer be mixed with milk at 5*a*. per seer, to make a mixture worth 3*a*. 6*p*. per seer ?

2. How must tea at 12*a*. per lb. be mixed with tea at 15*a*. per lb. to make a mixture worth 14*a*. 6*p*. per lb. ?

3. In what ratio must sugar worth Rs. 10 per maund be mixed with sugar worth Rs 13 per maund to make a mixture worth Rs. 11, 4*a*. per maund.

4. In what ratio must milk at 2*a*. per seer be mixed with milk worth 3*a*. 6*p*. per seer, to make a mixture worth 2*a*. 6*p*. per seer ?

5. How much wheat at Rs. 4, 10*a*. per maund be mixed with wheat at Rs. 5, 4*a*. per maund to make a mixture worth Rs. 4, 12*a*. per maund ?

6. A merchant buys sugar at Rs. 13 a maund and some more at Rs. 17 a maund. How must he mix them so as to make the mixture worth Rs. 15, 4*a*. a maund ?

7. How should I mix tea worth 15*a*. per seer with tea worth 9*a*. per seer so that by selling the mixture at 13*a*. 9*p*. per seer I may gain 10 per cent.?

8. In what ratio must a shopkeeper mix sugar worth Rs.5 per maund with sugar worth Rs. 7, per maund so that by selling the mixture at Rs. 7,3*a*. per maund he may gain 15 per cent. on his outlay ?

9. How much ghee at Rs. 75 per maund be mixed with vegetable ghee at Rs 40 per maund, so that by selling the mixture at Rs. 60 a maund the profit may be 20 per cent. ?

10. To mix some wheat at Rs. 5 per maund, with some at Rs. 6 per maund, and with some at Rs. $7\frac{1}{2}$ per maund, so as to make a mixture worth Rs. 7 per maund; how much of each quantity is to be taken in the mixture ?

11. How should a grocer mix some tea at 5*a*. per seer with some at 7*a*. per seer and with some at 10*a*. per seer so as to make a mixture worth 8*a*. per seer ?

12. A merchant buys sugars at Rs. 7, Rs. 9 and Rs. 11 per maund respectively ; how must he mix them so that by selling the mixture at Rs. 11 per maund, he may gain 10 per cent. ?

13. A grocer buys teas at 9*a*. 11*p*. and 13*a*. per lb. respectively ; how must he mix them so that by selling the mixture at 15*a*. per lb. he may gain 25 per cent.

14. A shopkeeper buys 32 seers of milk at 3*a*. per seer; how much water should he mix so that by selling the mixture at 2*a* 6*p*. per seer he may gain 6*a*. 6*p*. ?

15. A shopkeeper buys 25 seers of milk at 4*a*. per seer; how much water should he mix with it so that by selling the mixture at 3*a*. per seer he may lose only 1*a*. ?

16. A man buys 40 seers of milk at 3*a*. per seer; how much water should he mix with it so that by selling the mixture at 2*a*. 6*p*. per seer the profit may be 5*a*. ?

17. A person buys some quantity of milk at 3*a*. per seer and drinks a quarter of it himself. With the remainder he mixes 6 seers of water and sells it at 2*a*. 6*p*. per seer; his gain is 3*a*. How much milk does he buy ?

18. A wine merchant has four kinds of brandy, some at Rs. 9 per gallon, some at Rs. 8, some at Rs. 6, some at Rs. 4. He wants to make a mixture of 84 gallons worth Rs.7 per gallon, How much of each sort should be taken to make the mixture ?

19. To mix spirits worth Rs. 8, Rs. 6 and Rs. 3 per gallon for making a mixture worth Rs. 5 per gallon, how much of each quantity is to be taken ?

20. A grocer wishes to mix teas at 2*s*., 3*s*. 6*d*. and 4*s*. per lb. respectively how must he mix them (using the first two kinds in the proportion of 2 : 3 and the last two in the proportion of 3 : 4) so that by selling the mixture at 3*s*. 4*d*. per lb. $\frac{1}{20}$ of the receipts may be clear profit ?

21. Nine gallons are drawn from a cask full of wine; it is then filled with water. Nine gallons of the mixture are drawn and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 16 : 9. How much does the cask hold ?

22. Five gallons are drawn from a cask full of wine, it is then filled with water. Five gallons of the mixture are drawn and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 25 : 11. How much does the cask hold ?

23. There are four vessels of equal capacity, $\frac{1}{3}$ of the first, $\frac{1}{8}$ of the second, $\frac{1}{4}$ of the third and $\frac{1}{5}$ of the last is filled with spirit. The first is then filled with water and from the mixture the second is filled up, again from this second mixture the third is filled up and likewise the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

§ 5. Interchange of ingrediants.

Suppose there is a lump of two ingredients ; now if an equal quantity of another lump in which the proportion of the ingredients is interchanged, is added to the first lump, then evidently the quantities of the two ingredients become equal in the new compound. For instance, suppose there are two vessels ; in the first of them 4 gallons of wine are mixed with 9 gallons of water and in the second, 9 gallons of wine are mixed with 4 gallons of water. If we mix up the contents of the two vessels, then in the new mixture, there will be evidently 13 gallons of wine and 13 gallons of water.

Example 6. *The price of gold is £3, 17s. $10\frac{1}{2}d.$ per oz.* A lump of gold and silver weighing 18 lbs. is worth £ 637, 7s. but if the ratio of gold and silver be interchanged it would be worth only £259, 1s. Find the ratio of gold and silver in the lump and the price of silver per oz.

Sol. If the two lumps are added together, there would be 18 lbs. of gold and 18 lbs. of silver and the price of the two lumps together would be £637, 7s.+£259, 1s. or £ 896, 8s.

Now 18 lbs. of gold+18lbs. of silver=£896, 8s. but 18 lbs. of gold=£3, 17s. $10\frac{1}{2}d. \times 18 \times 12$=£841, 1s.

$\therefore$ 18 lbs. of silver=£55, 7s.

1 oz. of silver$=\frac{£55,\ 7s.}{18\times12}$=5s. $1\frac{1}{2}d.$ **Ans.**

Again cost of 1 oz. of the first lump$=\frac{£637,\ 7s.}{18\times12}$

$=$£2, 19s. $\frac{1}{6}d.=59\frac{1}{7\frac{1}{2}}s.$

Cost of 1 oz. of gold=£3, 17s. $10\frac{1}{2}d.=77\frac{7}{8}s.$

and cost of 1 oz. of silver=5s. $1\frac{1}{2}d.=5\frac{1}{8}s.$

$$\begin{array}{cc} & 59\frac{1}{7\frac{1}{2}} \\ 77\frac{7}{8} & \\ & 5\frac{1}{8} \end{array}$$

$$59\tfrac{1}{7\frac{1}{2}}-5\tfrac{1}{8} : 77\tfrac{7}{8}-59\tfrac{1}{7\frac{1}{2}}$$

$$53\tfrac{8}{9} : 18\tfrac{3}{36}$$

Hence gold : silver$=53\frac{8}{9} : 18\frac{3}{36}$

$=485\times4 : 679$

$=20 : 7.$ **Ans.**

EXERCISE 130.

1. The price of gold is Rs. 38, 15*a*. per oz. A lump of gold and silver weighing 18 lbs. is worth Rs. 7246, 8*a*., but if the weights of gold and silver are interchanged, it would be worth only Rs. 1717, 8*a*. Find the ratio of gold and silver in the lump and the price of silver per oz.

2. A person bought apples and pears for Rs. 2, 3*a*. If the numbers of apples and pears be interchanged, it would have cost him only Re. 1, 9*a*. If apples cost 9*a*. and pears 3*a*. a dozen, how many of each did he buy?

3. Two alloys are composed of silver and copper in the following ratios by weight:—98 : 2 in the first and 95 : 5 in the second. These two are melted together in the ratio of 5 : 3. Find the ratio of silver to copper in the resulting mixture.

4. Two vessels contain mixtures of wine and water: in one there is twice as much wine as water and in the other 3 times as much water as wine. Nine gallons from the first and six gallons from the second are drawn off to fill a third vessel. Find the ratio of wine to water in the third vessel.

5. From a vessel filled with 20 gallons of spirit, 2 gallons are removed and the vessel is then filled with water. What amount of spirit is left after this has been done twice?

6. Gold is woth £3, 17*s*. 11*d*., an ounce. An alloy containing 7 times as much gold as silver is worth 3 times as much as another alloy which contains 3 times as much silver as gold. In what ratio must these alloys be mixed to form one worth £2, 10*s*. an ounce? [*Burma 1924*]

7. Two tins contain originally 20 pints of milk and 10 pints of water respectively. Four pints of the liquid are now drawn from each tin and placed in the other, the liquid being thoroughly mixed. The same process is repeated a second time. Find the percentage of milk in each tin in the final mixtures. [*Burma 1923*]

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§6. Application of the Alligation Rule.

Example 1. Rs. 38, 12*a*. were divided among 120 children. Each girl had 8*a*. and each boy 4*a*.; how many boys were there?

Sol. The mean amount each child had $= \frac{\text{Rs. } 38,\ 12a.}{120} = \frac{6\,20}{120}a. = \frac{31}{6}d.$

girl	boy.
$8a.$	$4a.$
$\frac{31}{6}$	

$\frac{31}{6} - 4 : 8 - \frac{31}{6}$ or $\frac{7}{6} : \frac{17}{6}$ or $7 : 17$

$\therefore$ No. of boy $f = \frac{1}{2\frac{1}{4}} \times 120 = 85$. **Ans.**

Example 2. A person had 250 lbs of tea, some of it he sold at 10 p. c. profit, some at $12\frac{1}{2}$ p. c. profit and the rest at 15 p. c. profit. He gains $12\frac{9}{10}$ p. c. on the whole. Find how much of each quantity he sold on different rates.

Sol. The mean S. P. $= 112\frac{9}{10}$

(1)	(2)	(3)
110	$112\frac{1}{2}$	115

Now we can link (1), (3) and (2), (3)

$\therefore$ ratio in the (1) and (3) qua. $= 115 - 112\frac{9}{10} : 112\frac{9}{10} - 110$
$= \frac{21}{10} : \frac{29}{10} = 21 : 29$

and ratio in the (2) and (3) qua. $= 115 - 112\frac{9}{10} : 112\frac{9}{10} - 112\frac{1}{2}$
$= \frac{21}{10} : \frac{4}{10} = 21 : 4$

Now write these ratios and add :—

Since $7 + 7 + 11 = 25$
$250 \div 25 = 10$

(1)	(2)	(3)
21	.....	29
	21	4
21	: 21	: 33
$= 7$	: 7	: 11

$\therefore$ quantity at 10 p. c. $= 10 \times 7 = 70$ lbs.
,, at $12\frac{1}{2}$ p. c. $= 10 \times 7 = 70$ lbs.
and ,, at 15 p. c. $= 10 \times 11 = 110$ lbs. **Ans.**

Note. Other answers are also possible. The students can ch this answer supposing the cost price to be Re. 1 per lb.

How to get other answers:—

Ratio between (1) and (3)
(1) = 21 : 29
(3) = 42 : 58
(3) = 105 : 145 and so on

Take any pair, *e. g,*

(1)	(2)	(3)
42	.....	58
	21	4

$\therefore$ 42 : 21 : 62 will produce a different answer.

Ratio between (2) and (3)
(1) = 21 : 4
(2) = 42 : 8
(3) = 84 : 16 and so on

Again

(1)	(2)	(3)
105	.....	145
	84	16
105	: 84	: 161
$= 15$	: 12	: 23

will produce another answer and so on.

Note. The student should note carefully that if he will link the first pair with the first, the second with the second and so on, he will get the same answer but if he will link different pairs the answer will be different but all correct.

EXERCISE 131.

1. Rs. 78, 12*a.* were divided among 150 children, each boy got 12*a.* and each girl 6*a.* Find the number of each.

2. Divide Rs. 1580 among 180 claimants, so that some may receive Rs. 8, 5*a.* 4*p.* and others Rs. 9, 5*a.* 4*p.* Find the number of each kind of claimants.

3. A sum of £5, 10s. is made up of 75 coins which are either florins or shillings ; how many are there of each coin ?

4. A person has 150 lbs. of sugar, part of which he sells at 10 p. c. profit and the rest at $12\frac{1}{2}$ p. c. profit. He gains $11\frac{1}{6}$ p. c. on the whole. Find how much is sold at 10 p. c. profit.

5. A cloth merchant sold 55 yards of cloth for Rs. 88, 12*a.*, some at Re. 1. 8*a.* per yard and the rest at Re. 1. 12*a.* per yard ; how much is sold at Re. 1, 12*a.* per yard ?

6. A per son had 200 lbs of tea ; some of it he sold at 10 p. c. profit and some at 15 p. c. profit and the rest at 20 p. c profit. He gained $14\frac{1}{4}$ p. c. on the whole. How much did he sell on different rates ?

II. EQUATION OF PAYMENT

§7. Two or more debts incurred by a person at *different* times may be discharged by one payment; the time of payment of the total debts is called the *equated time* and the process by which we find this time is known as the *equation of payments*. In such cases, we *assume* that the sum of the interest of *all* the debts for their respective periods is equal to the interests of their sum for the *equated time*. Hence, to find the equated time we have the following simple rule :

Rule. *Multiply each debt by the corresponding time and then divide the sum of the products by the sum of the debts.*

Example. If Rs. 75 be due in 4 months, Rs. 125 in 5 months and Rs. 150 in 7 months, what is the equated time ?

Sol. The required time $= \frac{75\times4+125\times5+15\times7}{75+125+150}$

$= \frac{300+625+1050}{350}$

$= \frac{1975}{350} = \frac{79}{14} = 5\frac{9}{14}$ months **Ans.**

EXERCISE 132.

1. Rs.100 is due in three months, Rs. 210 in 2 months and Rs. 90 in 7 months. Find the equated time.

2. What is the equated time of payment when $\frac{1}{2}$ of a debt is due in 3 months, $\frac{1}{5}$ in 8 months and the remainder in 15 months ?

3. *A* owes *B* £4600, of which £1000, is to be paid in 50 days, £1300 in 40 days and the remainder in 140 days. Find the equated time.

4. *A* owes *B* Rs. 7300 to be paid in $5\frac{4}{7}\frac{4}{3}$ months ; he pays Rs. 1500 at the end of three months and Rs. 2100 at the end of five months ; when will the remainder be due ?

5. Out of a debt due 15 months hence, $\frac{1}{2}$ was paid at 4 months, $\frac{1}{4}$ at 6 months and $\frac{1}{8}$ at 12 months. How many months may the payments of the remaining portion of the debt be deferred ?

6. *A* man owes four debts ; the equated time of payment being 6 months. The first debt of £500 is due in 3 months, the second of £450 in 4 months, the third of £300 in 5 months. When is the fourth debt of £1000 due ?

III. PASTURE WITH GROWING GRASS.

§8. Examples on this subject can best be done by means of simultaneons equations. The following solutions illustrate the method :—

Example 1. If 29 sheep can feed on a field of uniformly growing grass in 7 days or 25 sheep can feed on the same field in 9 days, how many sheep will feed in 6 days ?

Sol. Denote amount of original grass a units and each day's growth by b units.

Now 29 sheep feeding for 7 days is equivalent to 29×7 $=203$ sheep for 1 day and similarly 25 sheep for 9 days is equivalent to 225 sheep for 1 day.

$\therefore$ by the question

$a+7b$, units of grass feed 203 sheep for one day, *i.e.*, equal to 203 units.

and $a+9b$ units of grass feed 225 sheep for 1 day, *i. e.*, equal to 225 units.

$\therefore$ omitting the common unit we have

$a+7b=203$ } whence $b=11$ units and $a=126$ units.
and $a+9b=225$ }

Now supposing the required number of sheep to be x and remembering that x sheep being fed for 6 days is equivalent to $6x$ sheep for 1 day, we have, from the question (as before), $a+6b=6x$.

Substituting values of a and b, we have

$x=32$ sheep. **Ans.**

Example 2. If 40 oxen eat the grass of a field growing uniformly in 12 days and if 25 oxen eat the grass of the same field in 20 days, find how long 30 oxen will take to consume it.

Sol. With the same symbols and with the same explanation as in Example 1, we have,

$a+12b=480$ } whence $b=\frac{5}{2}$
and $a+20b=500$ } and $a=450$

Now suppose that 30 oxen require x days, then

$a+xb=30x$,

or $450+\frac{5}{2}x=30x$

whence $x=\frac{900}{55}=\frac{180}{11}=16\frac{4}{11}$ days. **Ans.**

EXERCISE 133.

1. A field of 15 acres, grass growing uniformly, is consumed by 20 sheep in 96 days, but by 30 sheep in 60 days. How many sheep will consume it in 24 days?

2. A meadow of 50 acres with uniformly growing grass is consumed by 80 oxen in 60 days but by 60 oxen in 90 days. How many oxen will eat half of it in 30 days?

3. If 133 horses eat the grass of a meadow in 13 days and 112 horses could eat the grass of the same meadow in 16 days (the grass growing uniformly); in what time could 125 horses eat it?

4. If 25 sheep consume 10 acres of pasture (grass growing uniformly) in 30 days, and 30 sheep consume 8 acres of it in 18 days, how many acres will be consumed by 80 sheep in 60 days?

5. A field of uniformly growing pasture is consumed by 120 oxen in 20 days or by 150 oxen in 15 days. How many oxen will consume it in 12 days, if at the end of 8 days, 30 of them are removed?

6. A cistern constantly flowing is emptied by a number of waste pipes. If 10 of these are opened, the cistern is emptied in 15 minutes but if 25 be opened, it is emptied in 5 minutes. How much time will it take to be emptied if 15 are opened ?

7. If 23 sheep eat 8 acres of pasture in 26 days and 25 sheep consume 7 acres of the same in 20 days (the grass growing uniformly), how many acres of it would 33 sheep consume in $5\frac{7}{9}$ days ?

8. A leaky cistern is filled in 5 hrs. with 30 pails of 3 gallons each but in 3 hrs. with 20 pails of 4 gallons each, the pails being poured in at intervals. Find the capacity of the cistern and the time in which the water should flow out.

MISCELLANEONS EXERCISES II

1. Simplify $\dfrac{5\frac{1}{2}+1\frac{2}{33}\div 1\frac{3}{4}-\frac{5}{18}\div 1\frac{1}{18}}{1\frac{1}{3}\times\frac{2}{3}-\frac{22}{57}+\frac{1}{4}\div 2\frac{1}{4}}$.

2. A woman has a certain number of eggs ; she sells $\cdot\dot{3}$ of the number and one more to one person, $\cdot\dot{3}$ of the remainder to a second person, and $\cdot 5$ of the remainder to a third person : after these sales she has 15 eags left. How many had she at first ?

3. (*a*) The total expenses of a family when wheat is 16 srs. a rupee are Rs. $40\frac{1}{4}$; when wheat is 12 srs. a rupee they are Rs. $42\frac{1}{3}$; find its total expenses when wheat is 10 srs. a rupee; supposing other expenses remain the same.

(*b*) Find the square root of $1+\frac{1}{2}\,(\cdot 0345)^3$ correct to 5 places of decimals.

4. A garrison of 2000 men has provisions for 42 days. How long will the provisions last, if the garrison be increased by 800 men ?

5. Find the value of 55 mds., 22 srs. 11 chk. of ghee at Rs. 25. 13*a*. 4*p*. per mund.

6. *A* can do in 9 days as much work as *C* can do in 6 days, and *B* in 15 days as much as *C* in 12 days : what time would *B* take to finish a piece of work which *A* can do in 36 days ?

7. Find the least number which being divided by 8, 12 and 16 leaves in each case a remainder 3 but when divided by 15 leaves no remainder.

8. A room is 37 ft. 3 in. long; 18 ft. 9 in. wide and 14 ft. high. Find the cost of papering its walls with paper 2 ft. wide at 6*d.* per yard.

9. One boy runs 300 yards and another 285 yards in 1 minute. How many yards start must the second have so that they may run a dead heat in a mile race ?

10. The average age of the boys in a school of 650 boys is 15·6 years. 50 boys leave. thereby diminishing the average of the school to 15·2 yrs. ; find the average age of those who leave.

II

1. What is the greatest number consisting of 5 digits which can be added to 8321, so that the sum may be exactly divisible by 15, 20, 24, 27, 32 or 36 ?

2. Simplify $\left\{\frac{2}{3-\frac{1}{1-\frac{1}{2}}} - 5 - \frac{2}{\frac{3}{2}-\frac{1}{6}} \text{ of } \frac{1}{3}\right\} \div \frac{\frac{1}{2}+\frac{5}{4}}{1\frac{1}{2}}$

3. The cost of matting a room 16 ft. broad and 12 ft. high at 3 *a.* per sq. yard is Rs. 7, 9*a.* 4*p.* What will be the cost of papering its walls at the same rate allowing for six doors, each 6 ft by 3 ft. ?

4. A man buys an article and sells it at a gain of 10 p. c. If he had bought it at 10 per cant. less and sold it for Rs. 1 more he would have gained 25 per cent. Find the cost price.

5. A man bought two heaps of mangoes, one for Rs. 10, 5*a.* and the other for Rs. 18, 6*a.* 9*p.* If the price of each mango be the same, and not less than two annas and not more than three annas, find the total number of mangoes he bought.

6. *A* can do a piece of work in 25 days, *B* in 20 days and *C* in 24 days. They three work together for 6 days and then *B* leaves. In what time would *A* and *C* finish the remaining work ?

7. Find the value of $\frac{12+\sqrt{\cdot 009}}{1-\sqrt{4}}$ correct to 3 places of decimals.

8. If 5 compositors working 8 hours a day can compose a book of 240 pages in 21 days, in what time would 3 compositors working 10 hours a day compose a book of 360 pages ?

9. A merchant buys two kinds of tea at 1*s.* $11\frac{3}{4}$*d.* and 1*s.* 5*d.* per lb. respectively. In what proportion must he mix them so as to gain $37\frac{1}{2}\%$ by selling the mixture at 2*s.* $3\frac{1}{2}$*d.* per lb?

10. Find the least sum of money that must be added to Rs. 25316, 12*a*. 8*p*. to make the sum divisible by 221.

III

1. Reduce $\cdot 0416 \times \frac{\cdot 142857}{(\frac{5}{12}+\frac{3}{8})\times 40}$ of Rs. 5, 8*a*. to the fraction of 1*a*.

2. What least number must be added to $7\frac{1}{8}$ that the result being divided by $1\frac{1}{6}$, the quotient shall be an integer?

3. *A* and *B* complete a piece of work in 8 days, *B* and *C* do the same work in 12 days, and *A*, *B* and *C* finish, it in 6 days. In how many days will *A* and *C* complete the work?

4. If by selling sugar at 24*s*. a cwt. a grocer would lose 4 per cent., at what price per lb. must he sell it to gain 12%?

5. The population of a town is 50000. If the number of males were increased by 9 per cent. and the number of females by 4 per cent., the population would become 53200. Find the number of males and females in the town.

6. Make out a bill for the following articles supplied by Messrs. Mool Chand & Co. to L. Gujar Mal:—

10 lbs. of tea at Re. 1, 3*a*. per lb.; 6 seers of sugar at Rs. 2, 3*a*. per bag of 5 seers; 4 tins of coffee at Re. 1, 1*a*. per tin; 8 silk handkerchiefs at Rs. 3, 8*a*. per dozen; 3 mds. 37 seers of Portland cement at 8 seers per rupee; a child's perambulator price Rs. 30. Subtract 10 % discount for cash.

7. The area of a rectangular field is $\frac{3}{5}$ of an acre; and its length is twice its breadth, determine the length of its sides approximately.

8. A man has £5, 17*s*. consisting of sovereigns, half crowns and shillings in the proportion of 2, 3, 11. How many has he of each coin?

9. Find the number of complete kilometres in 850 miles taking a centimetre to be ·3937 inches. [*Burma* 1923]

10. A clock which gains $2\frac{1}{4}$ mts. in 24 hrs. is 3 minutes slow at noon on Sunday; when will it indicate correct time and what will it indicate at 6 o'clock on Monday evening?

IV

1. Find the value of $\frac{\cdot 49}{2\cdot 1}$ of $\frac{(3\frac{1}{3}-2\frac{1}{2})\div\frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3}\div(\frac{1}{2}+\frac{1}{4})}$ of £ 46.

2. Find the greatest number of 4 digits and the least number of 5 digits that have 135 for their G. C. M.

3. Multiply 675348 by 2739113 in three lines.

4. Find the value of 5 tons 15 cwt. 3 qr. 7 lb. of coal at £4, 13 *s.* 4*d.* per ton.

5. A candidate who gets 40 marks fails by $12\frac{1}{2}$ marks, the minimum to be secured being 35 per cent. Find the maximum.

6. If 100 horses consume a stack of hay 24 ft. long, 10 ft. high and 8 ft. broad in 8 days; for how many days will a stack 20 ft. long, 15 ft. high and 6 ft. broad, supply 150 horses ?

7. A cyclist starts for a ride at 8-55 A. M. By 10-34 A. M. he has ridden 16 miles 154 yards. How far will he have gone altogether by 1-16 P. M., his speed being the same throughout ?

8. Find the cost price of an article which if sold at 7% profit brings, 5*s.* more than if it were sold at 17% loss.

9. A multiplication sum having been worked is partially rubbed out, the figures that remain are the entire multiplicand 999 and the last three digits 193 in the product. Restore the complete work.

10. A person buys 80 tons of coal and after selling them again at 1*s.* 6*d.* per sack finds that he has gained £4 ; had he sold them for 1*s.* 4*d.* per sack he would have lost £6. Find the weight of each sack and the cost price per ton.

V

1. Find the missing digits indicated by the mark* in 46*389* if it is divisible by 45.

2. *A* and *B* have between them 132 horses, ·25 of *A*'s is equal to $\cdot\dot{1}4285\dot{7}$ of *B*'s. How many has each of them ?

3. A box made of board an inch thick, measures on the outside 20 in. long, 14 in. wide and 8 in. deep ; find the cubic contents of the interior and the cost of painting the outside at 9*p.* per sq. ft.

4. A tradesman demanded a price for a watch which was 40 per cent. above cost and gave the purchaser 10 per cent discount on the price asked, gaining thereby Rs.8, 2*a.* What was the cost price ?

5. A ship with 1200 men on board had sufficient provisions to last 17 weeks. The survivers of a wreck having been taken aboard, the provisions were consumed in 15 days. How many men were taken aboard ?

6. Find the square root of $1-(\cdot0678)^3$ to 4 places of decimals.

7. A man bought 4 sorts of wheat at an average price of Rs. 6 a maund. If the prices are such that they increase by a common difference of 5*a*. per maund, find the cost of each sort per maund.

8. Find, by practice, the price of $521\frac{2}{3}$ maunds of sugar at Rs. 22, 11*a* 11*p*. per md. (*Use two aliquot parts only*).

9. A school of boys and girls consists of 453 children ; the number of boys is $\cdot52$ of the number of girls. How many boys are there ?

10. The breadth of a room is two thirds of its length and three halves of its height and the contents are 5832 cubic ft. Find the dimensions of the room.

VI

1. Simplify $\frac{5}{7}+\cfrac{3}{5\frac{1}{2}+\cfrac{4\frac{1}{2}}{1-\cfrac{3}{8}}}$ of $25\frac{3}{7}+\cfrac{}{7\frac{2}{3}-5\frac{11}{12}}$

2. Express $\frac{3}{8}$ of 7*s*. 6*d*. $+1\cdot25$ of 5*s*. $-\cdot545$ of 9*s*. 2*d*. as a decimal of £10.

3. In a game at fives, out of 15 points *A* can give *B* 3 also *A* can give *C* 7 points ; how many points can *B* give *C* so as to make an even match ?

4. A cistern contains $243\frac{3}{4}$ cubic ft. of water. Find the length of the side of a second cistern 4 ft. 4 in. deep with a square base which contains four times as much water as the first.

5. What profit per cent. is made by selling an article at a certain price, if by selling at two-thirds of that price there would be a loss of 20 per cent.

6. *A* can do a certain piece of work in 72 days, *B* in 96 days, *C* in 120 days. *C* begins alone for the first day then *A* and *B* join and all three work for 12 days, after which *A* leaves off ; in how many more days will *B* and *C* finish it ?

7. Find the square root of $\cfrac{1000\cdot20001}{1000}$

8. A rectangular courtyard the sides of which are 5 : 11 costs Rs. 144, 6*a*. for paving at 10*a*. 6*p*. per sq. yard. Find the lengths of its sides.

9. Twelve bullocks can plough a square field, the length of a side of which is 90 yds. in $2\frac{1}{2}$ days. In how many days will 20 bullocks plough a square field, the length of a side being 150 yds. ? [*Burma 1923*]

10. The average weight of the 8 oars-men in a boat is increased by $2\frac{1}{2}$ lbs., when one of the crew who weighs 11 stones 12 lbs. is replaced by a new man. What is the weight of the new man ?

VII

1. (*a*) Simplify $\frac{\frac{1}{2}+\frac{1}{3} \text{ of } \frac{1}{4}+\frac{5}{6}}{\frac{1}{13} \text{ of } (1+5\frac{1}{2})+\frac{5}{6} \text{ of } \frac{1}{2\frac{1}{3}} \text{ of } (7-2\frac{2}{5})-\frac{1}{3}}$

(*b*) Express $\frac{3}{5}$ of Re. 1, 5*a*. as the decimal of Re. 1, 4*a*.

2. What is the greatest length which is contained whole number of times exactly in both $25\frac{15}{28}$ ft. and $21\frac{9}{20}$ ft. ?

3. A man left $\frac{1}{2}$ of his property to his eldest son, $\frac{1}{3}$ to his youngest, $\frac{1}{12}$ to his daughter and the remainder which was Rs. 120 to his wife : what was his whole property worth ?

4. A police man goes after a thief who has 528 yards' start ; if the policeman goes at the rate of a mile in 7 minutes and the thief at the rate of a mile in 10 minutes, how far will the thief have gone when he is overtaken ?

5. A hollow circular cylinder stands on a solid cubic pedestal of the same material whose edge is 6 ft. ; the internal and external diameters of the cylinder are 4 and $6\frac{1}{4}$ ft. ; the weight of the cylinder is the same as that of the pedestal ; find its height.

6. A number of men can be formed either into a solid square, or into a hollow square 9 deep, having 970 men in the front rank of each side ; how many men are there in each side of the solid square ?

7. A merchant buys 80 gallons of whisky at 18*s*. per gallon and 180 gallons at 15*s*. per gallon and mixes them. At what price must he sell the mixture to gain $8\frac{1}{3}\%$ upon his outlay ?

8. Find the sq. root of $7{\cdot}4538-6{\cdot}8\div 8{\cdot}5-2{\cdot}03\times 1{\cdot}17$.

9. A rectangular field 50 ft. long and 30 ft. wide has a path of uniform width running *outside* all round the sides. The area of the path is 425 sq. ft. ; find its width.

10. Find two least integers such that $\frac{5}{8}$ of the first shall be equal to $\frac{25}{42}$ of the second.

VIII

1. Multiply Rs.2, 1*a*. by $\frac{4/5+6/7+13/15+10/21}{1/5+5/7+13/15+13/21}$

2. Find the G. C. M. and L. C. M. of 49·383 and 14·569.

3. Find by practice, the time of buiding a wall 27 yds. long, 1 yd. thick and 6 ft. high of which one cubic yd. is built in 3 hrs. 18 min. 45 sec.

4. *A*'s rate of working is to *B*'s as 4 to 3 and *B*'s is to *C*'s as 2 to 1. How long will it take *C* to do what *A* would do in 6 days ?

5. Find by how much the square root of $9+\cfrac{1}{1+\cfrac{1}{7+\frac{1}{6}}}$ differs from $\frac{355}{113}$; which of these comes nearest to $3+\frac{1}{10}\sqrt{2}$?

6. In what proportion must a grocer mix one kind of tea at 12*a*. per ℔. with another at Re. 1, 5*a*. per ℔. in order that by selling the mixture at Re. 1, 8*a*. per ℔., he may make a profit of 20 per cent. ?

7. In an examination, *A* gets 10 p. c. less than the minimum number of marks required for passing, *B* obtains $11\frac{1}{9}$ p. c less than *A* and *C* $41\frac{3}{17}$ p. c. less than the number of marks obtained by *A* and *B* together. Does *C* pass or fail ?

8. A dealer buys 80 baskets of rice at Rs. 5, 12*a*. per basket , he mixes it with 32 baskets of another kind and sells the mixture at Rs. 6, 6*a*. per basket making a profit of 19 per cent. How much did he pay for the second kind of rice ?

9. *A* can beat *B* by 5 yds. in a 100 yards' race and *B* can beat *C* by 10 yds. in a 200 yards race : by how much can *A* beat *C* in a 400 yards race ?

10. I went for a walk between 4 and 5 and returned between 7 and 8. The hands had exactly changed places ; at what time did I go for the walk ?

IX

1. Simplify : $\frac{3\frac{7}{9}\times 1\frac{1}{17}+4\frac{11}{12}-4\frac{3}{16}}{5\frac{1}{9}-7\frac{7}{8}\div 28\frac{7}{20}+\frac{1}{3}}\times\frac{3\frac{2}{3}}{4\frac{1}{7}}+3\frac{5}{8}\times 5\frac{6}{7}-17\frac{1}{2}+\frac{5}{6}$.

2. Eight bells which toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively, begin tolling all simultaneously with the

clock striking. How many hours must elapse before all toll simultaneonly again with the clock striking ? (The clock is supposed to strike at hour only).

3. By how much does the difference of $1\frac{1}{2}\frac{1}{6}$ and $\frac{9}{13}$ fall short of their sum ? Express the defect as a decimal.

4. How long will it take to walk along the four sides of a square field which contains 16 acres 401 sq. yds. at 3 miles an hour ?

5. The length of a hall is 3 times the breadth. The cost of whitewashing the ceiling at $5\frac{1}{3}d.$ per. sq. yd. is £4, 12s. $7{\cdot}1d.$ and the cost of papering the walls at 1s. $9d.$ per sq. yd. is £35. Find the height of the hall.

6. In mixing tea, 1 lb. in every 100 lbs. is wasted. In what proportion must a dealer mix teas which cost him 1s. $9d.$ and 1s.$4d.$ per lb. respectively, so as to gain 10 per cent. by selling the mixture at 1s. $8d.$ per lb. ?

7. A hare is 500 leaps before a greyhound and takes 4 leaps to his 3, but 2 of the greyhound's leaps cover as much ground as 3 of the hare's ; in how many leaps will the greyhound catch the hare ?

8. I spent £5 in buying eggs at 2 a penny and the same sum in buying eggs at 3 a penny, I sold them all at the rate of 5 for two pence. How much did I gain or lose ?

9. If the four-penny loaf weighs 3 lbs. 9 oz. when wheat is at 9s. $4d.$ per bushel, what ought the six penny loaf to weigh when wheat is 11s. $1d.$ per bushel ?

10. (*a*) Extract the square roots of $5\frac{1}{1}\frac{1}{6}$ and 76·195441.

(*b*) A man bought a piece of land for Rs. 140 and intending to sell it, fixed such a price, that by selling it at $12\frac{1}{2}\%$ under the intended price, he would still have a gain of $12\frac{1}{2}$ per cent. on the prime cost. At what price did he intend to sell the land ?

X

1. How mony times is $\frac{1}{7}$ of $1\frac{1}{3}\frac{1}{3}$ of 2s. $2\frac{1}{4}d.$ contained in $2\frac{1}{4}$ of 3s. $4d.$ $+4\frac{4}{1\,1}$ of 1s. $1\frac{3}{4}d.$ $+\left(\frac{27\frac{5}{9}\text{ of }28\frac{1}{4}}{20\frac{2}{3}\text{ of }75\frac{1}{3}}+\frac{3\frac{6}{7}\text{ of }1\frac{1}{3}}{7\frac{5}{7}\text{ of }2\frac{2}{3}}\right)$ of 16s. $8d.$?

2. On measuring a distance of 32 yds. with a rod of a certain length, it was found that the rod was contained 41 times with $\frac{1}{2}$ an inch over. How many inches will there be over in measuring 44 yds. with the same rod ?

3. *A* who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{2}$ hrs. before *B* who goes the same road at $4\frac{1}{2}$ miles an hour. Where will he overtake *A* ?

4. If 3 fowls and 4 pigeons cost Rs. 2, 3*a*. 6*p*. and 5 fowls and 2 pigeons cost Rs. 2, 12*a* . , find what must be paid for 4 fowls and 3 pigeons.

5. A horse was sold for Rs. 60, at a certain loss. Had it been sold for Rs. 81, the gain would have been $\frac{3}{4}$ of the loss. Find the cost of the horse.

6. Four men undertake to complete a piece of work in 12 days, at the end of 5 days only $\frac{2}{9}$ of the work is accomplished; how many extra men (working at the same rate) must be employed to complete the work in the given time ?

7. (*a*) The sides of a rectangle are 3 : 4 and the area is 1452 sq. ft. Find its length and breadth.

(*b*) A packet of matches which cost 4*a*. to manufacturer was sold at 9*a*. per packet. The cost of manufacturer has risen $8\frac{1}{2}$ per cent, ; the matches are still sold at 9*a*. per packet, but each packet contains now only 10 boxes instead of 12. Compare the percentage profit made now by the manufacturer with what he made formerly. [*Burma* 1923]

8. One clock gains 2 minutes in 3 days, another loses 6 minutes in 6 days; if they are set right at 12 o' clock to-day, when will their times differ by a quarter of an hour ?

9 The subscription to a certain memorial fund amounted to Rs. 976, 9*a*. and each person subscribed as many annas as there where subscribers. Find the number of subscribers.

10. A contractor engaged to finish six miles of railway in 200 days, but after employing 140 men for 60 days he found that only one and a half miles were completed. How many additional men must be engaged that the work may be finished within the given time ?

CHAPTER XXV

SIMPLE INTEREST

§ 1. If I borrow a certain sum of money for a certain period from a money lender, I am expected to pay a certain extra sum of money at a fixed rate for the use of the money borrowed. The considertaion thus paid is called the **Interest**, the money borrowed is named the **Principal** and the **sum** of interest and principal together is termed the **Amount.**

§2. The **Rate** is the money paid for the use of a certain sum for a certain time. Thus if a certain sum is borrowed on the condition that for the use of every rupee in the loan for a month 3*p*. will be paid, it is said to have been borrowed *at the rate of 3p. per rupee per month.* Again, if it is borrowed on the condition that for the use of every Rs. 100 in the loan for one year, Rs. 6 will be paid, it is said to have been borrowed *at the rate of 6 per cent. per annum.*

§3. Per annum means for a year. The words *per annum* are generally omitted. When we say at the rate of 5 p. c. we mean 5 p. c. per annum.

§4. If the interest is paid as it falls due, it is called **simple interest.** *The word interest is always to be understood as simple interest.*

§ 5. **To find interest when the rate per rupee or £ per month is given.**

Example 1. Find the simple interest on Rs. 120 for 8 months at 3*p*. per rupee per month.

Sol. Int. on Re. 1 for 1 month $=3p.=\text{Re.}\frac{1}{64}$.

$\therefore$ „ Rs. 1 for 8 months $=\text{Re.}\frac{1}{64}\times 8$.

$\therefore$ „ Rs 120 for 8 months $=\text{Rs.}\frac{1}{64}\times 8\times 120$

$=\text{Rs. } 15.$ **Ans.**

Hence when rate per rupee or £ per month is given we have the following rule :

Rule. *Int.* $=$ *Principal* $\times$ *rate per Re.* $\times$ *time in months.*

Example 2. Find the simple interest on £320, 8*s*. 6*d*. for $1\frac{1}{2}$ years at 4*d*. per £ per month.

Sol Principal [illegible]
Rate [illegible]
Time [illegible]
∴ Interest [illegible]

[illegible]

[illegible] Ans.

EXERCISE 13:

Find the interest on: —

1. Rs. 192 for 4 months at 4*p*. per rupee per month.
2. Rs. 96 for 1½ years at 2*p*. per rupee per month.
3. Rs. 384 for 8 months at 5*p*. per rupee per month.
4. Rs. 800 for 2 months at 1*p*. per rupee per month.
5. Rs. 128 for 8 months at 3*p*. per rupee per month.
6. Rs. 50 for 8 months at 3*p*. per rupee per month.
7. Rs. 125 for 6 months at ½*a*. per rupee per month.
8. Rs. 175 for 9 months at 2*p*. per rupee per month.
9. Rs. 192 for 1½ years at 4*p*. per rupee per month.
10. Rs. 384 for 1¼ years at 5*p*. per rupee per month.
11. Rs. 320 for 1⅓ years at 1*p*. per rupee per month.
12. £360, 13*s*. 4*d*. for 1½ yrs. at 6*d*. per £ per month.
13. £350, 6*s*. 8*d*. for 2¼ yrs. at 5*d*. per £ per month.
14. Rs. 521, 10*a*. 8*p*. for 1¾ yrs. at 4*p*. per rupee per month.
15. Rs. 624, 8*a*. 4*p*. for 1½ yrs. at 6*p*. per rupee per month.

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§6. **To find the interest when the rate per cent. per month is given.**

Example 3. Find the simple interest on Rs. 520 for one year 3 months at 12*a*. per cent. per month.

Sol. Int. on Rs. 100 for 1 month = Re. $\frac{3}{4}$
∴ " " Rs. 100 for 15 months = Rs. $\frac{3}{4}\times 15$
∴ " " Re. 1 for " " = Rs. $\frac{3}{4}\times 15\times\frac{1}{100}$
∴ " " Rs. 520 for " " = Rs. $\frac{3}{4}\times 15\times\frac{1}{100}\times 520$
= Rs. $\frac{117}{2}$ = Rs 58, 8*a*. **Ans.**

Hence when the rate per cent. per month is given we have the following rule:

Rule. $Interest = \frac{Principal \times rate\ p.\ c. \times time\ in\ months}{100}$

Thus in the above question

Interest = Rs. $\frac{520 \times 12 \times 15}{16 \times 100}$ = Rs. $\frac{117}{2}$ = Rs. 58, 8*a*. **Ans.**

Example 4. Find the simple interest on Rs. 721, 5*a*. 8*p*. for 1 year 4 months at 8*a*. per cent. per month.

Sol. Principal = Rs. 721, 5*a*. 8*p*. = Rs. $\frac{346225}{48}$

Rate = Re. $\frac{1}{2}$ per cent. per month.

Time = 16 months.

$\therefore$ Interest = Rs. $\frac{346225}{48} \times \frac{1}{2} \times 16 \times \frac{1}{100}$ = Rs. $\frac{13365}{24}$

= Rs. 57, 11*a*. 4*p*. Ans.

EXERCISE 135.

Find the interest on:—

1. Rs. 525 for 2 years 8 months at 12*a*. p. c. per month.
2. Rs. 630 for 1 year 8 months at 8*a*. p. c. per month.
3. Rs. 780 for 2 years 1 month at 12*a*. p. c. per month.
4. Rs. 625 for $2\frac{1}{4}$ years at 5*a*. 4*p*. p. c. per month.
5. Rs. 675 for $2\frac{2}{3}$ years at 10*a*. 8*p*. p. c. per month.
6. £ 720 for $1\frac{1}{2}$ years at 15*s*. p. c. per month.
7. Rs. 960 for $1\frac{2}{3}$ years at 13*a*. 4*p*. p. c. per month.
8. Rs. 560, 7*a*. 6*p*. for 10 months at 12*a*. p. c. per month.
9. Rs. 995, 11*a*. 8*p*. for 1 year 3 months at Re. 1, 2*a*. per cent. per month.
10. Rs. 1531, 13*a*. 4*p*. for 2 years 1 month at 14*a*. p. c. per month.

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§7. To find interest when the rate per cent. per annum is given.

Example 5. Find the simple interest on Rs. 566, 10*a*. 8*p*. for 5 years at 3 per cent. per annum.

Sol. Rs. 566, 10*a*. 8*p*. $= \text{Rs. } \frac{1700}{3}$

$\because$ int. on Rs. 100 for 1 year $= \text{Rs. } 3$

$\therefore$,, ,, Re. 1 for 1 ,, $= \text{Re. } \frac{3}{100}$

$\therefore$,, ,, Re. 1 for 5 years $= \text{Re. } \frac{3}{100} \times 5$

$\therefore$,, ,, Rs. $\frac{1700}{3}$ for 5 years $= \text{Rs. } \frac{3}{100} \times 5 \times \frac{1700}{3}$

$= \text{Rs. } 85$. **Ans.**

Hence when the rate per cent. per annum is given we have the following rule :

Rule. $$Interest = \frac{Principal \times rate \times time}{100}$$

☞ Sometimes it is convenient not to reduce the principal to its highest denomination. If the fraction occurs in the rate % or in number of years it is still more convenient to multiply them together first.

Example 6. Find the simple interest and the amount of Rs. 829, 4*a*. 6*p*. for $2\frac{2}{3}$ years at $6\frac{1}{4}$ p. c.

Sol. Principal $=$ Rs. 829, 4*a*. 6*p*.

Time $= 2\frac{2}{3}$ years

Rate $= 6\frac{1}{4}$ p. c.

$\therefore$ inrerst $= (\text{Rs. } 829, 4a.\ 6p.) \times \frac{8}{3} \times \frac{25}{4} \times \frac{1}{100}$

$= (\text{Rs. } 829.\ 4a.\ 6p.) \times \frac{1}{6}$

$= \text{Rs. } 138, 3a.\ 5p.$ **Ans.**

Note. The amount may be obtained by adding the interest to the principal.

EXERCISE 136.

Find the simple interest on

1. Rs, 750 for $1\frac{1}{2}$ years at 4 per cent.
2. Rs. 800 for $2\frac{1}{2}$ years at 4 per cent.
3. Rs. 350 for 3 years at 4 per cent.
4. Rs. 825 for $2\frac{1}{2}$ years at 5 per cent.
5. Rs. 728 for $3\frac{1}{2}$ years at $6\frac{1}{4}$ per cent.
6. Rs. 533, 5*a*. 4*p*. for $3\frac{1}{2}$ years at $2\frac{1}{4}$ per cent.
7. Rs. 724, 2*a*. 8*p*. for $3\frac{1}{3}$ years at 5 per cent.
8. Rs. 538, 10*a*. 8*p*. for $4\frac{1}{2}$ years at 10 per cent.
9. Rs. 833, 5*a*. 4*p*. for $1\frac{2}{7}$ years at $3\frac{1}{2}$ per cent.
10. Rs. 531, 12*a*. 4*p*. for $2\frac{1}{2}$ years at 8 per cent.
11. Rs. 728, 13*a*. 4*p*. for 4 years at $6\frac{1}{4}$ per cent.
12. Rs. 525, 12*a*. 6*p*. for $4\frac{1}{2}$ years at 8 per cent.
13. Rs. 712, 6*a*. 2*p*. for $7\frac{1}{2}$ years at $6\frac{2}{3}$ per cent.
14. Rs. 631, 3*a*. 7*p*. for $5\frac{1}{3}$ years at $3\frac{3}{4}$ per cent.
15. Rs 749, 7*a*. 8*p*. for $2\frac{2}{3}$ years at $2\frac{1}{4}$ per cent.

§8. To find interest when time is given in months and days.

when the time is given in months and days, 12 months are reckoned to the year and 30 days to the month.

Example 7. Find the simple interest on £553, 2*s*. 6*d*. for 2 years 3 months 12 days at 8 per cent.

Sol. Principal =£553, 2*s*. 6*d*.

Rate=8 p. c.

Time=2 yrs. 3 months 12 days.

$=27\frac{2}{5}$ months$=\frac{137}{60}$ yrs.

$\therefore$ interest$=(£553.\ 2s.\ 6d.)\times 8\times\frac{137}{60}\times\frac{1}{100}$

$=£553,\ 2s.\ 6d.\times\frac{137}{750}$

£.	s.	d.	
553	2	6	
		17	
9403	2	6	
		8	$\because\ 137=17\times 8+1$
75225	2	0	
553	0	6	
10)75778	2	6	
15) 7577	16	3	
5) 505	3	9	
101	0	9	

750

$\therefore$ interest =£101, 9*d*. **Ans.**

Note. We can solve such questions by means of Aliquot parts also, e.g., thus :

£.	s.	d.	
553	2	6	
		8	Multiply by rate
4425	0	0	
		2	,, by time
8850	0	0	=int. of 2 yrs.
1106	5	0	=,, of 3 months.
122	18	4	=,, of 10 days,
24	11	8	=,, of 2 days.
10 | 10103	15	0	Divided by 100
10 | 1010	7	6	
101	0	9	Ans.

3 months$=\frac{1}{4}$ of a year
10 days$=\frac{1}{3}$ of 3 months
2 days$=\frac{1}{5}$ of 10 days

Aliter Working in decimals we shall proceed thus :—
principal = £ 553, 2*s*. 6*d*. = £ 553·125

8 p. c. of £ 553·125 = £ 44·25

2

	£ 88·50	= int. of 2 yrs.
3 months = $\frac{1}{4}$ of a yr.	£ 11·0625	= ,, ,, 3 months.
6 days = $\frac{1}{15}$ of 3 months	£ ·7375	= ,, ,, 6 days.
6 days = $\frac{1}{15}$ of 3 months	£ ·7375	= ,, ,, 6 days.
	£ 101·0375	

20

·7500*s*.

12

9·00*d*.

∴ £ 101. 9*d*. **Ans.**

EXERCISE 137.

Find the simple interest on :—

1. Rs. 2100 at 3*p*. per rupee per month for 5 months 20 days.
2. Rs. 1752 at 12*a*. per cent. per month for 6 months 18 days.
3. Rs. 1520 at 8*a*. per cent. per month for 2 years 3 months 15 days.
4. Rs. 3500 at $2\frac{3}{4}$ per cent. per annum for 1 year 2 months 15 days.
5. Rs. 1236, 13*a*. 7*p*. at $3\frac{3}{4}$ per cent for 3 years 2 months 12 days.
6. Rs. 1605, 7*a*. 6*p*. at 6 per cent. for 2 years 4 months 8 days.

Find the amount of:—

7. Rs. 1533, 5*a*. 4*p*. for 2years 7 months 15 days at 4%.
8. Rs. 1256, 10*a*. 8*p*. for 1 year 4 months 15 days at $6\frac{1}{4}$%.
9. Rs. 1666, 10*a*. 3*p*. for 3 years 6 months 20 days at $7\frac{1}{2}$%
10. Rs. 1526, 11*a*. 1*p*. for 1 year 2 months 12 days at $3\frac{1}{8}$%

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§9. **To Find interest when the time is given in days or years and days.**

When the time is given in days or in years and days, the year is taken to consist of 365 days. It will be convenient if

the student commits to memory the following multiplication table:—

$73 \times 1 = 73$
$73 \times 2 = 146$
$73 \times 3 = 219$
$73 \times 4 = 292$
$73 \times 5 = 365$

Example 8. Find the simple interest on Rs. 356, 5*a*. 8*p*. for 1 year 219 days at $6\frac{1}{4}$ per cent.

Sol. Principal = Rs. 356, 5*a*. 8*p*.
Rate = $6\frac{1}{4}\%$
Time = $1\frac{219}{365}$ or $\frac{8}{5}$ years.
$\therefore$ Interest = Rs. $356, 5a, 8p. \times (\frac{25}{4} \times \frac{8}{5} \times \frac{1}{100})$
= Rs. 356, 5*a*. 8*p*. $\times \frac{1}{10}$
= Rs. 35, 10*a*. 2*p*. **Ans.**

§10. When simple interest is to be calculated from one date to another, the first day is not taken into consideration.

Example 9. Find the simple interest on £151, 6*s*. 8*d*. from July 15, 1944, to December 8, 1944, at $12\frac{1}{2}$ per cent. per annum.

Sol Principal = £151, 6*s*. 8*d*.
Time = $\frac{146}{365} = \frac{2}{5}$ year
Rate = $\frac{25}{2}$ p.c.
$\therefore$ Interest = £151, 6*s*. 8*d*. $\times (\frac{2}{5} \times \frac{25}{2} \times \frac{1}{100})$
= £151, 6*s*. 8*d*. $\times \frac{1}{20}$
= £7, 11*s*. 4*d*. **Ans.**

July 16	
Aug. 31	
Sep. 30	
Oct. 31	
Nov. 30	
	146 days.

§11. The student should be on guard to count 29 days for February if the year be a leap one, for instance the time from 1st January 1944 to 19th October 1944 = 30 + 29 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 19 = 292 days or $\frac{292}{365} = \frac{4}{5}$ year.

EXERCISE 138.

Find the simple interest on :—

1.	Rs. 365	for 125 days at 4 p. c.
2.	Rs. 730	for 130 days at 5 p. c.
3.	Rs. 625	for 73 days at $4\frac{1}{2}$ p. c.
4.	Rs. 835	for 146 days at $6\frac{1}{4}$ p. c.
5.	Rs. 730	for 192 days at 10 p. c.
6.	Rs 533, 5*a*. 4*p*.	for 219 days at $2\frac{1}{4}$ p. c.
7.	Rs. 666, 10*a*. 8*p*.	for 292 days at $3\frac{1}{4}$ p. c.

8. £720 from 15th January to 29th March at 3 p. c.
9. £520 from July 1 to September 12 at 5 p. c.
10. £625 from August 1 to October 13 at $3\frac{1}{3}$ p. c.
11. £525 from November 15 to June 22 at 8 p. c..
12. £675 from August 4 to October 16 at $6\frac{1}{4}$ p. c.
13. £716, 13*s*. 4*d*. from July 15 to Dec. 8 at 5 p. c.
14. £728, 6*s*. 6*d*. from Dec. 16, 1941, to July 23, 1942, at $6\frac{1}{4}$ per cent.
15. Rs. 1666, 10*a*. 8*p*. from June 15, 1940, to August 27, 1942, at 4 per cent.
16. A person borrowed Rs. 7300 on November 15, 1943, at 4 per cent. What amount should he repay on July 6, 1944 to clear off the account ?
17. *A* borrowed Rs. 4500 on August 4, 1943, at $3\frac{1}{2}$ per cent. and paid back the sum with interest on Oct. 16, 1943. *B* also borrowed Rs. 3500 on the same day and paid the sum together with interest on Dec. 28, 1943, at 3 per cent. Who paid the interest more and by how much ?
18. *A* borrowed Rs 5000 on Sep. 15, 1942 at 4 per cent. and paid back Rs. 3160 on July 4, 1943. Find what amount should he pay on Nov. 27, 1943, to clear off the amount.
19. Find to the nearest penny the interest on £169, 15*s*. 11*d*. from 9th Nov. 1943 to 3rd March 1944 at $5\frac{3}{4}$ p. c.
20. Rs. 8545 were lent to a person for 2 years 3 months at $3\frac{1}{2}$ p. c.; after the expiry of that period he gave the horse and Rs. 172, 14*a*. $8\frac{2}{5}$*p*. in cash as interest ; find the price of the horse

INVERSE CASES OF INTEREST.

§12. **To find Principal.**

Consider the following examples when *Time. Rate per cent. and Interest or Amount are given.*

Case 1. When the interest is given.

Example 1. What sum of money will produce Rs. 99 interest in 3 years at 4 per cent. ?

Sol. Let us suppose the sum=Rs. 100.

$\therefore$ Interest on Rs. $100 = \frac{100 \times 3 \times 4}{100} =$ Rs. 3×4.

Now by unitary method we shall proceed thus :—

If the interest is Rs. 3×4, the sum $=$ Rs. 100

$\therefore$ If the interest is Re. 1, the sum $=$ Rs. $\frac{100}{3\times4}$

$\therefore$ If ,, ,, is Rs. 99. ,, $=$ *Rs. $\frac{100\times99}{4\times3}$

$=$ Rs. 825. **Ans.**

From the asterisked line we conclude that when time, rate per cent. and interest are given we have the following rule:

Rule. $Principal = \frac{Interest\times100}{Time\times Rate}$

Case 2. When the amount is given.

Example 2. What sum of money, if put out on interest at 4 p. c. will amount to Rs. 560 in 3 years?

Sol. Let us suppose the sum $=$ Rs. 100

$\therefore$ Interest on Rs. 100 $= \frac{100\times4\times3}{100} =$ Rs. 4×3

$\therefore$ Amount $=$ Rs. $(4\times3+100)$.

Now by unitary method we shall proceed thus:

If the amount is Rs. $(4\times3+100)$ the sum $=$ Rs. 100

$\therefore$ If ,, Re. 1 the sum $=$ Rs. $\frac{100}{4\times3+100}$

$\therefore$,, Rs. 560 the sum $=$ *Rs. $\frac{100\times560}{4\times3+100}$

$=$ Rs. 500 **Ans.**

From the asterisked line we conclude that when time, rate p. c. and amount are given we shall have the following rule:—

Rule. $Principal = \frac{Amount\times100}{Time\times Rate+100}$

Example 3. What principal will produce Rs. 172.8*a* interest in 5 years at $3\frac{3}{4}$ p. c.?

Sol. $\because$ Principal $= \frac{\text{Interest}\times100}{\text{Time}\times\text{Rate}}$

$\therefore$ reqd. principal $= \frac{\text{Rs. } 172.8a.\times100}{\frac{15}{4}\times5}$

$= \text{Rs. } \frac{3\frac{1}{5}\times100\times4}{2\times75} = \text{Rs. } 920.$ **Ans**

Example 4. What principal will amount to Rs. 170, 8*a*. in $1\frac{1}{2}$ years at 9% ?

Sol. $\because$ Principal $= \dfrac{\text{Amount} \times 100}{\text{Time} \times \text{Rate} + 100}$

$\therefore$ reqd. principal $=$ Rs. $\dfrac{1702,\ 8a. \times 100}{\frac{3}{2} \times 9 + 100}$ or $\dfrac{\frac{3405}{2} \times 100}{\frac{227}{2}}$

$=$ Rs. $\frac{3405}{2} \times 100 \times \frac{2}{227} =$ Rs. 1500 **Ans.**

EXERCISE 139.

1. What sum of money will produce Rs. 52 as interest in 4 years at $2\frac{1}{2}$ per cent. ?

2. What sum must be put out at simple interest for 3 years at $2\frac{1}{2}$% so as to produce Rs. 62, 8*a*. as interest ?

3. What sum will earn Rs. 189 in $1\frac{1}{2}$ years at 12% ?

4. What sum will earn Rs. 48 in 5 years at $1\frac{1}{2}$% ?

5. On what sum will the simple interest amount to Rs. 180 in $2\frac{1}{4}$ years at 15% ?

6. On what sum will the simple interest amount to Rs. 33, 4*a*. 6*p*. in $2\frac{1}{2}$ years at 5% ?

7. What principal will amount to Rs. 672 in 3 years at 4 per cent. ?

8. What principal will amount to Rs. 840 in 5 years at 8 per cent. ?

9. What principal in 3 years 6 months will become Rs. 1175 at 5 p. c.

10. What sum of money, if put out at simple interest at $3\frac{3}{4}$ p. c. will amount to Rs. 2805 in $4\frac{1}{2}$ years ?

11. What sum will amount to Rs. 8414, 4*a*. in $2\frac{1}{4}$ years at $3\frac{1}{2}$ p. c. simple interest ?

12. How much must I put in a bank so that in 7 years at $2\frac{1}{2}$ per cent, S. I. I may withdraw £8812, 10*s*. altogether ?

13. What sum should Tom deposit in the savings account so that in $2\frac{1}{2}$ years at $7\frac{1}{2}$ p. c. he may withdraw £199, 10*s*. altogether ?

14. What sum of money if put in a bank for $2\frac{3}{4}$ years at $3\frac{3}{4}$ p. c., simple interest will amount to Rs. 1765 ?

15. What sum of money in $2\frac{1}{2}$ years at 5 p. c. simple interest will amount to Rs. 883, 2*a*.

§13. **To find rate per cent.**

When Principal, Amount or Interest and Time are given, Rate per cent. is found as shown in the following example:—

Example 1. At what rate per cent. will the simple interest on Rs. 520 be Rs. 104 in 4 years ?

Sol. By unitary method we proceed thus:

$\because$ Interest on Rs. 520 for 4 years $=$ Rs. 104

$\therefore$ „ „ Re. 1 for 4 years $=$ Rs. $\frac{104}{520}$

$\therefore$ „ „ Re. 1 for 1 year $=$ Rs. $\frac{104}{520\times 4}$

$\therefore$ „ „ Rs. 100 for 1 year $=$ *Rs. $\frac{104\times 100}{520\times 4}$

$=5\%$. **Ans.**

Hence from the asterisked line we have the following rule:

Rule. *Rate per cent.* $= \frac{Interest\times 100}{Principal\times Time}$.

Example 2. At what rate per cent. will Rs, 720 amount to Rs. 774 in 3 years ?

Sol. interest $=$ Rs. 774 $-$ Rs. 720 $=$ Rs. 54.

$\because$ rate per cent. $= \frac{\text{Interest}\times 100}{\text{Principal}\times \text{Time}}$

$\therefore$ reqd. rate $= \frac{54\times 100}{720\times 3} = \frac{5}{2} = 2\frac{1}{2}\%$. **Ans.**

EXERCISE 140.

1. At what rate per cent. will Rs. 550 gain Rs. 66 in 3 years ?
2. The interest on Rs. 350 amounted to Rs. 42 in 3 years ; find the rate per cent.
3. At what rate per cent. will the interest on Rs. 750 be Rs. 150 in 4 years ?
4. At what rate per cent. will Rs. 600 gain Rs. 120 in 5 years ?
5. At what rate per cent. will Rs. 400 amount to Rs. 428 in 2 years ?
6. At what rate per cent. will Rs. 2000 amount to Rs. 2135 in 3 years ?
7. Rs. 800 amounted to Rs. 980 in 3 years ; find the rate per cent.

8. Rs. 600 amounted to Rs. 654 in $2\frac{1}{4}$ years; find the rate per cent.

9. At what rate per cent. will Rs. 1225 amount to Rs. 1439, 6*a* in $2\frac{1}{2}$ years?

10. The interest on £716 amounted to £134, 5*s*. in. $3\frac{3}{4}$ years; find the rate per cent.

11. At what rate per cent. will Rs. 1275, 13*a*. 4*p*. amount to Rs. 1371, 8*a*. 4*p*. in $2\frac{1}{2}$ years?

12. At what rate per cent. will Rs. 1327, 10*a*. gain Rs 398, 4*a*. $7\frac{1}{2}$*p*. in 6 years?

13. At what rate per cent. will Rs. 522, 1*a*. 4*p*. gain Rs. 130, 8*a*. 4*p*. in 5 years.

14. At what rate per cent. will Rs. 533, 5*a*. 4*p*. amount to Rs. 595, 13*a*. 4*p*. in $2\frac{1}{2}$ years?

15. Find the rate per cent. if Rs. 266, 4*d*. amount to Rs. 299, 8*a*. 6*p*. in 5 years.

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§14. **To find Time.**

The method of finding Time when Principal, Rate per cent. and Amount or Interest are given will be understood from the following examples:—

Example 1. In what time will the interest on Rs. 800 amount to Rs. 160 at 5 per cent. per annum?

Sol. Interest on Rs. 800 at 5 p. c. for 1 year = Rs. $\frac{800\times5}{100}$.

Now by unitary method we proceed as follows:—

$\because$ Rs. $\frac{800\times5}{100}$ is the interest for 1 year,

$\therefore$ Re. 1 is the interest for $\frac{100}{800\times5}$ years,

$\therefore$ Rs. 160 is the interest for* $\frac{100\times160}{800\times5}$ years.

$\therefore$ the required time = 4 years. **Ans.**

Hence from the asterisked line we have the following

Rule. $Time = \frac{Interest\times100}{Principal\times Rate\ per\ cent.}$

Example 2. In what time will Rs. 560 amount to Rs. 700 at 4 per cent.?

Sol Interest = Rs. 700 — Rs. 560 = Rs. 140.

$$\therefore \text{Time} = \frac{\text{Interest} \times 100}{\text{Principal} \times \text{Rate per cent.}}$$

$$\therefore \text{reqd. time} = \frac{140 \times 100}{560 \times 4} = \frac{25}{4} = 6\tfrac{1}{4} \text{ years.}$$ **Ans.**

EXERCISE 141.

1. In what time will Rs. 750 at simple interest produce Rs. 150 at 5 per cent.?

2. In what time will the interest on Rs. 825 amonnt to Rs. 99 at 4 per cent. ?

3. In what time will Rs. 1025 amount to Rs. 1312 at 7 per cent. ?

4. In what time will Rs. 720 at simple interest produce Rs. 120 at $2\frac{1}{2}$ per cent. ?

5. In what time will the interest on Rs. 575 amount to Rs. 362, 4*a*. at 9 per cent. ?

6. In what time will the interest on Rs. 721, 4*a*. amount to Rs. 144, 4*a*. at 5 per cent. ?

7. In what time will Rs. 729, 4*a*. amount to Rs. 957, 4*a*. 3*p*. at $6\frac{1}{4}$ per cent. ?

8. In what time will a sum of money double itself at 10 per cent. ?

9. In what time will a sum of money treble itself at 8 per cent. ?

10. In what time will Rs. 481, 10*a*. produce interest equal to the principal at 15 per cent.?

11. In what time will the interest on any sum of money at 5 per cent. be ·125 of the principal ?

12. In what time will Rs. 845, 4*a*. amount to Rs. 1056, 9*a*. at $1\frac{1}{2}$ *p*. per rupee per month. ?

13. In what time will a sum of money treble itself at $12\frac{1}{2}$ per cent. ?

14. A person borrowed Rs. 1525 at $2\frac{1}{2}$ per cent. on July 5 and after a certain period paid Rs. 7, 10*a*. as interest, find the date of payment.

15. on February 1, 1942, a person borrowed £500 at 5 p.c. promising to return it as soon as the interest amounted to £10; on what date did the loan expire ?

16. In how many years will 450 fr. amount to 576 fr. at $3\frac{1}{2}$ p. c. simple interest ?

☞ §15. **Some Important Typical Examples.**

Example 1. If Rs.566, 10*a*. 8*p*. amount to Rs. 680 in 4 years, what will Rs. 728, 5*a*. 4*p*. amount to in $2\frac{1}{2}$ years at the same rate p.c. per annum?

Sol. Int. on Rs.566, 10*a*. 8*p*.=Rs. 680—Rs. 566, 10*a*. 8*p*.
=Rs. 113, 5*a*. 4*p*. =Rs. $\frac{340}{3}$

$\therefore$ rate p. c. $=\frac{\frac{340}{3}\times 100}{566\frac{2}{3}\times 4}=\frac{340\times 100\times 3}{1700\times 4\times 3}=5$

Now int. on Rs. 728, 5*a*. 4*p*.$=\frac{(\text{Rs. } 728, 5a.\ 4p.)\times 5\times 5}{100\times 2}$

$=\frac{\text{Rs. } 728, 5a.\ 4p.}{8}$

=Rs. 91. 0*a*. 8*p*.

$\therefore$ the reqd. amount =Rs. 728, 5*a*. 4*p*.+Rs.91,8*p*.
=Rs. 819, 6*a*. **Ans.**

Example 2. A sum of money in $3\frac{1}{2}$ years at $6\frac{1}{4}\%$ amounts to £781, 19*s*. What will it amount to in 4 years at $7\frac{1}{2}\%$?

Sol. Amount =£781, 19*s*. =£$781\frac{19}{20}$=£$\frac{15639}{20}$.

Time $=\frac{7}{2}$ years

Rate $=\frac{25}{4}$ p.c.

$\therefore$ Principal= £ $\frac{\frac{15639}{20}\times 100}{(\frac{7}{2}\times\frac{25}{4})+100}=\frac{£15639\times 5}{\frac{975}{8}}$

$=$ £ $\frac{15639\times 5\times 8}{975}=\frac{3208}{5}$=£641, 12*s*.

Now Interest on £ 641, 12*s*. for 4 years at $7\frac{1}{2}\%$

$=\frac{(£641,\ 12s.)\times 4\times 15}{100\times 2}$

$=\frac{(£641,\ 12s.)\times 3}{10}$

$=\frac{£1924,\ 16s.}{10}$=£192, 9*s*. 7·2*d*.

$\therefore$ Amount=£641, 12*s*. +£192, 9*s*. 7·2*d*.
=£834, 1*s*. 7·2*d*. **Ans.**

Example 3. A certain sum of money amounts to Rs. 768 in 4 years and to Rs. 832 in 6 years. Find the sum and the rate per cent.

Sol. Rs. 832 —Rs. 768=Rs. 64.

i.e., Rs. 64 is the interest for 2 years.

$\therefore$ Interest for 4 years =Rs. 128.

$\therefore$ Principal = Rs. 768 — Rs. 128 = Rs. 640. **Ans.**

$\because$ Rate $= \dfrac{\text{Int.} \times 100}{\text{Principal} \times \text{Time}}$

$\therefore$ reqd. rate $= \dfrac{128 \times 100}{640 \times 4} = 5$ p. c. **Ans.**

Example 4. A certain sum amounts to £583 at $2\frac{1}{2}\%$ but at $\frac{1}{2}\%$ less rate to £572, 8*s*.,: find the time and sum.

Sol. Interest at $\frac{1}{2}$ p c. on the principal = £583 — £572, 8*s*.
= £10, 12*s*. = £$\frac{53}{5}$
= £$\frac{53}{5} \times 2 \times \frac{5}{2}$ = £53.

$\therefore$ Int. at $2\frac{1}{2}$ p. c. = £53

$\therefore$ Principal = £583 — £53 = £530

and time $= \dfrac{53 \times 100}{530 \times \frac{5}{2}} = 4$ years. } **Ans.**

Example 5. £320 amount to £384 at a certain rate in a certain time; also £625 amount to £775 in the same time at 1% higher rate; find the rate and time.

Sol. (1) Int. on £320 = £384 — £320 = £64

$$\therefore R \times T = \frac{64 \times 100}{320} = 20.$$

(2) Int. on £625 = £775 — £625 = £150

$$\therefore (R+1) \times T = \frac{150 \times 100}{625} = 24$$

$$\therefore \frac{R \times T}{(R+1) \times T} = \frac{20}{24} = \frac{5}{6}; \quad \text{or} \quad \frac{R}{R+1} = \frac{5}{6}$$

$\therefore$ $5(R+1) = 6R$ or $5R + 5 = 6R$

$\therefore R = 5\%$; higher $R = 6\%$ and $T = 4$ yrs. **Ans.**

Example 6. A person invested Rs. 1200 for 4 years and Rs. 900 for 3 years and the total interest from these investments was Rs. 375; find the rate per cent.

Sol. In the first case he invested Rs. 1200×4.
= Rs. 4800 for one year.
In the second case he invested Rs. 900×3
= Rs. 2700 for one year.

$\therefore$ total Investment for one year = Rs. 4800 + Rs. 2700
= Rs. 7500.

$\because$ Rate $= \dfrac{\text{Int.} \times 100}{\text{Principal} \times \text{Time}}$

$\therefore$ reqd. rate $= \frac{375 \times 100}{7500 \times 1} = 5$ p. c. **Ans.**

Example 7. A person invested £3200 at a certain rate per cent. and £2800 at 1 per cent. higher rate. The total interest from these investments in 4 years was £952; find the rates of interest.

Sol. Interest on £3200+£2800 for 4 years at the certain rate+ int. on £2800 for 4 years at 1 per cent. = £952.

or int. on £6000 for 4 years $+ \frac{2800 \times 4 \times 1}{100} =$ £952

or int. on £6000 for 4 years + £112 = £952

$\therefore$ int. on £6000 for 4 years at the certain rate = £840

$\therefore$ certain rate $= \frac{840 \times 100}{6000 \times 4} = 3\frac{1}{2}$ p. c.

and second rate $= 3\frac{1}{2} + 1 = 4\frac{1}{2}$ p. c. } **Ans.**

Example 8. A man had Rs. 1200, part of which he lent at 5 per cent. and a part at 4 p. c. He got Rs. 106 as interest after 2 years. How much did he lend at 5 per cent.?

Sol. If he had lent the whole sum at 5 per cent., the interest would have been Rs. $\frac{1200 \times 5 \times 2}{100} =$ Rs. 120; but the actual interest is Rs. 106, *i. e.* the interest would have been increased by Rs. 14.

This increase of Rs. 14 will be decreased in the interest of the second sum.

Interest on Rs. 100 at 5 per cent. for 2 years = Rs. 10.
Interest on Rs. 100 at 4 per cent. for 2 years. = Rs. 8.
i.e., there is an increase of Rs. 2 per hundred
$\therefore$ increase of Rs. 14 will be on Rs. $\frac{100}{2} \times 14 =$ Rs. 700.
$\therefore$ he lent Rs. 1200 — Rs. 700 = Rs. 500 at 5 p.c. **Ans.**

Aliter. Suppose he lent Rs. 1200 at 5 p. c. for 2 years.

$\therefore$ interest = Rs. $\frac{1200 \times 5 \times 2}{100} =$ Rs. 120

i. e., the int. will be increased by Rs. 120 — Rs. 106 = Rs. 14.

Now suppose he lent Rs. 1200 at 4 per cent. for 2 yrs.

$\therefore$ int. = Rs. $\frac{1200 \times 4 \times 2}{100} =$ Rs. 96

i, e. the int. will decrease by Rs. 106 — Rs. 96 = Rs. 10.

$\therefore$ the ratio between the two investments will be inverse of 14 : 10, *i. e.*, 10 : 14 or 5 : 7.

$\therefore$ investment at 5 p. c. $= \text{Rs.} \times \frac{5}{12} \times 1200 = \text{Rs. } 500$. **Ans.**

Aliter. The mean int. for 2 years per Rs, 100 $= \text{Rs.} \frac{106 \times 100}{1200} = \text{Rs.} \frac{53}{6}$

int. at 5 p. c.	int. at 4 p. c
Rs. 10	Rs. 8
Rs. $\frac{53}{6}$	
Rs. $\frac{53}{6} - 8$: Rs. $10 - \frac{53}{6}$	
or $\frac{5}{6} : \frac{7}{6}$	
or 5 : 7	

By the Alligation Rule, ratio between the investsments at 5 p. c. and 4 p. c. = 5 : 7

$\therefore$ investment at 5 p. c. $= \text{Rs.} \frac{5}{12} \times 1200 = \text{Rs. } 500$. **Ans.**

Example 9. A person had Rs. 1343, 5*a*. 4*p*., part of which he lent at 5 p c. for 4 years and the rest at $6\frac{1}{4}$ p. c. for 3 years the interest he received, in both the cases, was equal ; how much did he lend at $6\frac{1}{4}$ p. c ?

Sol. Suppose he lent Rs. 1343, 5*a* 4*p*. at 5 p. c. for 4 years

$\therefore$ int. $= \frac{\text{Rs } 1343,\ 5a\ \ 4p \times 5 \times 4}{100}$

$= \text{Rs. } 268,\ 10a.\ 8p. = \text{Rs. } \frac{806}{3}$.

Now suppose he lent Rs. 1343, 5*a* 4*p*. at $6\frac{1}{4}$ p c. for 3years,

$\therefore$ int. $= \frac{\text{Rs } 1343,\ 5a.\ 4p. \times 25 \times 3}{100 \times 4}$

$= \text{Rs.} \frac{4030}{16} = \text{Rs.} \frac{2015}{8}$

Since both the interests are equal,

$\therefore$ ratio bteween the two investments $= \frac{2015}{8} : \frac{805}{3}$

$= 6045 : 6448.$

$= 15 : 16.$

$\therefore$ investment at $6\frac{1}{4}$ p c, $= \frac{16}{31}$ of Rs. 1343, 5*a*. 4*p*.

$= \text{Rs. } 693.\ 5a.\ 4p.$ **Ans.**

Aliter. Suppose first investment $= \text{Rs. } 100.$

$\therefore$ int. $= \frac{100 \times 5 \times 4}{100} = \text{Rs. } 20$

$\therefore$ int. on the second investment at $6\frac{1}{4}$ p. c. for 3 yrs. is also Rs. 20.

$\therefore$ second sum $= \frac{\text{Int} \times 100}{\text{Time} \times \text{Rate}} = \frac{20 \times 100}{6\frac{1}{4} \times 3}$

$= \frac{20 \times 100 \times 4}{25 \times 3} = \text{Rs. } \frac{320}{3}$

$\therefore$ If first investment = Rs. 100, the second = Rs. $\frac{320}{3}$

$\therefore$ the ratio between the two investments = $100 : \frac{320}{3}$

$= 300 ; 320$

$= 15 : 16.$

We can now proceed further and get the answer.

Example 10. A person had Rs. 3500. He lent a part of it at 4 p. c. for 3 years and the remaining at $4\frac{1}{2}$ p. c. for $2\frac{1}{2}$ years. The interest in the first case is Rs. 48 more thon the interst received in the second case. Find both the sums

Sol. Principal when int. is Rs. 48, time 3 yrs. and rate 4 p. c. $= \frac{48 \times 100}{3 \times 4} =$ Rs. 400.

$\therefore$ remaining sum = Rs. 3500 − Rs. 400 = Rs. 3100.

Now divide Rs. 3100 into two such parts that the interest on the first part at 4 p. c. in 3 years may be equal to the interest on the second part at $4\frac{1}{2}$ p. c. in $2\frac{1}{2}$ yrs. [*See* Ex. 9] and then add Rs. 400 to the first part.

Example 11. What annual payment will discharge a debt of Rs. 848 due in 4 years, the rate of interest being 4% ?

Sol Let x rupees be the annual payment.

Amount of Rs. x in 3 yrs. at 4 p. c. $=$ Rs. $x + \frac{x \times 3 \times 4}{100} = \frac{112x}{100}$

Amount of Rs. x in 2 yrs. at 4 p. c. $=$ Rs. $x + \frac{x \times 2 \times 4}{100} = \frac{108x}{100}$

Amount of Rs. x in 1 yr. at 4 p. c. $=$ Rs. $x + \frac{x \times 1 \times 4}{100} = \frac{104x}{100}$

These amounts and the last payment of Rs. x, will discharge the debt

$\therefore \quad \frac{112x}{100} + \frac{108x}{100} + \frac{104x}{100} + x =$ Rs. 848

$\therefore \quad \frac{424}{100}x =$ Rs. 848

$\therefore \quad x =$ Rs. $848 \times \frac{100}{424} =$ Rs. 200 **Ans.**

Example 12. Divide Rs. 945, 12*a*. 4*p*. into three such parts that the amount of these parts after $2\frac{1}{2}$, 4 and 5 years respectively may all be equal, the rate of interest being 5% per annum.

Sol. Amount of first part $= \frac{\text{first part} \times 5 \times 5}{100 \times 2} + \text{first part}$

$= \frac{\text{first part}}{8} + \text{first part}$

$= \text{first part}(\frac{1}{8} + 1) = \frac{9}{8} \times \text{first part}$

Similarly amount of 2nd part $= \frac{6}{5} \times$ second part

and ,, ,, 3rd part $= \frac{4}{5} \times$ third part

$\therefore \frac{9}{8} \times$ first part $= \frac{6}{5} \times$ 2nd part $= \frac{5}{4} \times$ third part

$\therefore$ first part : 2nd part : 3rd part $= \frac{8}{9} : \frac{5}{6} : \frac{4}{5} = 80 : 75 : 72$

Now divide the given sum in the ratio of 80 : 75 : 72.

EXERCISE 142.

1. Find the simple interest on Rs. 365, 8*a*. from August 14, 1944, to March 21, 1945, at $3\frac{1}{8}$ per cent.

2. Find the simple interest on Rs. 756, 4*a*. from January 5, 1944, to May 30, 1944 at $3\frac{3}{4}$ per cent.

3. Find the amount of Rs. 2560 at $2\frac{1}{4}$ p. c. from June 15, 1944, to January 20, 1945.

4. Inder borrowed Rs. 785 for 3 years at 3*p*. per rupee per month and Sundar borrowed Rs. 665 for 4 years 2 months at the same rate ; who will pay more interest and by how much ?

5. What sum of money will amount to Rs. 540, 10*a*. in $2\frac{1}{2}$ years at $3\frac{1}{4}$ per cent. ?

6 At what rate p. c. will Rs. 325 amount to Rs. 349, 6*a*. in $2\frac{1}{2}$ years ?

7. In what time will Rs. 521, 15*a*. 11*p*. double itself at 8 per cent. ?

8. The interest of a certain sum amounts to Rs. 70, 5*a*. in 5 years at $2\frac{1}{4}$ p. c. ; find it.

9. At what rate p. c. will the interest on Rs. 457, 8*a*. amount to Rs. 114, 6*a*. in 8 years ?

10. In what time will Rs. 965 amount to Rs. 1206, 4*a*. at 5 per cent. ?

11. Amount of a certain sum in 3 years is Rs. 545 and in 4 years Rs. 560 ; find the sum and the rate per cent.

12. Amount of a certain sum in 3 years is Rs. 575 and in 5 years, Rs. 625 ; find the sum and the rate p. c.

13. A certain sum amounts to Rs. 600 in 4 years ; if the rate of interest be increased by one p. c. it amounts to Rs. 620. Find the sum and the rate per cent.

14. A certain sum amounts to Rs. 456 in $3\frac{1}{2}$ years and Rs. 464 in 4 years ; find it and also find the rate per cent.

15. What sum of money will amount to Rs. 1062 in $5\frac{1}{4}$ years at $3\frac{3}{7}$ per cent ?

16. What sum of money will produce Rs. 21, 8*a*. in 146 days at 5 per cent ?

17. Rs. 1000 were borrowed on July 1, 1947, at $3\frac{1}{2}$ p. c. and after a time the account was cleared by paying Rs. 1014; find the date of payment.

18. At what rat per cent. will the interest of a sum in $2\frac{1}{2}$ years amount to $\frac{1}{12}$ of the principal ?

19. At what rate per cent. will the interest of a sum in $2\frac{2}{3}$ years amount to $\frac{2}{15}$ of the principal ?

20. What sum of money will amount to Rs. 829, 8*a*. $7\frac{1}{5}p$, from July 15 to September 26 at $2\frac{3}{4}$ per cent. ?

21. What sum of money will produce Rs. 121, 4*a*. 6*p*. from March 13 to July 16 at $9\frac{1}{8}$ per cent. ?

22. If Rs. 621, 4*a*. amount to Rs. 745, 8*a*. in 4 years, what will Rs. 662, 10*a*. 8*p*. amount to in $2\frac{1}{2}$ years at the same rate p. c. per annum simple interest ?

23. If Rs. 525 amount to Rs. 616, 14*a*. in $4\frac{2}{3}$ yrs., what will Rs. 780 amount to in 4 years at the same rate p. c. per annum simple interest ?

24. A person lent Rs. 1200 for 4 years and Rs. 900 for 3 years and thus got Rs. 375 as interest; find the rate p. c.

25. A person lent Rs. 800 for 5 years and Rs. 900 for 4 years and thus got Rs. 304 as interest; find the rate p. c.

26. A person lent Rs. 625 for 4 years, Rs. 300 for 5 years and Rs. 820 for 2 years and thus got Rs. 282 as interest; find the rate per cent.

27. A sum of money in 4 years at 5 p. c. amounts to £864, 12*s* ; what will it amount to in 3 years at $6\frac{1}{4}$ p. c. ?

28. A person had Rs. 2000, part of which he lent at 5 p. c. and the remainder at $4\frac{1}{2}$ p. c. He got Rs. 282 as interest after 3 years. How much did he lend at $4\frac{1}{2}$ p. c. ?

29. A person had Rs. 2100. He lent a part of it at 5 p. c. and the remainder at 4 p. c.; after three years he found that the difference between the interests was Rs. 9 only. Find both the sums.

30. Interest of a certain sum in 5 years at 5 p. c. is more than the interest of the same sum in 6 years at $3\frac{1}{2}$ p. c. by Rs. 260; find the sum.

31. Interest of a certain sum in 8 years at 5 per cent. is greater than the interest of the same sum in 9 years at 3 per cent. by Rs. 169 ; find the sum.

32. Divide Rs. 2000 into two such parts that if the first be put out at S I. for 6 years at $3\frac{1}{2}$ p c. and the second for 3 years at $4\frac{1}{2}$ p. c the interest on the first sum shall be double that on the second.

33. A person invested £2500 at a certain rate per cent. and £1500 at 1 p. c. higher rate and the total interest from these investments in 3 year was £525 ; find the rates.

34. A person invested £1200 at a certain rate per cent. and £1600 at $2\frac{1}{2}$ p. c. less rate and the total interest from these investments in 4 years was £680 ; find the rates.

35. A person had Rs. 1566, 10*a*. 8*p*.; part of which he lent at 5 p. c. for 3 years and the rest at $4\frac{1}{2}$ p. c. for $4\frac{1}{2}$ years. The interest realised in both the cases was the same. How much did he lend at 5 p. c. ?

36. What annual payment will discharge a debt of Rs. 1620 due in 5 years at 4 p. c.?

37. Divide £182, 3*s*. 4*d*. into three parts in such a way that their amounts after 1, 2 and 3 years respectively may be equal, the rate of interest being 4 per cent.

38. *A*, *B* and *C* borrowed Rs. 9746 in all from a money lender at 5 p. c. interest and paid back equal amounts in full settlement of their respective debts after 2, 5 and 6 years respectively. Find the sum borrowed by each.

39. £450 amount to £ 504 at a certain rate per cent. and in a certain time, also £720 amount to £828 in the same time at 1 p. c. higher rate. Find the rate and time.

40. £360 amount to £432 in a certain time at a certain rate per cent. ; also £ 540 amount to £621 in a year less at the same rate ; find the rate and time.

41. A certain sum amounts at 5 p c. to £ 780 but at $\frac{3}{4}$ p. c. less rate to £ 760, 10s. Find the rate and sum.

42. Chandra Mohan Gupta bought a fountain pen for Rs. 10 on an instalment system agreeing to pay the price in 12 monthly instalments of Re. 1 each. What rate of interest has he charged ?

43. Two sums of money differing by Rs.200 are lent out, the larger at 4 p. c. and the smaller at 6 p. c. per annum ; if the difference in the amount of two sums be Rs. 170 at the end of five years, find the sums that were lent.

44. Three years ago, a man borrowed Rs. 2000 from a bank at 5 p.c. per annum. At the end of the year he paid Rs. 500. Similarly he paid Rs. 500 at the end of the second year. What sum must he now pay to clear off the debts.

CHAPTER XXVI

COMPOUND INTEREST

§ 1. We have mentioned in Chapter XXV that when the consideration for the use of money borrowed is paid yearly, half-yearly or monthly as agreed upon and the principal remains the same, the consideration thus paid is called the simple interest. But sometimes the interest is not paid as stated above and is added yearly or half-yearly to the principal and this amount becomes the principal for the next period. This system of adding the interest to the principal continues till the end of the specified time. Money, in this case, is said to be at compound interest and the difference between the final amount and the original principal is called the Compound Interest.

Thus simple interest on Rs. 200 at 5 per cent. in 2 years = Rs. 20.

But the compound interest as stated above will be reckoned as follows :—

Interest on Rs. 200 at 5 per cent. in one year = Rs. 10

Now the principal for the next year would be Rs. 200 + Rs. 10 = Rs. 210.

Interest on Rs. 210 at 5 per cent. in one year = Rs. 10, 8*a*.

∴ final amount = Rs. 210 + Rs. 10, 8*a*. = Rs. 220, 8*a*.

∴ compound interest = Rs. 220, 8*a*. − Rs. 200

= Rs. 20, 8*a*. Ans.

Note 1. Compound interest is evidenty always more than the simple interest.

2. The compound interest might also be obtained by adding together the interests for the 1st year and 2nd. year.

§2. It is convenient to work sums in compound interest in decimals. The following examples will illustrate the method clearly :—

Example 1. Find the compound interest on Rs. 500 for 3 years at 5. p. c.

Sol. Rs. 500 =Principal for 1st year.

5 (multiply by 5 and divide by 100)

Add { 25.00 =interest for 1 year,
500

525 =Principle for 2nd year

5 (multiply by 5 and divide by 100)

Add 26·25

525

551·25 =Principal for 3rd year

5 (multiply by 5 and divide by 100)

Add 27·5625 =Interest for 3rd year.

551·25

578·8125 =Total amount

500 =Principal

78·8125 =Compound int. for 3 yrs

=Rs. 78, 13*a*. **Ans,**

Note. Division by 100 is done mentally as explained in Art 5. Chapter VIII.

Aliter. Amount of Rs. 100 at the end of 1 year=Rs. 105

$\therefore$ " " Re. 1 " " " " =Rs. $\frac{105}{100}$

$\therefore$ " " Rs. 500 " =Rs. $\frac{105}{100}$ of Rs. 500

$\therefore$ " " Rs. 500 at the end of 2 years

$=\frac{105}{100}$ of ($\frac{105}{100}$ of Rs. 500)

$=(\frac{105}{100})^2$ of Rs. 500

$\therefore$ " " Rs. 500 at the end of 3 years

$=\frac{105}{100}$ of $(\frac{105}{100})^2$ of Rs. 500)

$=(\frac{105}{100})^3$ of Rs. 500.

Hence the following **Fromulae—**

$$\text{Amount} = \text{Principal}\left(1+\frac{\text{rate}}{100}\right)^{\text{No. of yrs.}}$$

$$\text{Compound intererst} = \text{Principal}\left\{\left(1+\frac{\text{rate}}{100}\right)^{\text{No. of yrs.}}-1\right\}$$

or briefly $A = P\left(1+\frac{r}{100}\right)^n$ ... ...(1)

C. I. $= P.\left\{\left(1+\frac{r}{100}\right)^n - 1\right\}$ ....(2)

We can apply decimals to this second method also, thus:—

Amount $= (\frac{105}{100})^3$ of Rs. 500.
$= \text{Rs } \frac{105}{100}\times\frac{105}{100}\times\frac{105}{100}\times 500$
$=$ Rs. $1{\cdot}05\times 1{\cdot}05\times 1{\cdot}05\times 500$
$=$ Rs. 578·8125.
$\therefore$ C.I. $=$ Rs. 578·8125 $-$ Rs. 500
$=$ Rs. 78·8125 $=$ Rs. 78, 13*a*. Ans.

Exampie 2. Find the compound interest on Rs. 512, 8*a*. for 3 years at $4\frac{1}{2}$ per cent.

Sol. Rs. 512, 8*a*. $=$ Rs. 512·5; $4\frac{1}{2} = 4{\cdot}5$.

Rs. 512·5 = Principal for Ist year
4·5
25625
20500
Add { 23·0625 = Interest for 1 year
512·5
535·5625 = Principal for 2nd year
4·5
26778125
21422500
Add { 24·1003125 = Interest for 2nd year
525·5625
559·6628125 = Principal for 3rd year
4·5
27983140625
22386512500
Add { 25·1848265625 = Interest for 3rd year
559·6628125
Subtract { 584·8476390625 = Total amount
512·5
72·347690625 = Compound interest
= Rs. 72, 5*a*. 6*p*. nearly. Ans.

Or by formula :

$$\text{Amount} = \left(1+\frac{4\frac{1}{2}}{100}\right)^3 \times \text{Rs. } 512,\ 8a.$$

$$= \frac{209}{200}\times\frac{209}{200}\times\frac{209}{200}\times 1025/2$$

$$= \text{Rs. } \frac{9357562225}{16000000} = \text{Rs. } \frac{9357{\cdot}562225}{16}$$

$$= \text{Rs. } 584{\cdot}8476390625$$

But Principal = Rs. 512·5

$\therefore$ C. I. = Rs. 72·3476390625 = Rs. 72, 5*a*. 6*p*. **Ans.**

§3. Interest payable half-yearly or quarterly.

When the interest is payable half-yearly, calculate the compound interest for *double* the number of years at *half* the rate per cent. and when it is payable quarterly, calculate it for 4 times the number of years at $\frac{1}{4}$ the rate per cent.

Example 3. Find the compound interest on Rs. 666, 10*a*. 8*p*. for $1\frac{1}{2}$ years at 5 per cent. per annum payable half-yearly.

Note. Since there are two half years in a year therefore C. I. in $1\frac{1}{2}$ years at 5 p. c. payable half yearly is the same as C. I. in 3 years at $2\frac{1}{2}$ per cent.

Sol. $\because$ Rs. 666, 10*a*. 8*p*. = Rs. $\frac{2000}{3}$.

$\therefore$ find the C. I. on Rs. 2000 at $2\frac{1}{2}$ per cent. in 3 years and divide the result by 3.

By formula C. I. $= \text{Rs. } \left\{\left(1+\frac{2\frac{1}{2}}{100}\right)^3 - 1\right\} \times 2000$

$$= \text{Rs. } \left\{\frac{41}{40}\times\frac{41}{40}\times\frac{41}{40} - 1\right\} \times 2000$$

$$= \text{Rs. } \left\{\frac{68921}{64000} - 1\right\} \times 2000$$

$$= \frac{4921}{64000}\times 2000 = \text{Rs. } \frac{4921}{32}$$

$$= \text{Rs. } 153,\ 12a.\ 6p.$$

Now divide this result by 3.

$\therefore$ required C. I. = Rs. 51, 4*a*. 2*p*. **Ans.**

Example 4. Find the compound interest on Rs. 333, 5*a*. 4*p*. for $1\frac{1}{2}$ years at 8 p. c. per annum payable quarterly.

Note. Since there are 4 quarters in a year, therefore, C. I. in $1\frac{1}{2}$ years at 8 p.c. payable quarterly is the same as C. I. in 6 years at 2 p.c.

Sol. ∵ Rs. 333, 5*a*. 4*p*. = Rs. $\frac{1000}{3}$;

∴ find the C. I. on Rs. 1000 for 6 years at 2 per cent. and then divide the result by 3.

C. I. = Rs. $\{(1+\frac{2}{100})^6 - 1\} \times 1000$

= Rs. $\{\frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} \times \frac{102}{100} - 1\} \times 1000$

= Rs. $\{1{\cdot}02 \times 1{\cdot}02 \times 1{\cdot}02 \times 1{\cdot}02 \times 1{\cdot}02 \times 1{\cdot}02 - 1\} \times 1000$.

= Rs. $(1{\cdot}126162419264 - 1) \times 1000$

= Rs. $\cdot 126162419264 \times 1000$

= Rs. 126·162419264 = Rs. 126, 2*a*. 7*p*. nearly.

∴ required C. I. = Rs. 42, 0*a*. 10*p*. nearly. **Ans.**

§4. **To find C. I. when the given time is not an exact number of years.**

Example 5. Find the compound interest on Rs. 1800 for 2 years 9 months at 4 per cent.

Sol.

Rs. 1800 = Principal for the 1st year.

4

Add { 72·00 = Interest for the 1st year.

1800

1872 = Principal for the 2nd year

4

Add { 74·88 = Interest for the 2nd year.

1872

1946·88 = Principal for the third year.

3 (See note below.)

Add { 58·4064 = Int. for 9 months of the 3rd year.

1946·88

Subtract { 2005·2864 = Total amount

1800

205·2864 = Required C. I.

= Rs. 205, 4*a*. 7*p*. nearly. **Ans.**

Note. Interest of Rs. 100 for 9 months at 4 p c. would be Rs. 3 ; ence in calculating the interest for 9 months of the 3rd year we have aken rate 3 p. c. instead of 4.

EXERCISE 143.

Find the compound interest on:—

1. Rs. 100 for 2 years at 5 per cent.
2. Rs. 200 for 2 years at 4 per cent.
3. Rs. 250 for 3 years at 5 per cent.
4. Rs. 500 for 2 years at 6 per cent.
5. Rs. 750 for 3 years at 5 per cent.
6. Rs. 1500 for 3 years at 4 per cent.
7. Rs. 2100 for 3 years at 5 per cent.
8. £1200 for 2 years at $2\frac{1}{2}$ per cent.
9. £1500 for 3 years at $3\frac{3}{4}$ per cent.
10. £1800 for 4 years at $3\frac{1}{2}$ per cent.

Find to the nearest pie or penny the compound interest on:—

11. Rs. 625, 4*a.* for 2 years at 5 per cent.
12. Rs. 821, 4*a.* for 3 years at 6 per cent.
13. £521, 5*s.* for 2 years at $3\frac{1}{2}$ per cent.
14. £625, 10*s.* for 3 years at $4\frac{1}{2}$ per cent.
15. Rs. 833, 5*a.* 4*p.* for 2 years at 5 per cent.
16. Rs. 566, 10*a.* 8*p.* for 3 years at $2\frac{1}{2}$ per cent.
17. Rs. 1266, 10*a.* 8*p.* for 3 years at 8 per cent.
18. £1233, 6*s.* 8*d.* for 2 years at 6 per cent.
19. £1666, 13*s.* 4*d.* for 2 years at 4 per cent.
20. £1353, 6*s.* 8*d.* for 3 years at 10 per cent.

Find the amount at compound interest of:—

21. Rs. 512, 8*a.* for $2\frac{1}{2}$ years at 5 per cent.
22. Rs. 621, 4*a.* for $2\frac{3}{4}$ years at 4 per cent.
23. Rs. 1600 for $2\frac{1}{4}$ years at $4\frac{1}{2}$ per cent.
24. Rs. 1533, 5*a.* 4*p.* for $2\frac{1}{2}$ years at 5 per cent.
25. £1626, 13*s.* 4*d.* for $3\frac{1}{2}$ years at 5 per cent.
26. £1582, 10*s.* for $2\frac{1}{2}$ years at 8 per cent.
27. Find the compound interest on Rs. 1000 at 5 p.c. for $1\frac{1}{2}$ years payable half-yearly.
28. Find the amount at compound interest on Rs. 1250, for 2 years at 6 p. c. payable quarterly.
29. Find the difference between the simple and compound interest on Rs. 1600 in $2\frac{1}{2}$ years at $3\frac{1}{2}$ p. c. per annum.
30. Find the amount at compound interest on £1200 for 3 years if the interest for the first, second and third years be 3, 5 and 4 p. c. respectively.

INVERSE CASES OF COMPOUND INTEREST.

§4. **To find Principal.**

First case. *When Interest, Time and Rate are given.*

Example 1. The compound interest on a certain sum of money in 3 yrs. at 5 p.c. amounted to Rs. 78, 13*a*., find the sum.

Sol Let Rs. 100 be the Principal.

$\therefore$ interest $= \text{Rs.} \left\{ \left(1+\frac{5}{100}\right)^3 - 1 \right\} \times 100$

$= \text{Rs.} \left\{ \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} - 1 \right\} \times 100$

$= \text{Rs.} \left\{ \frac{9261}{8000} - 1 \right\} \times 100$

$= \text{Rs.} \frac{1261}{8000} \times 100 = \text{Rs.} \frac{1261}{80}$.

If the int. is Rs. $\frac{1261}{80}$ the sum $=$ Rs. 100

$\therefore$,, ,, ,, is Re. 1, ,, $= \text{Rs.} 100 \times \frac{80}{1261}$

$\therefore$,, ,, ,, is Rs. 78 13*a*. ,, $= \text{Rs.} 100. \times \frac{80}{1261} \times \frac{1261}{16}$

$= \text{Rs.} 500$ **Ans.**

From the above solution we deduce the following rule :

Rule. $Principal = \dfrac{Given\ C.\ I. \times 100}{C.I.\ on\ Rs.\ 100}$

EXERCISE 144.

1. What sum of money must I invest at 5 p. c. compound interest, so that I may gain, Rs. 51, 4*a*. in 2 years ?

2. What sum will produce Rs. 128, 2*a*. in 2 years at 5 p.c. per annum compound interest ?

3. What sum put out at compound interest at 4 p. c. would produce Rs. 187, 4*a*. 8·832*p*. in 3 years ?

4. What sum of money must I invest at 4 per cent. compound interest, so that I may gain interest Rs. 390, 3*a*. $2\frac{2}{5}$*p*. in 3 years ?

5. What sum of money must I invest at 5 per cent. compound interest, so that I may agin Rs. 2586, 1*a*. $2\frac{2}{5}$*p*. in 4 years?

Second case. *When Amount, Time and Rate are given.*

Example 2. Find what sum, lent at compound interest, will amount to Rs. 1157, 10*a*. in 3 years at 5 p, c.

Sol. Let Rs. 100 be the Principal

$\therefore$ Amount = Rs. $(1+\frac{5}{100})^3+100$

= Rs. $\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}+100=\frac{9261}{80}$.

If the amount is Rs. $\frac{9261}{80}$ the sum = Rs. 100

$\therefore$,, ,, Re. 1. ,, = Rs. $100\times\frac{80}{9261}$

$\therefore$,, ,, Rs. 1157, 10*a*. ,, = Rs. $100\times\frac{80}{9261}\times\frac{9261}{8}$

= Rs. 1000 **Ans.**

From the above solution we have the following rule ;

Rule. $Principal = \frac{Given\ amount \times 100}{Amount\ on\ Rs.\ 100}$

EXERCISE 145.

1. What sum will amount to Rs. 882 in 2 years at 5 per cent. per annum compound interest ?

2. Find what sum will amount to Rs. 4134, 6*a*. in 2 years at 5 per cent. per annum compound interest.

3. The amount of a certain sum at compound interest in $1\frac{1}{2}$ years at 5 per cent. per annum amounts to Rs. 861 ; find the sum.

4. What sum put out at compound interest at 5 per cent. would amount in 3 years to £ 810, 6*s*. 9*d*. ?

5. Find what sum will amount to Rs. 7364, 10*a*. 9*p*. in 2 years at $3\frac{1}{2}$ per cent. per annum compound interest.

6. What sum lent at compound interest will amount to Rs. 16143, 12*a*. in $2\frac{1}{2}$ years at 5 per cent. per annum ?

7. What sum will amount to £1591, 13*s*. 2·16*d*. in 3 years at compound interest ; the interest for the first, second and third years being 3, 2 and 1 per cent. respectively ?

8. What sum will amount to Rs. 650 in one year and Rs. 676 in two years at compound interest ?

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§To find Time.

Consider the following example when *Principal, Amount and Rate are given.*

Example 3. In how many years will Rs. 625 amount to Rs. 676 at 4 per cent. compound interest ?

Sol. $\because$ Principal $\left(1+\frac{\text{Rate}}{100}\right)^n$ = Amount

$\therefore \quad 625(1+\frac{4}{100})n=676$

$\therefore$* $\quad (1+\frac{4}{100})n=\frac{676}{625}$

$\therefore \quad (\frac{26}{25})n=(\frac{26}{25})^2$ or $n=2$

$\therefore$ the required time is 2 years. **Ans.**

Or we may proceed from*thus

$(1+\frac{4}{100})n=\frac{676}{625}$ or $(\frac{26}{25})n=\frac{676}{625}$

Now divide $\frac{676}{625}$ by $\frac{26}{25}$ and then the quotient again by $\frac{26}{25}$ and so on.

$\frac{676}{625} \div \frac{26}{25}=\frac{26}{25}$.....1st year

$\frac{26}{25} \div \frac{26}{25} \quad =1$.....2nd year.

Now the division terminates $\therefore$ No. of years=2. **Ans.**

☞ 36. Sometimes the number of years contains fractional part also. The following examples will illustrate the method of solving such questions.

Example 4. In what time will £6250 amount to £7592, 14*s.* 6·48*d.* at 6 p. c. compound Interest ?

Sol. £7592, 14*s.* 6·48*d.* $=£\frac{7592727}{1000}$

$\because$ Principal $\left(1+\frac{\text{Rate}}{100}\right)^n$ = Amount

$\therefore$ $6250(1+\frac{6}{100})n \quad =\frac{7592727}{1000}$

$\therefore \quad (1+\frac{6}{100})n \quad =\frac{7592727}{1000} \times \frac{1}{6250}$

or $\quad (\frac{53}{50})n \quad =\frac{7592727}{6250000}$.

Now $\frac{7592727}{6250000} \div \frac{53}{50}=\frac{7592727}{6250000} \times \frac{50}{53}=\frac{143259}{125000}$ [one year

$\frac{143259}{125000} \div \frac{53}{50}=\frac{143259}{125000} \times \frac{50}{53}=\frac{2703}{2500}$ [two years

$\frac{2703}{2500} \div \frac{53}{50} \quad =\frac{2703}{2500} \times \frac{50}{53} \quad =\frac{51}{50}$ [three years

Now the last quotient is less than the divisor ; therefore reqd. No. of years=3 years+a fraction of a year.

Now find by the simple interest the time in which £1 will amount to $£\frac{51}{50}$ at 6 p. c, *i.e.*, the int. of £1 $=£\frac{51}{50}-£1=£\frac{1}{50}$.

$\therefore$ Time $=\frac{\text{Interest} \times 100}{\text{Principal} \times \text{Rate}}=\frac{\frac{1}{50} \times 100}{1 \times 6}=\frac{1}{3}$ year

$\therefore$ reqd. time $=3\frac{1}{3}$ yrs. **Ans.**

Note. Such questions can easily be worked out in decimals.

EXERCISE 146.

1. In what time will Rs. 500 amount to Rs. 551, 4*a.* at 5 per cent. compound interest ?

2. In what time will Rs. 1000 amount to Rs. 1157, 10*a.* at 5 per cent. compound interest ?

3. In what time will Rs. 24000 amount to Rs. 27783 at 5 p. c. compound interest ?

4. Rs. 500 produce Rs 40, 12*a.* $9\frac{3}{5}p.$ at 4 per cent. compound interest ; find the time.

5. In what time will £12500 amount to £15185, 9*s.* 0·96*d.* at 6 per cent. compound interest ?

6. In what time will £15625 amount to £20470, 6*s.* 4·8*d.* at 8 p. c. compound interest ?

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§ To find Rate per cent.

Observe the following examples *when Principal, Interest or Amount and Time are given.*

Example 5. At what rate p. c. compound interest will Rs. 500 amount to Rs. 578, 13*a.* in 3 years ?

Sol. $\because$ Principal $\left(1+\frac{r}{100}\right)^n$ = Amount

$\therefore \quad 500\left(1+\frac{r}{100}\right)^3 = \frac{9261}{16}$

or $\quad \left(1+\frac{r}{100}\right)^3 = \frac{9261}{16}\times\frac{1}{500}$

or $\quad \left(1+\frac{r}{100}\right)^3 = \frac{21\times21\times21}{20\times20\times20}$

or $\quad 1+\frac{r}{100} = \frac{1}{20}$ By extracting cube root.

$\therefore \frac{r}{100} = \frac{1}{20}; \quad \therefore r = 5.$

Rate = 5 p. c. Ans.

EXERCISE 147.

1. At what rate per cent. compound interest, will Rs. 5000 amount to Rs. 6655 in 3 years ?

2. At what rate p. c. compound interest, will Rs. 500 amount to Rs. 551, 4*a*. in 2 years ?

3. At what rate per cent. compound interest will Rs. 625 amount to Rs. 676 in 2 years ?

4. At what rate p. c. compound interest, will Rs. 1250 produce Rs. 128, 2*a*. in 2 years ?

5. Find the rate per cent ; if Rs. 250 amount to Rs. 270 6*a*. $4\frac{4}{5}p$. at compound interest in 2 years.

6. A sum of Rs. 235 put out at compound interest for 3 years produces Rs. 29, 7*a*. $4\frac{3}{7}\frac{2}{5}p$.; find the rate per cent. of interest.

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§ 1. Some Important Typical Examples.

Example 1. The difference between the simple and the compound interest on a certain sum of money for 3 years at 5 p. c. is Rs. 133, 7*a*.; find the sum.

Sol. S. I. on Re. 1 for 3 yrs. at 5 p. c. $=$ Rs. $\frac{5\times 3}{100}$ $=$ Rs. 15

Amount of Re. 1 at the end of 3 years at comp. interest
$=$ Rs. $(1+\frac{5}{100})^3$ $=$ Rs. $(1{\cdot}05)^3$ $=$ Rs. 1·157625
$\because$ comp int. on Re. 1 $=$ Re. ·157625
$\therefore$ diff. between S. I. and C. I. of Re 1 $=$ Re. ·007625
But the given difference $=$ Re. 133, 7*a*. $=$ Rs. 133·4375
$\therefore$ the required sum $=$ Rs. 133·4375 ÷ ·007625
$=$ Rs. 17500. Ans.

Example 2. If the C. I. on a certain sum for 2 years at 3 p. c. be Rs. 57, 1*a*. 6*p*., what will be the S. I. ?

Sol. C. I. on Re. 1 $=$ Re. $(\frac{103}{100})^2 - 1 =$ Re. $\frac{609}{10000}$.
and S. I. on Re. 1 $=$ Rs. $\frac{1}{100}\times 2\times 3 =$ Re. $\frac{6}{100}$
$\therefore$ S. I. : C. I. $= \frac{6}{100} : \frac{609}{10000} = 6\times 10000 : 609\times 100 = 200 : 203$
$\therefore$ S. I. $= \frac{200}{203}\times$ C. I.
$= \frac{200}{203}\times$ Rs. 57, 1*a*. 6*p*.
$=$ Rs. $\frac{200}{203}\times\frac{1827}{32} =$ Rs. $\frac{225}{4} =$ Rs. 56 4*a*. Ans.

Example 3. The interest on a given sum of money for one year is Rs. 26, 4*a*. and the compound interest for 2 years is Rs. 53, 13*a*. : find the rate per cent.

Sol. Simple int. for 2 yrs. = Rs. 26, 4*a*. × 2 = Rs. 52, 8*a*.

and compound interest for 2 years = Rs. 53, 13*a*.

∴ difference between the C. I. and S. I. = Re. 1, 5*a*.

∵ in S. I. and C. I. the int. for the 1st year is the same

∴ Re. 1, 5*a*. is the int. on Rs. 26, 4*a*. for one year.

∴ rate $= \frac{\frac{21}{16} \times 100}{26\frac{1}{4} \times 1} = 5$ p. c. **Ans.**

Example 4. A sum of money put out at compound interest for 2 years amounts to £ 540, 16*s*. and in 3 years to £ 562, 8*s*. 7·68*d*. Find the rate of interest.

Sol. Evidently £ 562, 8*s*. 7·68*d*.—£ 540, 16*s*. *i.e.* £ 21, 12*s*. 7·68*d*, (£ $\frac{2704}{125}$) is the interest on £540, 16*s*. (£$\frac{2704}{5}$) for 1 year

∴ Interest on £ 100 for 1 yr. $= \frac{5}{2704} \times 100 \times \frac{2704}{125} = 4$.

∴ reqd. rate is 4 p. c. **Ans.**

Example 5. What annual payment will discharge a debt of Rs. 6305 in 3 years, the rate of interest being 5 per cent. compound interest ?

Sol. Suppose the annual payment is Rs. 100.

∴ Rs. 100 are to be paid after one year.

∴ Principal = Rs. $100 \times \frac{100}{105} =$ Rs. $\frac{2000}{21}$.

Again, Rs. 100 are to be paid after 2 years,

∴ Principal = Rs. $100 \times \frac{100}{105} \times \frac{100}{105} =$ Rs. $\frac{40000}{441}$.

Again, Rs. 100 are to be paid after 3 years,

∴ Principal is Rs. $100 \times \frac{100}{105} \times \frac{100}{105} \times \frac{100}{105} =$ Rs. $\frac{800000}{9261}$.

Total sum = Rs. $\frac{2000}{21} + \frac{40000}{441} + \frac{800000}{9261} =$ Rs. $\frac{2522000}{9261}$.

If the principal is Rs. $\frac{2522000}{9261}$ the annual payment is Rs. 100, if the principal be Rs. 6305 then the annual payment will be = Rs. $\frac{9261}{2522000} \times 100 \times 6305$

= Rs. $\frac{92610}{4} = 2315, 4a$. **Ans.**

Example 6. A man borrows a certain sum and pays back in 2 years in two equal instalments. If compound interect is reckoned at 4 per cent. and if he pays back annually Rs. 676, what sum did he borrow ?

Sol. Since the annual instalment=Rs. 676 and rate=4%.

$\therefore$ The reqd. sum=Rs. $676\times\frac{100}{100+4}$+Rs. $676\times\left(\frac{100}{100+4}\right)^2$

=Rs. 650+Rs. 625

=Rs 1275. **Ans.**

Example 7. Divide Rs. 5854, 8*a*. bstween *A* and *B* so that *A*'s share at the end of 7 years may be equal to *B*'s share at the end of 9 years, C. I. being at 4 p. c.

Sol. By the question

(*A*'s share) $(1+\frac{4}{100})^7=(B\text{'s share})(1+\frac{4}{100})^9$

i.e., $\frac{A\text{'s share}}{B\text{'s share}}=(1+\frac{4}{100})^2=\frac{676}{625}$,

Now divide Rs. 5854, 8*a*. in the ratio of 676 : 625

$\because$ 676+625=1301.

$\therefore$ *A*'s share=$\frac{676}{1301}$ of Rs. $5854\frac{1}{2}$=Rs. 3042

and $\therefore$ *B*'s share=Rs. 5854, 8*a* −Rs. 3042=Rs. 2812, 8*a*. } **Ans.**

EXCRCISE 148.

1. On what sum will the difference between the S. I. and C. I. for three years at $2\frac{1}{2}$ p. c. amounts to £ 3, 15s. $7\frac{1}{2}d$.

2. The difference between the simple and the compound nterest on a certain sum for 3 years at $3\frac{3}{4}$ per cent. is £3, 8s. $4\frac{1}{8}d$. Find the sum.

3. A sum of money put out at C. I. amounts to Rs. 2420, in 2 years and to Rs. 2662, in 3 years. Find the sum and the rate.

4. A certain sum put out at C. 1. amounts in 2 years to £ 270·4 and in 3 years to £ 281·216. Find the sum and the rate.

5. On what sum of money will the C. I. for 2 years be the same as the S. I, on £ 943 for 10 years, the rate of interest being 5 per cent. ?

6. I borrow money at 3% simple interest payable yearly and lend it immediately at 5 p. c. interest payable half yearly receiving C. I. for the second half-year and gain thereby Rs. 660 at the end of the year. What is the sum of money which I borrow ?

7. The C. I. on a sum for 3 years at 5 per cent. is £ 331, 0s. 3*d*.; what is the S. I. ?

8. The C. I on a certain sum for 2 years at 5% is £ 17, 1s. 8*d*; find the C. I. on the same sum for the same time at 4 per cent.

9. The difference between simple and compound interest on a sum of money for 3 years at 5 per cent. is £7, 12*s*. 6*d*. Find the sum.

10. I buy a house on the condition that I shall pay £500 now, £ 425 one year hence and £289 two years hence. What would be the cash value of the house; compound interest being calculated at $6\frac{1}{4}$ p. c. ?

11. The interest on a given sum of money for one year is £26 and the compound interest for 2 years is £53·04 ; find the rate per cent.

12 What annual payment will discharge a debt of Rs. 8200 due in 2 years at the rate of 5% compound interest ?

13. A sum of £ 12193, 15*s*. is borrowed to be paid in 3 equal annual instalments. Find the instalments, if the rate is 4 per cent compound interest.

14. Divide £ 820 into two parts so that their amounts after 2 and 3 yrs. respectively at 5% compound interest may be equal.

15. What sum of money will amount to Rs. 3528 in 2 years at 5 p. c. and what will it amount to in 2 more years?

16. A sum of money is borrowed and paid back in two equal instalments of Rs. 8820 allowing 5% per annum compound interest. What is the sum borrowed ? [*Burma. 1924.*]

17. Divide £ 6375 between *A* and *B* so that *A*'s share at the end of 3 years and *B*'s share at the end of 2 years if put out at 4 per cent. compound interest may be equal.

18. The third year's interest of a sum put at 4 p. c. compound interest was 16*s*. 7·68*d*. more than that of the preceding year. Find the sum.

19. Divide £6305 into three sums such that their amounts by compound interest at 5 p. c. per annum for 2, 3 and 4 years respectively shall be equal.

20. *A* placed a sum of money for 3 years at 5 p.c. simple interest, and *B* placed out an equal sum at the same rate for the same time at compound interest and thereby gained £ 26, 13*s*. 4*d*. more than *A*. What money was placed out by each ?

21. *A* man left Rs. 45909 for his sons *A* and *B* who are 15 and 13 years old in such a way that if their shares be invested at 2% compound interest, they shall receive equal amounts on reaching 18 years of age. How did he divide the money?

CHAPTER XXVII

PRESENT WORTH AND DISCOUNT

§1. *A* sells a horse worth Rs. 500 to *B* but *B* instead of paying to *A* the price of the horse in cash money, promises to pay after 6 months. Evidently *A* will charge something more than Rs. 500 from *B*, namely the interest of this money for 6 months. Suppose that the rate of interest is 4 p. c., therefore the interest on Rs. 500 for 6 months is Rs. 10 and hence *B* shall pay Rs. 510 for the horse after 6 months, but if he had paid cash then he would have paid Rs. 500 only for the horse. In this case, Rs. 500 is said to be the **Present Worth** of Rs. 510 due 6 months hence and Rs. 10 is called the **Discount** on Rs. 510. Evidently, thorefore, *the interest on the Present Worth is the discount due some time hence, i.e.*,

Discount = Interest on Present Worth,
and Amount = Present Worth + Discount.

§2. **Def.** The Present Value or Present Worth of a sum due at the end of a certain period, is the sum which with its interest for the given period, will be together equal to the sum due.

The **sum due** is also called **Amount.**

The **True discount** (briefly, **discount**) is the difference between the sum due and its present worth. Hence **discount** is the deduction made for the payment of a sum of money before it is due. It is of two kinds :

(*i*) True Discount.
(*ii*) Banker's or commercial discount.

§3. **To find the discount.**

Observe the following examples :—

Example 1. Find the discount on Rs. 360 due at the end of 4 years at 5 p. c.

Sol. ∵ the int. on Rs. 100 for 4 yrs. at 5 p. c. = Rs. 20
∴ the amount of Rs. 100 = Rs. 120,
i.e, the discount on Rs. 120 = Rs. 20,
∴ ,, ,, ,, Re. 1 = Rs. $\frac{20}{120}$.
∴ ,, ,, ,, Rs. 360 = Rs. $\frac{20}{120} \times 360$
= Rs. 60. **Ans.**

Note. Interest is calculated on the Principal but the discount is calculated on the amount.

§4. **To find the Present Worth.**

The following solved examples will illustrate the method :—

Example 2. Find the present worth of £ 721 due 9 months hence at 4 p. c.

Sol. ∵ The int. on £ 100 for 9 months at 4 p.c.=£ 3,

∴ amount of £ 100 after 9 months=£103 ;

i. e., The present Worth of £ 103=£ 100 ;

∴ ,, ,, of £ 1 =£ $\frac{100}{103}$

∴ ,, ,, of £721 =£$\frac{100}{103}\times 721$

=£ 700. **Ans.**

§5. When Present Worth and Discount are reckoned at **Compound interest**, we shall find the amount of Rs. 100 or £ 100 at C. I. for the given time at the given rate, and proceed as before.

Example 3. Find the Present Worth and Discount of Rs. 2112, 8*a.* due 2 yrs. hence at 4 p. c. Compound Interest.

Sol. At 4 p. c. C. I. for two yrs. Re. 1 amounts to Rs. $(1{\cdot}04)^2$, *i e.*, Rs. 1·0816.

Present Worth of Rs. 1·0816=Re. 1

∴ ,, ,, ,, Re. 1=Re. $\frac{1}{1{\cdot}0816}$

∴ ,, ,, ,, Rs. 2112·5=Rs. $\frac{2112{\cdot}5}{1{\cdot}0816}$

P. W.=Rs. 1953, 2*a.* **Ans.**

Discount=Rs. 2112, 8*a.*—Rs. 1953, 2*a.*

= Rs. 159, 6*a.* **Ans.**

EXERCISE 149.

Find the discount on (1—5) —

1. Rs. 1270 due 7 months hence at 10 p.c . S. I.
2. £ 1051, 5*s.* due a year hence at 5 $\frac{1}{8}$ p. c. S. I.
3. £ 520, 17*s.* 6*d.* due 3$\frac{1}{2}$ yrs. hence at 4$\frac{1}{2}$ p. c. at S. I.
4. Rs. 135, 14*a.* 8*p.* due 3$\frac{2}{3}$ yrs. hence at 4$\frac{1}{2}$ p. c. S. I.
5. £ 2450, 18*s.* 9*d.* due 3$\frac{1}{2}$ yrs. hence at 3$\frac{3}{4}$ p. c. C. I.
6. What discount at 5 p. c. simple interest must be allowed for present payment of £ 2200 due 2 years hence ?

Find the present worth of (7—11) :—

7. £ 870 due $2\frac{1}{2}$ years hence at $3\frac{1}{2}$ p. c. S. I.

8. Rs. 223, 7*a*, due 10 months hence at $3\frac{3}{4}$ p. c. S. I.

9. £ 769, 5s. due $8\frac{1}{3}$ years hence at 4 p. c. S. I.

10. £ 587, 18*s*. 9*d*. due 6 months hence at $4\frac{1}{2}$ p. c. S. I.

11. £ 9724, 1s. due 4 yeats hence at 5 p. c. C. I.

12. Find the present worth of Rs. 545, 11*a*. 8*p*. due 2 yrs. hence at 4 p. c. compound interest.

13. If the discount on a sum of money due 6 months hence at 8 per cent. be £ 7, 10*s*. $11\frac{1}{4}$*d*., find the Present Worth of the sum.

14. What must be paid in cash in order to clear off a bill of Rs. 297, 7*a*. 6*p*. due in 3 years 6 months, simple interest being calculated at 4 p. c. per annum ?

15. If money is worth $5\frac{1}{4}$ p. c. per annum, what sum of money will discharge a debt of Rs. 8175 which falls due after 5 months ?

16. Find the present worth of £ 241, 12*s*. 4*d*. 146 days hence at $4\frac{1}{3}$ p. c. per annum ?

17. Find the difference between the interest on Rs. 2466. 10*a*. 8*p*. for $2\frac{3}{4}$ years at $5\frac{1}{2}$ p. c. and the discount on Rs. 2839, 12*a*. due $2\frac{3}{4}$ years hence at the same rate. Explain the result.

18. What sum of money paid down will discharge a debt of £ 1000 due in two equal half-yearly instalments, simple interest being at 5 per cent. ?

19. If Rs. 6760 be due three years hence, allowing compound interest at 4 per cent. per annum, what sum will be due at the end of the first year ?

INVERSE CASES OF DISCOUNT

§6. All inverse questions on discount can be easily solved just like those on interest, if it is remembered that the interest on present worth is equal to the discount on amount due.

§7. **To find the Sum or Amount.**

Example 1. What is the sum due $8\frac{1}{3}$ years hence, whose discount at 4 per cent. is £ 192, 6*s*. 3*d*. ?

Sol. Suppose the sum is £100.

$\therefore$ int. for $8\frac{1}{3}$ yrs. at 4 p. c. $= £\frac{25}{3} \times 4 = £\frac{100}{3}$

$\therefore$ the amount of £100 $= (100 + \frac{100}{3}) = £\frac{400}{3}$

and $\because$ £192, 6*s*. 3*d*. $= £192\frac{5}{16} = £\frac{3077}{16}$

$\therefore$ (*a*) By "Rule of Three," we have

$£\frac{100}{3} : £\frac{3077}{16} :: £\frac{400}{3} :$ required amount.

$\therefore$ the amount $= £\frac{3}{100} \times \frac{3077}{16} \times \frac{400}{3} = £\frac{3077}{4} = £769,\ 5s.$ **Ans.**

(*b*) By Unitary Method:—

$£\frac{100}{3}$ is the discount on $£\frac{400}{3}$

$\therefore$ £1 is ,, ,, ,, on $£\frac{400}{3} \times \frac{3}{100}$

$\therefore £\frac{3077}{16}$ is ,, ,, on $£\frac{400}{3} \times \frac{3}{100} \times \frac{3077}{16}$

$= £\frac{3077}{4} = £769,\ 5s.$ **Ans.**

Aliter. Present worth $= \frac{£192, 6s. 3d.}{4 \times 8\frac{1}{3}} \times 100$

$= £576,\ 18s.\ 9d.$

$\because$ sum due = P. W. + Discount

$\therefore$ reqd. sum = £576, 18*s*. 9*d*. + £192, 6*s*. 3*d*.

= £769, 5*s*. **Ans.**

6 **Example 2.** If the present value of a bill due 8 months hence at $4\frac{1}{2}$ per cent. is Rs. 66, 10*a*. 8*p*. find the bill.

Sol. $\because$ interest on present worth = discount on the sum

$\therefore$ discount $= \frac{(\text{Rs. } 66,\ 10a.\ 8p.) \times 8 \times 9}{100 \times 12 \times 2}$

$= \text{Rs. } \frac{200 \times 8 \times 9}{3 \times 100 \times 12 \times 2} = \text{Rs. } 2$

$\therefore$ bill = Rs. 66, 10*a*. 8*p*. + Rs. 2

= Rs. 68, 10*a*. 8*p*. **Ans.**

Example 3. On what sum of money due at the end of 2 years does the discount at 5 per cent. compound interest amount to £54, 13*s*. 4*d*.?

Sol. Amount of £100 for 2 yrs. at C. I. $= £100(\frac{105}{100})^2$

$= £100 \times \frac{105}{100} \times \frac{105}{100} = £\frac{441}{4}$.

Discount of $£\frac{441}{4} - £100 = £\frac{41}{4}$.

$\because$ $£\frac{41}{4}$ is the discount on $£\frac{441}{4}$

$\therefore$ £1 is ,, ,, on $£\frac{441}{4} \times \frac{4}{41}$

£54, 13*s*. 4*d*. or $£\frac{164}{3}$ is ,, on $£\frac{441}{4} \times \frac{4}{41} \times \frac{164}{3}$

= £588. **Ans.**

EXERCISE 150.

What is the sum due (1–3):—

1. 9 months hence, whose discount at 4 per cent. is Rs. 1200, 2*a*. ?

2. $3\frac{1}{2}$ years hence whose discount at $4\frac{1}{2}$ per cent. is £70, 17*s*. 6*d*. ?

3. 245 days hence whose discount at $3\frac{1}{7}$ per cent. is £ 12, 16*s*. 8*d*.

4. If the discount of a bill due 1 year 8 months hence at $4\frac{1}{2}$ per cent. be £ 35, 5*s*., find the sum.

5. What sum due 9 months hence at $3\frac{1}{2}$ per cent. is worth Rs. 850 now ?

6. If the present value of a bill due 15 months hence at $2\frac{2}{3}$ per cent. be £631, 10*s*., find the sum.

7. On what sum of money due at the end of 2 yrs. at 4% compound interest does the discount amount to £34 ?

8. The true discount on a bill due 8 months hence at $2\frac{1}{2}$ per cent. is £ 176, 14*s*. 8*d*., find the amount of the bill.

§8. To find Time.

Example 4. When is the sum due, if the discount on Rs. 23, 5*a*. 4*p*. at $1\frac{2}{3}$ per cent. amounts to Rs. 1, 7*a*. 4*p*.?

Sol ∵ Amount – Discount = Present worth.

∴ here the P. W. is Rs. 23, 5*a*. 4*p*. – Re. 1, 7*a*. 4*p*.
= Rs 21, 14*a*. = Rs. $\frac{175}{8}$,

and ∴ int. on Rs. $\frac{175}{8}$ at $1\frac{2}{3}$ p. c. is Re. 1, 7*a*. 4*p*. or Rs. $\frac{35}{24}$.

$$\because \text{Time} = \frac{\text{Int.} \times 100}{\text{Principal} \times \text{Rate}}$$

$$\therefore \text{Reqd. time} = \frac{\frac{35}{24} \times 100}{\frac{175}{8} \times \frac{5}{3}} = \frac{35 \times 100}{24} \times \frac{3 \times 8}{175 \times 5}$$

$$= 4 \text{ years.} \quad \textbf{Ans.}$$

Rule. $Time = \frac{Discount \times 100}{P.\ W. \times Rate}$

EXERCISE 151.

When is the sum due, in the (questions 1—5) if :—

1. P. W. of Rs. 9265 at $4\frac{1}{2}$ p. c. be Rs. 8500 ?
2. P. W. of £ 658 at $3\frac{3}{4}$ p. c. be £ 640 ?
3. P. W. of £ 241, 12*s*. 4*d*. at $4\frac{1}{3}$ p. c. be £ 237, 10*s*. ?
4. Discount on Rs. 2516, 4*a*. at $6\frac{1}{4}$ p. c. be Rs. 482, 14*a*. 8*p*.
5. Discount on £ 355, 5*s*. at $4\frac{1}{2}$ p. c. be £5, 5*s*.
6. If the present worth of £ 668, 5*s*. $7\frac{1}{2}$*d*. at 5 per cent. be £ 568, 15*s*., find when the sum is due ?
7. If Rs. 165, 14*a*. $1\frac{7}{17}$*p*. be the discount of a debt of Rs. 2820 at simple interest at the rate of $3\frac{3}{4}$ p. c., how many months before due was the debt paid ?

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§9. To find Rate.

Example 3. At what rate per cent. does the discount on £ 1378 in one year four months amount to £53 ?

Sol. ∵ Present worth = Amount − Discount

∴ in this case P. W. = £ 1378 − £53 = £1325 and the interest on it in 1 year 4 months is £ 53.

$$\therefore \text{Rate} = \frac{\text{Int.} \times 100}{\text{Principal} \times \text{Time}} = \frac{53 \times 100}{1325 \times 1\frac{1}{2}} = 3 \text{ years. Ans.}$$

Hence the **Rule:** $Rate = \dfrac{Discount \times 100}{P.W. \times Time}$

EXERCISE 152.

1. If the discount on Rs. 2261, 5*a*. 4*p*. due $1\frac{1}{2}$ years hence be Rs. 128, what is the rate ?
2. What is the rate of interest if the discount on £387, 7*s*. $7\frac{1}{5}$*d*. payable at the end of 3 years be £ 41, 10*s*. $1\frac{1}{5}$*d*. ?
3. What is the rate if the present worth of Rs. 8175 due in 5 months is Rs. 8000 ?
4. What is the rate of interest, when discount on Rs. 41204, 4*a*. 8*p*. due 9 months hence is Rs. 1200, 2*a*. ?
5. The P. W. of Rs. 1321, 8*a*. due $2\frac{1}{4}$ years hence is Rs. 1200, find the rate of interest.
6. What is the money worth when the discount on £481, 8*s*. $2\frac{1}{4}$*d*. due 4 months hence is £5, 3*s*. $2\frac{1}{4}$*d*. ?

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§10. Some Important Typical Examples.

(a) To find the sum, when the difference between interest and discount on the same sum, rate and time are given.

Example 1. If the difference between the interest and the discount on a certain sum of money for 2 years at 5 p. c. is Rs. 2, 13*a*. 4*p*., find the sum.

Sol. Suppose the sum =Rs. 100

$\therefore$ Interest for 2 years at 5 p. c. =Rs. 10

$\therefore$ Amount ,, ,, ,, ,, =Rs. 110

$\therefore$ Discount on Rs. 110 =Rs. 10

$\therefore$ Discount on Re. 1 =Rs. $\frac{10}{110}$

$\therefore$ Discount on Rs. 100 =Rs. $\frac{10}{110} \times 100 = \frac{100}{11}$.

$\therefore$ Diff. between discount and interest on Rs. 100
=Rs. 10—Rs. $\frac{100}{11}$=Re. $\frac{10}{11}$; Rs. 2, 13*a*. 4*p*.=Rs. $\frac{17}{6}$.

$\because$ Re. $\frac{10}{11}$ is the difference on Rs. 100

$\therefore$ Re. 1 ,, ,, on Rs. $100 \times \frac{11}{10}$

$\therefore$ Rs. $\frac{17}{6}$,, ,, on Rs. $100 \times \frac{11}{10} \times \frac{17}{6}$=Rs. $\frac{935}{3}$
=Rs. 311, 10*a*. 8*p*. **Ans.**

An Important Rule.

The difference between the interest and the discount on a sum of money is equal to the interest on the discount.

$\because$ Sum =Present Worth+Discount.

$\therefore$ Interest on sum=Interest on P.W +Int. on Discount, but interest on P. W. =Discount on the sum.

$\therefore$ Interest on sum=Discount on sum+Int. on Discount.

i.e. **Interest on sum—Discount on sum=Int. on Discount.**

It follows, therefore, that interest is always greater than the discount.

(b) To find sum and time when interest and the discount on the same sum and rate are given.

Example 2. The interest on a sum of money at 4 p. c. is £ 67, 4*s*. and discount on the same sum for the same time at the same rate is £ 60. What is the sum and time?

Sol. $\because$ Int. on sum—Discount=Int. on Discount.

$\therefore$ £ 67, 4*s*.—£ 60, *i.e.* £ $7\frac{1}{5}$=Int. on £60.

$\therefore$ £ 1=Int. on £ $60 \times \frac{5}{36}$.

$\therefore$ £$67\frac{1}{5}$=Int. on £ $60 \times \frac{5}{36} \times \frac{336}{5}$=£560.

i. e., the required sum=£560. **Ans.**

Hence the **Rule** : $Sum = \frac{Interest \times Discount}{Interest - Discount}$

Now the question is "In what time the interest on £560 at 4 p. c. will be £67, 4*s*.?

$$\therefore \text{Time} = \frac{\text{Interest} \times 100}{\text{Principal} \times \text{Rate}} = \frac{(£67,\ 4s.) \times 100}{£560 \times 4}$$

$$= \frac{336 \times 100}{5 \times 560 \times 4} = 3 \text{ years.} \quad \textbf{Ans.}$$

(c) To find Sum and Rate when Interest and Discount on the same sum and time are given.

The method is the same as in Example 2.

(d) To find the time when interest and discount on different sums are given.

Example 3. If the interest on Rs. 8500 at $4\frac{1}{2}$ per cent. be equal to the discount on Rs. 9265 for the same time at the same rate, when is the latter sum due ?

Sol. Int. on Rs. 8500=Discount on Rs. 9265.

$\because$ Rs. 8500 is the P. W. of Rs. 9265

$\therefore$ Int. on Rs. 8500=Rs. 9265.—Rs. 8500=Rs. 765 ;

$$\therefore \text{Time} = \frac{765 \times 100}{8500 \times 4\frac{1}{2}} = 2 \text{ yrs.} \quad \textbf{Ans.}$$

(c) To find the rate when interest and discount on different sums are given.

The method is the same as in Example 3 above.

(f) To find discount for a greater period when the discount for the less period is given.

Example 4. If Rs. 8 be allowed as discount off a bill of Rs. 80 due 6 months hence, how much should be allowed off a bill of the same amount due 14 months hence ?

Sol. Rs. 8 is 6 months' discount on Rs. 80.

$\therefore$ Rs. 8 is 6 months' interest on Rs. 72.

$\therefore$ Rs. 20 is 15 months' interest on Rs. 72.

i.e., Rs. 20 is 15 months' discount on Rs. 92.

Discount on Rs. 92=Rs. 20.

$\therefore$ „ on Rs. 1 =Re. $\frac{20}{92}$.

$\therefore$ „ on Rs. 80 =Rs. $\frac{20}{92} \times 80$=Rs. $\frac{400}{23}$.

$\therefore$ reqd. discount =Rs. 17, 6*a*. $3\frac{3}{23}p$. **Ans.**

EXERCISE 153.

1. If the interest on Rs. 50 at $4\frac{1}{2}$ p. c. be equal to discount on Rs. 59 for the same time at the same rate, when is the latter sum due?

2. If the discount on £2830, 15*s*. $7\frac{1}{2}$*d*. be equal to the interest on £2784, 7*s*. 6*d*. for the same time, find the time, the rate of interest being 5 per cent.

3. The difference between the interest and discount on a certain sum for 9 months at 4 p. c., is Rs.2,13*a*. Find the sum.

4. The interest on Rs. 250 at 3 p. c. is equal to the discount on Rs. 265 at the same rate and for the same time. Find the time.

5. The interest on Rs. 1462, 8*a*. at a certain rate for 4 years is equal to the discount on Rs. 1725,12*a*. for the same time at the same rate. What is the rate per cent.?

6. The discount on a certain sum at $2\frac{1}{2}$ p.c. is £19, 12*s*. and interest on the same sum for the same time and rate is £26, 19*s*. Find the time.

7. If the present value of a bill of £3820,6*s*, 3*d*. due 5 months hence is £3750, what is the present value of a bill of £3876, 11*s*. 3*d*. due 9 months hence at the same rate? Find also the rate of interest.

8. At what rate per cent. will the interest on £3729, 7*s*. 6*d*. in 4 years, be equal to the discount on £4661, 14*s*. $4\frac{1}{2}$*d*. for the same time?

9. The interest on a certain sum of money for a certain time is Rs. 36 and the discount for the same time is Rs. 30; find the sum.

10. If Rs. 12 be allowed at 6 months' discount off a bill of Rs. 132 and at the same rate of interest Rs. 40 be allowed off a bill of Rs. 240; for how long a period had the latter bill to run?

11. The discount on Rs. 275 for a certain length of time is Rs. 25; what is the discount on the same sum (*i*) for double the length of time, (*ii*) for half the length of time?

12. A tradesman marks his goods with two prices, one for cash money and the other for credit of 6 months; what ratio should the two prices bear to each other, the rate of interest being $7\frac{1}{2}$ per cent.? If the credit price of an article be Rs. 332, what is its cash price?

13. A man bought a house for Rs. 1100 and sold it at once for Rs. 1217, 8*a*. allowing the buyer 5 months' credit. Money being worth $3\frac{1}{2}$ p. c. per annum, what was the gain per cent.?

14. If Rs. 10 be allowed as discount off a bill of Rs. 50, and at the same rate Rs. 25 be allowed off a bill of Rs. 150, due at the end of 8 months, for how long a period had the first bill to run?

15. If £6 be allowed as a discount off a bill of £56 for 8 months find the amount of another bill, of which £ 30 is allowed as discount for 20 months.

16. If Rs. 5 be allowed as discount off a bill of Rs. 125 due a certain time hence, what should be the discount allowed off, if the bill had twice as long to run?

17. If £ 2652, 5*s*. be due 3 years hence, what sum will be due at the end of 1 year, if compound interest is allowed at 3 per cent.?

18. A grocer buys 480 mds. of sugar for Rs. 6135 payable at the end of 3 months and on the same day sells them at Rs. 12, 11*a*. per maund ready money. What per cent. does he gain or lose by the transaction reckoning interest at 9 per cent. per annum?

19. I buy a horse for £ 40 and sell it for £45 at a credit of 8 months. What do I gain per cent. reckoning money worth 6 per cent. per annum?

20. Eight copies of a book can be bought for a certain sum payable at the end of a year, but for a cash payment 10 copies can be had at the same price. What is the money worth?

21. The interest on £ 720 for a certain time is £ 18; find the discount on the sum for the same time.

22. The interest on a certain sum for two years is £ 60, while the discount on the same sum due three years hence is £80. Find the sum and the rate per cent.

23. £ 159, 7*s*. is due 4 years hence and £ 173, 18*s*. 5 years hence. What sum at the present time is equivalent to both these sums, calculating interest at $3\frac{1}{2}$ per cent.?

24. A book sent from England costs me (including 1*s*. 6*d*. postage) 16*s*. 1*d*., my book-seller allowing me two pence in the shilling as discount on the published price, what is the published price?

25. The interest on a certain sum of money for 3 months at 5 per cent. exceeds the discount on the same sum due 3 [illegible] at the same rate by 11*a*. 3*b*. Find the sum.

§11. Banker's Discount.

In all commercial transactions when a man buys goods, he frequently does not pay *"cash"* for them but gives what is known as a *Bill of Exchange,* (Hundi) which is an agreement for paying the price after a certain time. The following is a form of a bill of exchange, which is always drawn on a stamped paper.

Bill of Exchange. 10100 Due on June 7, 1946.

STAMP

Lahore.
January 4, 1946.

Five months after date pay to myself or order the sum of One Hundred only Rupees Ten thousand and for value received.

To
L. Brij Lal,
Calcutta.

Accepted, payable at The Punjab National Bank Ltd., Calcutta.
Brij Lal.

Harprasad.

Here Brij Lal has bought from Harprasad goods worth Rs. 10100 (*face value* of the bill) but does not pay in cash. He promises to pay later on, say after 5 months. Harprasad, therefore draws up the bill of exchange as above and forwards it to Brij Lal for his "acceptance." If the latter agrees to it, he signs his name accross the bill with the note that the money can be obtained from the bank named there, on the date agreed upon. Brij Lal is said to "accept" the bill, which he returns to Harprasad. It is said to be *drawn* "at five months."

The date when the bill is drawn is January 4. Five months (always calendar months) from date will be June 4.

But Harprasad cannot legally claim the money till three days after this, called "Three Days of Grace," pass.

The date when the bill "matures," *i.e.*, when the money becomes legally due, is June 7. Harprasad, therefore, cannot claim the money of the bill till June 7. when he will present it to the Punjab National Bank and receive Rs. 10100 of Brij Lal's money. The whole transaction is then closed.

But there is another way, which is very generally adopted, in which Harprasad might proceed. Suppose in the interval between January 4 and June 7, Harprasad wants money very badly—he may then take the bill to the banker or a broker and ask him to discount it. If the banker is willing and can pay the money, then he keeps the bill with him and, in exchange pays over to Harprasad some money in cash. On the 7 June, when the bill matures" the banker presents the bill to the Punjab National Bank and receives Rs. 10100 of Brij Lal's money and here the whole transaction is closed.

Now the question arises,—how much does the banker (or broker) who "discounts" the bill pay to Harprasad? If he pays the *true* present value of the Rs. 10100, he makes no profit for himself. It is, therefore, the custom for the Banker to pay in cash the face value of the bill *minus the simple interest* on it for the interval the bill has yet "to run" *i.e.*, the number of days from the date on which the banker discounts it up to June 7.

Banker's discount is, therefore, the same as the simple interest on the bill for the unexpired time.

It is also known *Practical or Commercial Discount.*

§12. The person who draws the bill is called the **drawer** and the person on whom it is drawn is called the **drawee** or **acceptor**, in the above example L. Harprasad is the drawer and Mr. Brij Lal the drawee. The person to whom the money is to be paid is called the payee.

A Promissory Note is also a contract (or promise) for a certain sum of money on a certain date to a person named or to his order, or to bearer. It differs from a bill of exchange

only in this respect that it is written by the debtor instead of by creditor. The form of a promissory note is given below :—

To Mr. Harparsad	January 4, 1946.
STAMP	Five months after date, I promise to pay to Harprasad or his order, the sum of Ten Thousand and One Hundred Rupees for value received.
Rs. 10100	Brij Lal

The bill above leads to the following question :—

Example 1. Harprasad draws, a bill, whose face value is Rs. 10100 on January 4 at 5 months and offers it to a banker on March 26 to be discounted. If the banker's rate of discount be 5 p. c per annum, what will he pay for the bill ?

Sol. ∵ the bill is drawn on January 4 ∴ it is legally due on June 7. Since it is discounted on March 26 ∴ it has yet "to run, from March 26 to June 7, *i.e.*, 73 days.

Now the banker's discount=simple interest on Rs. 10100 for 73 days at 5 per cent.=Rs. 101 ;

∴ the banker discounts (or buys) the bill for Rs. 10100 *minus* Rs. 101=Rs. 9999. **Ans.**

Note 1. The banker keeps the bill with him up to June 7 when on presenting it to the bank, he gets Rs. 10100.

Note 2. The true present value of Rs. 10100 for 73 days at 5 p. c. can be found to be Rs. 10000 and hence true discount is Rs. 100. Since the banker pays for the bill only Rs 9999 instead of Rs. 10000 he makes a gain of Re. 1.

The fact is expressed by saying that banker's discount is greater than true discount by Re. 1.

Note 3. One month from January 31 (the last day of the month) will lead to February 28 or 29 (the last day of the month) i.e., the word month always means calendar month.

Example 1. A banker discounts a 2 months' bill of Rs. 1000 at 3 p. c. discount. What rate of interest per annum is he charging ?

Sol. Banker's discount=Rs. $\frac{1000\times3\times2}{100\times12}$=Rs. 5.

∴ he advances Rs. 995 in payment of the bill and in 2 months time he gets Rs. 1000, *i.e.*, interest on Rs. 995 for 2 months is Rs. 5.

$$\therefore \text{ rate of interest} = \frac{5 \times 100}{995 \times \frac{1}{6}} = 3\tfrac{3}{199} \text{ per cent. Ans.}$$

§13. The following facts should be clearly understood :—

1. The discount on a bill=int. on the true P. W. of the bill.
2. Banker's discount on a bill=interest on the bill.

and ∴ banker's discount−true discount=interest on (bill—true P. W.)=int. on true discount ; and

∴ banker's discount=true discount+int. on true discount
=amount of true discount.

Example 3. If Rs. 120 is the true discount on Rs. 4120 for 9 months, for how long is Rs. 120 the true discount on Rs. 6120 at the same rate ?

Sol ∵ Rs. 120 is the true discount on Rs. 4120.

∴ Rs. 4000 is the true P. W. of Rs. 4120.

∴ Rs. 120 is the interest on Rs. 4000 for 9 months at a certain rate.

$$\therefore \text{ Rate} = \frac{120 \times 100}{4000 \times \frac{9}{12}} = 4\%.$$

Again, Rs. 120 is the true discount on Rs. 6120

∴ Rs. 120 is the int. on Rs. 6000 for a certain time at 4 per cent. ;

$$\therefore \text{ required time} = \frac{120 \times 100}{6000 \times 4} \text{year} = 6 \text{ months. Ans.}$$

Example 4. The banker's discount on £604 at 5 p. c. is the same as the true discount on £611, 11*s.* for the same time at the same rate. Find the time.

Sol. ∵ int. on £604=true discount on £611, 11*s.*
=int. on true P. W. of £611, 11*s.*

∴ £604 =true P. W. of £611, 11*s.*

∴ £7. 11*s.* =true discount on £611, 11*s.*

i.e., £7, 11*s.*=int. on £ 604 at 5 p. c. for a certain time;

$$\therefore \text{ the required time} = \frac{£\frac{151}{20} \times 100}{604 \times 5} \text{ year.}$$

=3 months. Ans.

EXERCISE 154.

1. What rate of interest do I get for my money when, in discounting a bill due in 10 months I deduct as discount 4 p. c. of total amount of the bill ?

2. At what rate p. c. will the banker's discount on £6452, 10*s*. 8*d*. due 4 years hence amount to £ 1613, 2*s*. 8*d*. ?

3. The difference between banker's discount and true discount on a sum of money for 3 months at 5 per cent. is Rs. 16, 10*a*. 8*p*. Find the sum.

4. Find the banker's discount on a bill of £342, 17*s*. 10*d*. drawn on May 1 at 4 months and discounted on June 15, rate of interest being $1\frac{1}{2}$ per cent.

5. A banker in discounting a bill due in 6 months at 6 per cent. charges 9*s*. more than the true discount. Find the amount of the bill.

6. The present worth of a certain sum due in 3 years at 5 p. c. compound interest is £416, 13*s*. 4*d*. ; what is the discount ?

7. If the true discount on a sum due in 1 year, be $\frac{400}{419}$ of the banker's discount, what is the rate p. c. of simple interest ?

8. The true discount on a certain sum due 7 months hence is £8, 15*s*. and the banker's discount on the same sum for 7 months is £8, 18*s*, $\frac{3}{4}$*d*, ; find the sum and the rate per cent. per annum.

9. How much less than the true P. W. will a banker give for a bill of Rs. 9504, due in $7\frac{1}{2}$ months, interest at 5 p. c.?

10. What will a banker retain on discounting a bill of Rs. 12750, drawn on the 4th March at 10 months and discounted on 14th August at 5 per cent. ?

11. A bill was drawn on May 14 at 2 months and was discounted on July 2 at $8\frac{1}{9}$ per cent. If the Banker's gain was 1*s*. 4*d*., for what sum was the bill drawn ?

12. A person discounting a bill 8 months before it is due at 6 per cent. receives £2, 7*s* 6*d*. less than the amount of the bill ; what was the bill drawn for ?

13. A bill for £126, 5*s*. was drawn on March 9 at 5 months, interest being calculated at 5 p. c.; discount on the bill was £1. 5*s*. 3*d*. On what date was the bill discounted?

14. A banker discounted a bill 9 months before due and found that he would have $\frac{12}{13}$ of what he deducted as discount, if he had reckoned true discount. Find the rate at which interest was calculated.

15. A bill-broker pays £242, 19*s*. 8*d*. for a bill of £243, 6*s*. 8*d*. discounted 12 days before it is nominally due. At what rate did he reckon the discount?

16. A bill for Rs. 2625 is discouuted and the discount charged amounts to Rs. 18, 6*a*. If the rate of interest be $3\frac{1}{2}$ per cent., find how many days the bill has yet to run.

17. The interest on a certain sum for 10 years is greater than the true discount on the same sum, due in 10 years by one-quarter of the discount. Find the rate which is the same in both cases.

18. The true discount on a sum of money for 3 months is Rs. 150, and the commercial discount for the same time and rate is Rs. 151, 2*a*. Find the sum of money and the rate.

19. The true discount on a certain sum due in 3 months at 4% is £17, 5*s*. Find the banker's discount and the sum.

20. The true discount on a sum of money for 6 months is £ 12 and the banker's discount for the same time and at the same rate is £12. 10*s*. : what is the sum and the rate of interest ?

CHAPTER XXVIII

BANKRUPTCY, TAXES, RATES, COMMISSION, BROKERAGE AND INSURANCE.

§1. Bankruptcy or Insolvency.

A tradesman becomes Bankrupt or insolvent when his debts exceed his property. His debts are called liabilities and his property is called his assets. By property we mean cash, book debts, *i. e.*, the money other persons owe to the bankrupt and immovable property such as buildings, machinery land, etc.

It is clear that the bankrupt cannot pay to his creditors the full amount of their money. If his assets are $\frac{3}{4}$ of his liabilities he shall pay $\frac{3}{4}$ of their money, *i.e.* 12*a.* per rupee or 15*s.* per £. This 12*a.* per rupee or 15*s.* per £ is called **dividend.**

Suppose a tradesman owes his creditors Rs. 6000 and his assets are Rs. 2000; what dividend shall he pay in the rupee ?

We shall solve the example thus :

For Rs. 6000 he can pay Rs. 2000

∴ for Re. 1 he shall pay Rs. $\frac{2000}{6000}$ = Re. $\frac{1}{3}$.

i. e. 5*a.* 4*p* **Ans.**

Note. Dividend in the lb. or Re. = Assets ÷ Liabilities

whence Assets = Liabilities × Dividend

and Liabilities = Assets ÷ Dividend.

§2. We shall now solve some examples for illustration.

To find dividend in the rupee or £.

Example 1. A bankrupt's assets amount to Rs. 5497, 8*a.* and his liabilities are Rs. 7330. What can he pay in the rupee, and what shall a creditor receive for his claim of Rs. 1275 ?

Sol. (*i*) **Dividend** in the Re. = Assets ÷ Liabilities

= Rs. 5497, 8*a.* ÷ Rs. 7330

= Re. $\frac{3}{4}$ or 12*a.* **Ans.**

(*ii*) For Re. 1 he can pay Re. $\frac{3}{4}$.

∴ for Rs. 1275 he shall pay Rs. $1275 \times \frac{3}{4} = \frac{3825}{4}$

i.e., Rs. 956, 4*a.* **Ans.**

To find Liabilities.

Example 2. A banker's assets were Rs. 5433, 5*a*. 4*p*. and he paid a dividend of 10*a*. 8*p*. in the rupee; what were his liabilities?

Sol. Liabilities = Assets ÷ dividend

$$= \text{Rs. } 5333,\ 5a.\ 4p. \div 10a.\ 8p.$$
$$= \text{Rs. } \frac{16000}{3} \times \frac{3}{2}$$
$$= \text{Rs. } 8000. \quad \textbf{Ans.}$$

To find assets.

Example 3. A bankrupt's liabilities were Rs. 1866, 10*a*. 8*p*. and he paid 13*a*. 4*p*. in the rupee; find his assets.

Sol. Assets = Liabilities × Dividend

$$= \text{Rs. } 1866,\ 10a.\ 8p. \times 13a.\ 4p.$$
$$= \text{Rs. } \frac{56000}{3} \times \frac{5}{6}$$
$$= \text{Rs. } \frac{140000}{9} = \text{Rs. } 1555,\ 8a.\ 10\tfrac{2}{3}p. \quad \textbf{Ans.}$$

Example 4. A bankrupt can pay 10*a*. in the rupee, had he Rs. 2000 more, he could have paid 12*a*. in the rupee. Find the amount of his debts and assets.

Sol. ∵ 12*a*. − 10*a*. = 2*a*. = Re. $\frac{1}{8}$

∴ he could have paid Re. $\frac{1}{8}$ more on Re. 1 of his debts

∴ he could have paid Re. 1 more on Rs. 8 of his debts.

∴ ,, ,, Rs. 2000 ,, Rs. 2000 × 8 ,,

Hence his debts are Rs. 2000 × 8 = Rs. 16000
and his assets are 10*a*. × 16000 = Rs. 10000 } **Ans.**

Example 5. A bankrupt has book debts, equal in amount to his liabilities; but on Rs. 600 of such debts he can recover only 12*a*. in the rupee and on Rs. 2400 only 4*a*. in the rupee and the expenses of the liquidation are Rs. 366, 10*a*. 8*p*. If he pays 13*a*. 4*p*. in the rupee, what is the amount of his liabilities?

Sol. (*i*) On Re. 1 the bankrupt loses (16*a*. − 12*a*.) or 4*a*.

∴ on Rs. 6000 he will lose Rs. $6000 \times \frac{1}{4}$ = Rs. 1500.

(*ii*) On Re. 1 he loses (16*a*. − 4*a*.) or 12*a*.

∴ on Rs. 2400 he will lose Rs. $2400 \times \frac{3}{4}$ = Rs. 1800.

∴ The total loss including the expenses of bankruptcy
= Rs. 1500 + Rs. 1800 + Rs. 366, 10*a*. 8*p*.

$$= \text{Rs. } 3666,\ 10a.\ 8p. = \text{Rs. } \frac{11000}{3}.$$

His creditors' loss = (16*a*. − 13$\frac{1}{3}$*a*.) or $\frac{8}{3}$*a*. in the rupee

$\frac{8}{3}a.$ = Re. $\frac{1}{6}$.

Now Re. $\frac{1}{6}$ is the loss on Re. 1 of liabilities

$\therefore$ Re. 1 ,, ,, on Rs. 6 ,, ,,

$\therefore$ Rs. $\frac{11000}{3}$,, ,, on Rs. $6 \times \frac{11000}{3}$,, ,,

$\therefore$ his liabilities = Rs. 22000. **Ans.**

§3. **Preferential Claims.**

Rent, servant's wages, etc. are called preferential claims and they are always paid in full before the dividend is declared.

Expenses of winding up a concern *i.e.*

Expenses incurred in getting a bankruptcy sanctioned in collecting the book debts, and in arranging payment to the creditors are also, like the preferential claims, deducted before the dividend is declared.

Example 6. The assets of a bankrupt amount to Rs. 5621 and the liabilities to Rs. 7682. If Rs. 525, 8*a*. be the preferential claims and Rs. 324, 8*a*. the expenses of winding up, find the dividend declared in the rupee.

Sol. Assets after paying the preferential claims and expenses of winding up = Rs. 5621 − (Rs. 324, 8*a*. + Rs. 525, 8*a*.) = Rs. 4771.

and liabilities after paying the preferential claims

= Rs. 7682 − Rs. 525, 8*a*. = Rs. 7156, 8*a*.

$\therefore$ Dividend $= \frac{\text{Rs. } 4771}{\text{Rs. } 7156,\ 8a.} = \text{Rs. } \frac{4771 \times 2}{14313}$

= Re. $\frac{2}{3}$ *or* 10*a*. 8*p*. **Ans.**

Example 7. A creditor receives on a debt of £ 1280, a dividend of 11*s*. 3*d*. in the £ and he receives a further dividend of 3*s*. 6*d*. in the £ on deficiency. What amount does he receive in all ?

Sol. The first payment = 11*s*. 3*d*. or £ $\frac{9}{16}$ on £ 1 of debts,

$\therefore$ The deficiency = $1 - \frac{9}{16}$ or £ $\frac{7}{16}$ on £1 of debts.

The second payment = 3*s*. 6*d*. or £ $\frac{7}{40}$ on £ 1 of debts.

$\therefore$ The 2nd payment = $\frac{7}{40} \times \frac{7}{16}$ or £ $\frac{49}{640}$ on £1 of debts.

$\therefore$ both payments = $\frac{9}{16} + \frac{49}{640}$ or £ $\frac{409}{640}$ on £1 of debts.

Now in £1 of debt the creditor receives £ $\frac{409}{640}$,

$\therefore$,, £ 1280 ,, ,, ,, ,, £ $\frac{409}{640} \times 1280$

i.e., £ 818. **Ans.**

EXERCISE 155.

1. A bankrupt's assets amount to Rs. 3216 and his liabilities are Rs. 6432. What can he pay in the rupee ?

2. A bankrupt's assests amount to Rs. 1200 and his debts to Rs. 1600 ; how much can he pay in the rupee ?

3. A bankrupt's debts amount to Rs. 1624, 10*a*. and assets to Rs. 541. 8*a*. 8*p*. only ; what dividend can he pay in the rupee ?

4. A bankrupt owes Rs. 1575, 8*a*. but his assests are Rs. 1050, 5*a*. 4*p*. How much in the rupee can he pay ?

5. A bankrupt's effects amount to Rs. 7521 and debts to Rs. 10028. What dividend can he pay in the rupee and what amount of money will he pay to a creditor for a claim of Rs. 7128 ?

6. A bankrupt owes *A* Rs. 26520, *B* Rs. 46338 and *C* Rs. 15114, 8*a*., his estate is worth Rs. 29324, 2*a*. 8*p*., how much can he pay in the rupee and what will each creditor receive ?

7. A bankrupt's libilities amount to Rs. 38700 and his creditors lose Rs. 12900 : what dividend in the rupee does he pay and what will he pay to a creditor for a claim of Rs. 8972 ?

8. A bankrupt's assets are Rs. 21000 and he can pay a dividend of 13*a*. 4*p*. in fhe rupee ; find the libilities ?

9. A bankrupt's libilities amount to Rs. 18533, 5*a*. 4*p*. and he pays a dividend of 12*a*. in the rupee : find his assets.

10. A creditor received 16*s*. 8*d*. in the £ and thereby lost £ 150; how much was due to him ?

11. A bankrupt's effects amount to Rs. 15126, 3*a*. and he declares a dividend of 9*a*. in the rupee. What amount does he owe ?

12. A bankrupt's effects amount to Rs. 20136, and liabilities to Rs. 26521, 4*a*. after paying the expenses of winding up the concern he declares a dividend of 12*a*. in the rupee: find the amount of expenses.

13. A bankrupt can pay a dividend of 13*a*. 4*p*. in the rupee, had he Rs. 1200 more he could have paid 14*a*. 8*p*. in the rupee. Find the amount of his debts and assets.

14. A bankrupt can pay a dividend of 13*a*. 4*p*. in the rupee;had he Rs. 500 less the dividend would have decreased

by 2*a*. 8. in the rupee. Find the amount of his debts and assets.

15. A bankrupt has book debts equal in amount to his liabilities, but on £1400 of such debts he can only recover 15*s*. in the £ and the expenses of the winding up are £ 110; if he pays his creditors 16*s*. 8*d*. in the £, what is the amount of his liabilities?

16. A bankrupt has book debts equal in amount to his liabilities. but on £ 500 of such debts he can only recover 12*s*. in the £ and on £ 800 only 10*s*. in the £ and the expenses of the winding up are £150; if he pays his creditors 18*s*. in the £, what is the amount of his liabilities?

17. The assets of a bankrupt amount to Rs. 1680, 8*a*., the liabilities to Rs. 2184, 4*a*. If the preferential claims be Rs. 324, 4*a*. and the expenses for winding up the concern Rs. 116, 4*a*. what dividend can he declare in the rupee?

18. The effects of a bankrupt amount to £ 2137, 12*s*. 4*d*. and the debts to £ 2625, 10*s*. If the preferential claims be £225, 10*s*. and the expenses for winding up the concern £112, 2*s*. 4*d*., how much can be paid in the £ to the creditors?

19. A creditor receive on a debt of £ 480 a dividend of 10*s*. in the £ and he receives a further dividend of 6*s*. 8*d*. in the £ on the deficiency; what amount does he receive in all?

20. A creditor receives on a debt of £ 384 a dividend of 12*s*. 6*d*. in the £ and he receives a further dividend of 3*s*. 9*d*. in the £ on the deficiency. What amount does he receive in all?

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§4. Incomes and Taxes.

The tax levied by the Government on the annual total **gross income** of a person or a company at the rate of so many pies or pence in every rupee or pound respectively is called the **Income Tax.** What remains after the payment of the income-tax is called the **net-income.** It follows therefore that

Net income = gross income − income-tax.
Gross income = net income + income-tax.

§5. Rates.

The sums of money paid at the rate of so many pies or pence in a rupee or a pound respectievly to the local authori-

ties such as Municipalities. District Boards, etc., by the house-holder or land-owner are called **Rates.**

The *rates* are charged on the annual value (called also the rateable value) of the property.

Example 1. A man's gross income is Rs. 1600; find the income-tax at the rate of 4*p.* in the rupee.

Sol. $4p. = \frac{1}{3}a. =$ Re. $\frac{1}{48}$;

If the gross income is Re. 1, the income-tax $=$ Re. $\frac{1}{48}$.

$\therefore$ If the gros income is Rs. 1600, the income-tax $=$ Rs. $\frac{1}{48} \times 1600$

i.e., Rs. 33, 5*a.* 4*p.* **Ans.**

From the above solution we learn that the

Income-tax = gross income × the rate of tax.

Example 2. A man's gross income is £ 2100. Find his *net income* when he pays the income-tax at the rate of 5*d.* in the £.

Sol. $5d. = \frac{5}{12}s. = £\frac{1}{48}$.

Income-tax = gross-income × the rate of tax

$= £2100 \times \frac{1}{48} = £\frac{175}{4} = £43,\ 15s.$

$\because$ net income = gross-income − income tax,

$\therefore$ reqd. net income $= £2100 - £43,\ 15s.$

$= £\ 2056,\ 5s.$ **Ans.**

Aliter. $£1 - 5d. = £\ 1 - £\frac{1}{48} = £\ \frac{47}{48}$

If gross income is £ 1, the net income $= £\frac{47}{48}$

,, ,, ,, £ 2100 ,, ,, $= £\frac{47}{48} \times 2100$

i.e., £ $\frac{8225}{4}$ = £ 2056, 5*s.* **Ans.**

Example 3. If the income-tax be at the rate of 5*p.* in a rupee and a man has to pay a tax of Rs. 15, 8*a.* 4*p.*, find the amount of his *gross income*.

Sol. $5p. = \frac{5}{12}a. =$ Re. $\frac{5}{192}$; Rs. 15, 8*a.* 4*p.* $=$ Rs. $\frac{745}{48}$.

If the income-tax is Re. $\frac{5}{192}$, gross income = Re. 1

,, ,, ,, Re. 1 ,, ,, $=$ Rs. $\frac{192}{5}$

,, ,, ,, Rs. $\frac{745}{48}$,, ,, $=$ Rs. $\frac{192}{5} \times \frac{745}{48}$

$=$ Rs. 596. **Ans.**

From the solution we conclude that

Gross income = Income-tax ÷ rate of tax.

Example 4. A man's net income after the payment of income-tax at the rate of 7*p.* in the rupee is Rs. 1665. Find the *gross income.*

Sol. Re. 1−7*p.*=185*p.*=Re. $\frac{185}{192}$.

If the net income is Re. $\frac{185}{192}$, gross income=Re. 1.

" " " Re. 1 " " =Rs. $\frac{192}{185}$

" " " Rs. 1665 " " =Rs. $\frac{192}{185}\times 1665$

=Rs. 1728. **Ans.**

Here gross income=given net income ÷ net income of Re. 1.

Example 5. A man's net income after payment of income-tax at the rate of 6*p.* in the rupee is Rs. 2421, 14*a.* What will it be, if the income-tax be reduced by $1\frac{1}{2}p.$ in the Re. ?

Sol. (*i*) Re. 1−6*p.*=Re. $\frac{31}{32}$; Rs. 2421, 14*a.*=Rs. $\frac{19375}{8}$

(*ii*) 6*p.*−$1\frac{1}{2}p.$=Re. $\frac{3}{128}$; Re. 1−Re. $\frac{3}{128}$ =Re. $\frac{125}{128}$.

If net income is Re. $\frac{31}{32}$; the gross income=Re 1,

∴ if net income is Re. 1, the gross income =Rs. $\frac{32}{31}$

∴ if net income is Rs. $\frac{19375}{8}$, " " =Rs. $\frac{32}{31}\times\frac{19375}{8}$

=Rs. 2500.

Also if gross income is Re.1, the net income=Rs. $\frac{125}{128}$

∴ if gross income is Rs. 2500, the net-income

=Rs. $\frac{125}{128}\times 2500$=Rs. $\frac{78125}{32}$=Rs. 2441, 6*a.* 6*p.* **Ans.**

EXERCISE 156.

1. A man's gross income is Rs. 1500 and he pays Rs. 26, 8*a.* as income-tax. Find the net income.

2. A man's net income is Rs. 1123, 8*a.* 4*p.* ; if he pays Rs. 21. 5*a.* 4*p.* as income-tax, what is his gross income ?

3. A man's gross income is Rs. 1875. Find the income-tax at the rate of 4*p.* in the rupee.

4. A man's gross income is Rs. 2800. Find his net income after paying an income-tax at the rate of 7*p.* in the rupee.

5. A man's salary is Rs.400 per mensem. Find the annual net income after paying an income-tax at 4*p.* in the rupee.

6. A man pays income-tax of Rs. 56, 7*a.* 6*p.* at the rate of 6*p.* in the rupee ; find his net income.

7. A man pays an income-tax of Rs. 39. 1*a.* at the rate of 5*p.* in the rupee ; find his net income.

8. A man after paying an income-tax at 6*p.* in the rupee, has Rs. 1550, 8*a.* 6*p.* What was his gross income ?

9. A person after paying an income-tax at 5*d*. in the £ has £ 2447, 18*s*. 4*d*. Find his gross income.

10. A man's net income after paying an income-tax at the rate of 4*d*. in the £ is £ 2261, 14*s*. What will it be, if the income-tax be 5*d*. in the pound ?

11. A person's net income after paying an income-tax at 7*p*. in the rupee is Rs. 1252, 9*a*. 8*p*. What will it be, if the income-tax be reduced by 1*p*. in the rupee ?

12. When the income-tax is 5*d*. in the £ a man pays £15 less than when the income-tax is 7*d*. in the £. Find his income.

13. When the income-tax is 9*p*. in the rupee, a man has to pay Rs. 19, 3*a* 4*p*. more than when the income-tax is reduced by 2*p*. in the rupee. Find his income.

14. A person's gross income is £ 1125 and his net income is £ 1106, 5*s*. Find the rate of income-tax.

15. Income up to Rs. 1500 being subject to an income-tax of 4*p*. in the rupee and income above Rs. 1500 to 6*p*. in the rupee, find what income above Rs 1500 a man must have to be just as rich as a man with Rs. 1488 a year.

16. Incomes upto £ 1600 being subject to an income-tax of 5*d*. in the £ and income avove £ 1600 to 8*d*. in the £, find what income above £ 1600 a man must have to be just £ 15 a year richer than the man with £ 1584 a year.

17. Incomes up to £ 1200 are subject to an income-tax of 7*d*. in the £ and incomes above £1200 to an income-tax of 10*d*. in the £. Find what income above £ 1200 a man must have to be just 10*s*. a year poorer than the man with £ 1190 a year.

18. The rent of a man's bunglow is Rs. 720 per annum. It is assessed to the rates at half of this ; the school rate is 2*a*. 9*p*. in the rupee ; the roads rate is 3*a*. 3*p*. What does hs pay altogether for his residence ?

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§6. **Commission or Brokerage.**

A person who buys or sells goods, property, etc., for another person or company is called an **agent** or more commonly **a broker**, and the remuneration (certain percentage) he receives in return of his labours is called **commission** or **brokerage.**

§7. **Insurance.**

Insurance is a contract made between a company and a person, by which the former undertakes to pay to the latter or

his heirs, the losses caused to him by some accident such as fire, shipwreck or death of a person and the latter promises to pay yearly, half-yearly, quarterly or monthly, as aggreed upon, a certain percentage of the sum insured.

The Company which makes such contracts is called the **Insurance Company** and the person or property protected against accidents by such contracts is called the **insured** ; The sum of money (a certain percentage paid yearly, half-yearly, etc.) is called the **premium.**

§8. When a man insures so as to recover not only his property but also the premium and other expenses of insurance it is said to be **covered.**

All questions under Commissions, Brokerage and Insurance are solved by the Unitary Method. We shall now solve some examples by way of illustrations.

Example 1. A broker sells 28 shares of a bank at Rs. 25 per share ; find his brokerage at $1\frac{1}{2}$ per cent.

Sol. Value of 28 shares $=$Rs. $28\times25=$Rs. 700.

$1\frac{1}{2}$ per cent. $=\frac{1\frac{1}{2}}{100}=\frac{3}{200}$ of the whole.

$\therefore$ brokerage$=$Rs. $\frac{3}{200}\times700=$Rs. $\frac{21}{2}=$Rs. 10, *Sa.* Ans.

Example 2. An agent sells a house and pays the owner Rs. 3840 after deducting his commission at 4% ; find the value of the house.

Sol. If he pays Rs. 96, the value of the house$=$Rs. 100.

$\therefore$ If he pays Re. 1 the value of the house $=$Rs. $\frac{100}{96}$.

$\therefore$ If he pays Rs. 3840, the value of the house

$=$Rs. $\frac{100}{96}\times3840=$Rs. 4000. Ans

Example 3. For what sum should goods worth Rs. 3750 be insured at $6\frac{1}{4}\%$ so that in case of loss the owner may recover the premium as well as the value of the goods.

Sol. If the goods worth Rs. 100$-$Rs. $6\frac{1}{4}$ *i.e.*, Rs. $93\frac{3}{4}$ be insured for Rs. 100 he will recover the loss as well as premium

Now Rs. $93\frac{3}{4}$ must be insured for Rs. 100,

$\therefore$ Re. 1 ,, ,, ,, Rs $100\times\frac{4}{375}$,

$\therefore$ Rs. 3750 ,, ,, ,, Rs. $100\times\frac{4}{375}\times3750$

which is equal to Rs. 4000.

$\therefore$ covered insurance$=\frac{100}{100-\text{rate}}\times$value of the property.

EXERCISE 157.

1. Find the commission on goods worth Rs. 324 at $1\frac{1}{4}\%$.

2. I purchase goods worth Rs. 521 and pay a brokerage at $1\frac{1}{2}\%$. What do the goods cost me altogether ?

3. I purchased goods worth Rs. 5325, and paid Rs. 106, 8*a*. as commission ; find the rate per cent. of commission.

4. If the brokerage on goods at 5*a*. 4*p*. per cent. amonnts to Rs. 12, 5*a*. 4*p*., find the value of the goods.

5. Commission at $1\frac{1}{8}$ per cent. amounts to Rs. 6, 12*a*. On what sum is the commission paid ?

6. A broker after deducting his brokerage at 12*a*. per cent. pays Rs. 14887, 8*a* to the owner of the house. At what price was the house sold ?

7. A man whose life is insured for Rs. 2000 has to pay an annual premium of Rs. 70. What is the rate per cent. ?

8. A man insures his life for Rs. 3000 ; what is the annual premium at 3 per cent. ?

9. A man insures his life for Rs. 2000 ; what is the annual premium at Rs 3, 10*a*. 9*p*. per cent. ?

10. For what sum must a house-holder insure his house worth Rs. 6998, 4*a*. at 7 per cent. so that he may in case of loss, be able to recover both the value of the house and the premium ?

11. For what sum must a cargo worth Rs. 23500 be insured at 6 per cent. so that in case of loss, both cargo and premium may be recovered ?

12. A house is insured for Rs. 15000 so as to recover both the value of the house and the amount of the premium at 3%. Find the actual value of the house.

13. A cargo is insured for £ 2000 so as to recover both the value of the cargo and the amount of the premium at 5 per cent.; find the actual value of the cargo.

14. A man's annual income is Rs. 5700 ; after paying insurance premiums which are exempted from income-tax, he pays Rs. 166, 10*a*. 8*p*. as income-tax at $3\frac{1}{3}$ per cent. What amount of money does he pay for premium ?

15. A man's annual income is Rs. 4100. He pays no income-tax on the money paid for premium. On the remainder he pays Rs. 133, 5*a*. 4*p*. as income-tax at 4 per cent. What amount of money does he pay for premiums ?

CHAPTER XXIX
EXCHANGE

§1. **Exchange** means the process of paying or receiving the money of one country for an equivalent sum in another.

This process can be effected by remitting

(*i*) specie or coined money ;

(*ii*) bullion or gold, silver in bars ;

(*iii*) a bill of exchange.

The payment of a sum of money, by means of specie or bullion is a costly process ; remittances, therefore, are usually made by sending bills of exchange.

§2. **Def.** A written order, to some person at a distance, to pay a certain sum at a particular time, to another person or to his order, is called a bill of exchange.

Bills of exchange are of two kinds—**foreign bills** and **inland bills.**

§3. The quality between the standard coins of two countries obtained by compairing their weight and fineness, is called the Par of Exchange between those countries. Thus, if it is found that the quantity of pure silver in one rupee is exactly equal to that of pure silver in 1·25*s*. then Re.=1·25*s*.

§4. **The rate or cousre of exchange** is such a variable sum of money of one country as is proposed to be given for a fixed sum of that of another ; thus, for instance £ 1. a fixed sum of English money, is given for variable sum of French money, *viz*, 25·16 francs, 25·18 francs, 25·23 francs,etc., more or less than the par of exchange.

Note that when the rate of exchange is *above* the par of exchange it is said to be **at a premium**. and when it is *below* the par of exchange, it is said to be at a **discount.**

§5. **Arbitration of exchange** is the method of determining a course of exchange called the direct or arbitrated rate, between any two places, when the rate of exchange between each of them and some other place or places is known.

Example 1. What is the exchange value of £ 78, 11*s*. 7½*d*. at 2*s*. ¾*d*. per rupee ?

Sol. £78, 11*s.* $7\frac{1}{2}d.$=£$78\frac{93}{160}$; 2*s.* $\frac{3}{4}d.$=£$\frac{33}{320}$.

∴ Reqd. value=£$78\frac{93}{160}\div$£$\frac{33}{320}=\frac{12573}{160}\times\frac{320}{33}$

=Rs. 762. **Ans.**

Example 2. If Rs. 160, 4*a.* 8*p* are equivalent to £15, 15*s.* $6\frac{57}{64}d.$, find the exchange value per rupee.

Sol. Rs. 160, 4*a.* 8*p.*=Rs. $160\frac{7}{24}$.

£15, 15*s.* $6\frac{57}{64}d.=315\frac{147}{256}s.$

∴ Reqd. exchange value=$315\frac{147}{256}s.\div$ Rs. $160\frac{7}{24}$.

$\frac{80787}{256}\times\frac{24}{38847}=\frac{63}{32}s.=1s.\ 11\frac{5}{8}d.$ per rupee. **Ans.**

Example 3. I pay Rs. 51000 to a bank for a bill of exchange payable in London. The rate of exchange is 1*s.* $10\frac{1}{2}d.$ for the rupee and the bank charges me 2 per cent. on the amount payable in England. How much will my agent in London receive?

Sol. 1*s.* $10\frac{1}{2}d.$=£$\frac{3}{32}$.

∴ The exchange value=£$51000\times\frac{3}{32}$

Now the agent receives £100 out of £102 sent by me.

∴ the sum received by the London agent

$=51000\times\frac{3}{32}\times\frac{100}{102}$

=£$\frac{23750}{2}$=£4687, 10*s.* **Ans.**

Example 4. If the par of exchange be two English shillings for the Indian rupee, but if an India bill of exchange for Rs. 540, 12*a.* be negotiated in London for £51, 10*s.*, how much per cent. below par is the rate of exchange ?

Sol. Rs. 540, 12*a.*=Rs. $\frac{21603}{4}$, £51, 10*s.*=£$\frac{1023}{2}$

∴ rate of exchange=£$\frac{1023}{2}\div$ Rs. $\frac{21603}{4}=\frac{1023}{2}\times\frac{4}{21603}$

=£$\frac{2}{21}$ or $\frac{40}{21}s.$

∴ 2*s.*−$\frac{40}{21}s.$ or $\frac{2}{21}s.$ is below par on every 2*s.*

∴ Reqd. percentage=$\frac{2}{21}\times100\div2$.

=$\frac{100}{21}$ or $4\frac{16}{21}\%$. **Ans.**

Example 5. When the rate of exchange is 1*s.* $7\frac{3}{4}d.$ for the rupee, what is the nearest sum of Indian money equivalent to £79, 3*s.* $7\frac{3}{4}d.$?

Sol. Since Re. 1=1*s.* $7\frac{3}{4}d.=\frac{79}{48}s.$ or $\frac{79}{4}d.$

∴ Rs. 2=3*s.* $3\frac{1}{2}d.$

Now £79, 3*s.* $7\frac{3}{4}d.$=£79+3*s.* $3\frac{1}{2}d.+4\frac{1}{4}d.$

=Rs. $(79\times20)\div\frac{79}{48}+$Rs. 2+Re. $\frac{17}{4}\div\frac{79}{4}$.

=Rs. 960+Rs. 2+3*a.* 5*p.* nearly

=Rs. 962, 3*a.* 5*p.* **Ans.**

Example 6. An American merchant sends goods worth 58870 dollars to an English merchant when the course of exchange is 3·36 dollars to a pound sterling but receives his money after 2 months when the rate of exchange is 3·48 dollars to a sovereign. How much does the English merchant gain or lose by the 2 months' delay in payment ?

Sol. The exchange value in the 1st case $=£58870\times\frac{100}{336}$

2nd case $=£58870\times\frac{100}{348}$

$\therefore$ gain $=£58870\times\left(\frac{100}{336}-\frac{100}{348}\right)$

$$=£58870\times100\times\frac{12}{336\times348}=£\frac{3625}{6}$$

$=£604,\ 3s.\ 4d.$ **Ans.**

Example 7. The exchange between London and Calcutta is *1s.* $7\frac{1}{2}d.$ per rupee. that between Paris and London is 25·2 franks for a pound sterling and that between Rome and Paris is 11 lira for 10 francs (1) Find the arbitrated rate between Italy and India. (2) If a Calcutta merchant owes Rs. 4000 to a merchant of Rome what sum in lira will liquidate the debt ?

Sol. $1s\ 7\frac{1}{2}d.=£\frac{13}{160}$

Re. $1=£\frac{13}{160}$

£ $1=25\cdot2$ francs

francs $10=11$ lira

How many lira = rupee 1

$\therefore$ Reqd. No. of lira $=\dfrac{13\times252\times11}{160\times10\times10}=2\cdot25225.$

$\therefore$ Rs. $4000=2\cdot25225\times4000=9009$ lira. **Ans.**

Example 8. If the short rate of exchange in London on Paris is 25·2 and the rate of discount for two months' bills on London is at $4\frac{1}{2}\%$ per annum, what debt in paris can be discharged by a person in London who has a two months' bill on London for £2400

Sol. Discount on £1 $=£1\times\frac{9}{2}\times\frac{1}{100}\times\frac{2}{12}$

$=£\frac{3}{400}.$

$\therefore$ Present value of £1 $=£\left(1-\frac{3}{400}\right)$ or $£\frac{397}{400}$

Hence $£\frac{397}{400}=25\cdot2$ francs

$\therefore$ The amount of reqd. debt $=£2400\times\frac{397}{400}\times\frac{252}{10}$ francs

$=60026\cdot4$ francs. **Ans.**

Example 9. How much sterling should be given for 1920 marks each equivalent to $11\frac{1}{4}d.$ at par, when English money bears a premium of 4% in Germany ?

Sol. Since 1 mark $= 11\frac{1}{4}d.$ or $£\frac{45}{960}$

$\therefore$ 1920 marks $= £\frac{45}{960} \times 1920$ or £ 90

$\therefore$ The reqd. sum $= £90 \times \frac{104}{100}$

$= £93,\ 12s.$ **Ans.**

Example 10. What is the short rate of exchange, when the price in Hamburg of six months' bills in Calcutta is 1·18 marks per rupee and the rate of discount in Calcutta is $3\frac{1}{3}$ p. c.

Sol. Present value of Re. 1 $= \text{Re. } (1 - 1 \times \frac{10}{3} \times \frac{1}{100} \times \frac{6}{12})$.

$= \text{Re. } \frac{59}{60}$.

$\therefore$ Re. $\frac{59}{60} = 1{\cdot}18$ marks.

Hence the reqd. short rate $= 1{\cdot}18 \div \frac{59}{60}$

$= 1{\cdot}2$ marks. **Ans.**

EXERCISE 158.

1. How much Indian money can be paid for £1560, when the rate of exchange is 1*s.* 4*d.* per rupee.

2. Find the exchange value in pound sterling of Rs. 19920 at 1*s.* $10\frac{1}{2}d.$ for the rupee.

3. A merchant of Bombay indents ; from London goods worth £4840. What must he pay, when the course of exchange is 1*s* 10*d* a rupee ?

4. If London exchanges with Lahore at a gain of $8\frac{1}{3}\%$ when the course of exchange is Rs. 24, 6*a.* per £, what is the rate of exchange ?

5. Bombay exchanges with England at 1*s.* 10*d.* a rupee and with New York at Rs. 4, 2*a.* a dollar ; find the estimated rate of exchange between England and New York.

6. If the exchange between England and Spain be $10\frac{2}{3}d.$ per franc and that between England and India be 1*s.* 4*d.* per rupee, what is the arbitrated rate between Spain and India ?

7. A New York dollar at par of exchange is worth 4*s.* 8*d.* What is the value of 391 dollars when exchange on London is at a premium of $6\frac{1}{4}$ per cent. ?

CHAPTER XXX

STOCKS AND SHARES

§1. When the Government of a country require large sum of money for any purpose. *e.g.*, to construct a railway, or to carry on a war, etc., where does it get money from ? Evidently it must *borrow* or *contract a loan* from the people. The people lend money to the Government which give to the *Lenders* in exchange for their money, *Bonds or acknowledgments* for the amount lent but it reserves to itself the option of the *Time* of paying off the *Principal* on the clear understanding to pay the interest of the money regularly at fixed periods. *Stock* is the term applied to this money lent to the Government of a country or to a Trading firm at some fixed rate of interest.

Tnus, if the Government of India borrows 50 crores of rupees at 5 per cent. and if a person *A* lends Rs. 1000 of this amount, *A* is said to have Rs 1000, 5 per cent Stock. He receives a document to this effect and is entitled to receive the interest *viz.*, Rs. 50 upon this document from year to year until the Government repays the loan to *A*. This interest is paid *half-yearly* and the document may be sold and transferred from one person to another like any other kind of property.

§2. Suppose the holder of a stock wants cash money for some urgent business or for a change in his circumstances, what will he do? He cannot demand the payment of the loan from the Government before the due date but he can sell his stock to another person where by his claim to half-yearly interest is transferred to that person.

The cash value of stock is not affected by any change in the rate of interest, since it is fixed once for all by the Government at the time, when the money is borrowed. But it varies from time to time, sometimes even twice or thrice daily, owing to political or commerical causes ; for instance, if the current rate of interest is smaller, the investment is free from all possible risks and the number of investors is large, then holder of

Rs. 100 stock could sell it for more than Rs. 100. Similarly if the current rate of interest is greater or if the investment is risky, then the same stock is to be sold at perhaps less than Rs. 100. Hence remember the following terms :—

(1) **At par.** If by selling a Rs. 100 stock, a person receives Rs. 100 cash, the stock is said to be at par.

(2) **At a premium or above par.** If the selling price of Rs. 100 stock is more than Rs. 100 cash, the stock is said to be at a premium or above par.

(3) **At a discount or below par.** If the selling price is less than Rs. 100 cash, it is said to be at a discount or below par.

§3. **Brorerage.**

Stock is generally bought and sold through a broker, who usually *charges* $\frac{1}{8}$ *per cent. on the stock* bought or sold. Thus if the market value of Rs. 100 stock is Rs. 102, the seller of a stock receives Rs. $(102-\frac{1}{8})$ and the purchaser has to pay Rs. $(102+\frac{1}{8})$. This charge of a broker is called *brokerage*. Hence

The brokerage must be added to the price of stock which is bought and subtracted from the price of that which is sold through a broker. The broker's charge is often quoted "*brokerage* $\frac{1}{8}$," the words 'per cent.' being omitted. **Unless the brokerage is expressly stated it need not be considered when doing exam les in stock.**

§4. Money borrowed by a Government is called **National Debt.** Money lent to the Government of India is said to be invested in **Government of India Securities** Money lent to the Government of England is said to be invested in **Funds.** A large portion of the National Debt of England is called **Consols.**

§5. Students should very carefully distinguish between the *paper or nominal value* of stock and the *cash or actual price* as well as between the *amount of stock purchased* and the sum *invested* for its purchase. Thus if the 3 per cent. stocks are at 95, a man who invests Rs. 950 will be able to purchase Rs. 1000 stock *i.e.* the amount of stock held by the man is Rs. 1000, but the *actual* or *cash* value of that stock is only Rs. 950.

The student should also clearly understand the meaning of an expression of the type "Rs. 4000 in the 5 per cents." It does *not* mean that the man invested Rs. 4000 in the 5 per cent. stock but that he holds stock of the *nominal value* of Rs. 4000 in the 5 per cents. He might have actualy invested more or less than Rs. 4000 according as the stock was at a premium or at a discount.

§6. **Shares.**

Suppose a big railway line or some such work is to be done which requires a large amount of money. Evidently one or two individual cannot provide all the money. Several persons meet together and decide to start a company. These men, called **promoters**, first decide the amount of money or capital required for the purpose and then elect a few responsible men called **directors** who are considered fit to manage the affairs of the company which is called **a joint stock company.** The directors then divide the capital into a large numbr of equal parts or **shares** and invite the public to **subscribe** to the fund. A man can take as many shares as he likes and thus he becomes **a share-holder.** The whole amount of the shares is not generally paid in one instalment ; for instance, suppose that the capital of a Railway Company Rs. 1,000,000 is divided into 100,000 shares of Rs 10 each ; since the construction cannot be completed in a few days or months, the whole amount is not required at once. The company, therefore, might ask its share-holders to pay at first only Rs. 3 per share and the remaining Rs. 7 when called **upon.** The Rs. 300,000 thus raised is called the **paid up capital** of the company.

Now when the railway has been oompleted, it begins to work and thus earns money, a portion of which is spent in paying working expenses and the remainder is usually divided amongst the share-holders. Profit thus divided among share-holders is called **dividend.**

§7. **Different Varieties of Stock.**

When a company finds that the capital subscribed by its share-holders is not sufficient, it does not issue more shares but it usually borrows money at a fixed rate and agrees to pay interest on this money before paying dividends to its share-holders, who subscribed to its original capital. Money thus raised is called **preference stock,** whereas the original capital is called ordinary or **deffered stock.**

Again certain companies issue what are called **debentures.** The holder of a debenture stock receives a fixed rate of interest and not a share of its profits, but if the interest is not regularly paid, he can have claim on the property of the company.

§8. Warning against common mistakes.

In solving questions on stock, the student must clearly distinguish between **cash** and **stock**, as explained in Art 5.

A stock is often denoted by the rate of interest it yields. Hence "5 *per cents.* at 98" means :—

(*i*) There is a certain stock which pays a dividend of Rs. 5 on every Rs. 100 stock.

(*ii*) the market value of Rs. 100 stock is Rs. 98, *i.e.*, a person can purchase a Rs. 100 stock for Rs. 98 only and therefore

(*iii*) there is a dividend of Rs. 5 on an investment of Rs. 98, *i.e.*, by spending Rs. 98 only, a person can have an income of Rs. 5.

§9. All examples in stock can be solved by the principal of *Rule of Three* or *Unitary Method.* We shall have now solve a few questions of different types on stock.

(*a*) **Amount to be invested.**

Exampie 1. What sum of money will be required to buy Rs. 16,000 stock in $3\frac{1}{2}$ per cents. at 91 ?

Soi. Rs. 91 are required for buying a Rs. 100 stock

$\therefore$ Rs. 100 stock : Rs. 16000 stock : : Rs. 91 : reqd. sum

$\therefore$ the reqd. sum = Rs. $\frac{16000 \times 91}{100}$ = Rs. 14560. **Ans.**

or by *Unitary Method* :—

$\because$ cost of Re. 100 stock = Rs. 91

$\therefore$ cost of Re. 1 ,, = Re. $\frac{91}{100}$

$\therefore$ cost of Rs. 16000 ,, = $\frac{91}{100} \times 16000$ = Rs. 14560. **Ans.**

Example 2. What sum must I invest to buy £ 1700 stock in $4\frac{1}{2}$ per cent. at 101 $\frac{3}{8}$, brokerage being $\frac{1}{8}$ p. c. ?

Sol. Here £ $101\frac{3}{8} + £\frac{1}{8}$ *i.e.*, £ $101\frac{1}{2}$ is required for buying a £ 100 stock ;

$\therefore$ £ 100 stock : £ 1700 stock : : £ $101\frac{1}{2}$: required sum

$\therefore$ required sum = £ $\frac{1700 \times 101\frac{1}{2}}{100}$ = £ 1725, 10*s.* **Ans.**

EXERCISE 159.

How much must be invested to purchase (1—9) :—

1. Rs. 2000 stock at 92 ?
2. £ 2400 stock at 103 ?
3. Rs. 4500 stock in the $3\frac{1}{2}$ per cents. at 88 ?
4. £ 5550 stock in the 4 per cents. at $97\frac{5}{8}$?
5. £ 650 stock in 3 per cents. at $90\frac{3}{8}$ through a broker ?
6. Rs. 3075 in the $4\frac{1}{2}$ per cents. bonds at par (brokerage as usual) ?
7. £ 8833, 6*s*. 8*d*. in the 5 per cents. at $3\frac{3}{4}$ premium. brokerage $\frac{1}{8}$ p. c. ?
8. Rs. 1229, 2*a*. 8*p*. in the 4 per cents. at $\frac{3}{8}$ discount, brokerage as usual ?
9. Rs. 1250 stock st $50\frac{1}{2}$ premium (dividend $5\frac{1}{2}$ p. c.) ?
10. When a certain stock is selling at 19% premium, what money will a person require to buy £ 2025 stock at $5\frac{1}{2}$ p. c., brokerage being $\frac{1}{8}$ p. c. ?

(*b*) Amount obtained by the sale of stock.

Example 3. A person has Rs. 10,000 stock in a certain company whose shares are at 135 per cent. premium. What amount does he get by selling his stock through a broker, brokerage being $\frac{1}{8}$ per cent.

Sol. Here by selling Rs. 100 stock, the man gets Rs. 235 —Re. $\frac{1}{8}$=Rs. $234\frac{7}{8}$, *i.e.*, Rs. $\frac{1879}{8}$.

$\therefore$ by selling Re. 1 stock he will get Rs. $\frac{1879}{8}\times\frac{1}{100}$.

$\therefore$ „ Rs. 10000 „ „ Rs $\frac{1879}{8}\times\frac{1}{100}\times 10000$

=Rs. 23487, 8*a*. **Ans.**

EXERCISE 160.

How much money do I derive from the sale of (1—3) :

1. Rs. 16000 in the $3\frac{1}{2}$ per cents. at $88\frac{1}{8}$?
2. £ 15850, 16*s*. 8*d*. in the $3\frac{1}{2}$ per cents. at $98\frac{3}{8}$, brokerage being as usual ?
3. £ 9533, 6*s*. 8*d*. in the 3 per cents. at $95\frac{5}{8}$, brokerage being as usual ?

4. A person has Rs. 5000 stock in a certain company whose shares are at 116% premium. What amount does he get by its sale through a broker, brokerage being $\frac{1}{8}$ p. c. ?

5. A person has Rs. 7500 stock in a certain company whose shares are at $8\frac{7}{8}$ p. c. discount. What amount does he get by its sale through a broker ?

(c) **Amount of stock purchased.**

Example 4. How much stock can be purchased by investing £ 506, 9*s*. 2*d* in the $3\frac{1}{4}$% at $89\frac{1}{4}$, brokerage $\frac{1}{8}$% ?

Sol. Here in order to purchase £ 100 stock a person should invest £ $(89\frac{1}{4}+\frac{1}{8})$, *i.e.*, £ $\frac{715}{8}$; £ 506, 9*s*. 2*d*. = £ $\frac{12155}{24}$.

$\therefore$ amount of stock bought for £ $\frac{715}{8}$ = £ 100

$\therefore$ " " " " for £ 1 = £ $100 \times \frac{8}{715}$

$\therefore$ " " " for £ $\frac{12155}{24}$ = £ $100 \times \frac{8}{715} \times \frac{12155}{24}$

= £ 566, 13*s*. 4*d*. **Ans.**

Example 5. I have Rs. 3600 of 3% stock ; I sell out at 110 and buy $4\frac{1}{2}$% stock at 120 ; how much $4\frac{1}{2}$% stock at 120 do I hold ?

Sol. By selling Rs. 100 stock I get Rs. 110 ;

$\therefore$ by selling Rs. 3600 stock I get Rs. $\frac{110}{100} \times 3600$ = Rs. 3960.

Again by investing Rs. 120, I buy Rs. 100, $4\frac{1}{2}$ p. c. stock

$\therefore$ by investing Rs. 3960 I buy Rs. $\frac{100}{120} \times 3960$ " "

i.e. Rs. 3300 stock. **Ans.**

EXERCISE 161.

Find the amount of stock purchased by investing (1—4) :—

1. Rs. 8280 in the $3\frac{1}{2}$ per cents. at 92.

2. £ 821, 5*s*. in the $4\frac{1}{2}$ per cents. at $82\frac{1}{8}$.

3. Rs. 5062, 8*a*. when the price is at premium of Rs. $12\frac{3}{8}$ and brokerage as usual.

4. £ 6909, 18*s*. in the 3 per cents. at £ $92\frac{3}{4}$, brokerage being 2*s*. 6*d*. per £ 100.

5. Find the price of 4% paper when Rs. 5230 stock can be bought for Rs. 5753 cash.

6. A person invests 3000 fr. in 5 per cents. at 87·73 ; what amount of stock will he hold ?

7. A man with £ 5330 purchased stock at £ 91 and when the price of the stock increased by £ $1\frac{3}{4}$ he sold it, and with the money thus obtained, he again purchased Debentures at £ $102\frac{1}{2}$. How much debenture stock did he purchase ?

(d) **Gain or loss by sale of stock.**

Example 6. A person invests £14340 in consols when they are at $89\frac{1}{2}$ and sells when they are at $93\frac{3}{8}$. What is his gain ? (Brokerage as usual.)

Sol. Here, he bought the consols at £$(89\frac{1}{2}+\frac{1}{8})$, *i.e.* £$89\frac{5}{8}$, and when he sold he got £ $(93\frac{3}{8}-\frac{1}{8})$, *i.e.*, £ $93\frac{1}{4}$. so that for every £$89\frac{5}{8}$ or £$\frac{717}{8}$ that he invested he gained £$(93\frac{1}{4}-89\frac{5}{8})$ *i.e.*, £ $3\frac{5}{8}$ or £ $\frac{29}{8}$.

$\therefore$ for investing £ 14340 his gain $= £\ \frac{8}{717}\times\frac{29}{8}\times 14340.$

$= £\ 580.$ **Ans.**

EXERCISE 162.

1. A person investing £ 13597, 10*s.* in the purchase of the 3 per cent. at $92\frac{3}{8}$ and afterwards sells it at $94\frac{1}{4}$. Brokerage being as usual, what profit does he make ?

2 I lay out Rs. 2653, 8*a.* in buying $3\frac{1}{2}$ per cent. consols at $97\frac{3}{4}$ and then sell it at $96\frac{1}{8}$. Find my loss, usual brokerage being charged on each transaction.

3. I invest Rs. 126540 in the 3 per cents. at $99\frac{3}{4}$; at what price must I sell to gain Rs. 15817, 8*a.*?

4. A man invests Rs. 3,750 in buying a stock at $93\frac{5}{8}$ and sold it at $95\frac{5}{8}$. If the whole transaction was done through brokers, how much did he gain ?

5. How much stock in the 4% should be bought at 96, in order that by selling out at par Rs. 250 may be gained ?

6. How much must I invest in the 3 per cent. stock when they are at 10 per cent. below par, that by selling when they are at 5 per cent. premium, I may gain Rs. 1125 ?

7. A person invests £ 1365 in the 3 per cents. at 91 ; he sells out £ 1000 stock when they have risen to $93\frac{1}{2}$ and the remainder when they have fallen to 85. How much does he gain or lose by the transaction ?

8. A person invests Rs. 7560 in the 3 per cent. stock at $94\frac{1}{2}$ and when it falls to 90, he sells out $\frac{1}{4}$ of his stock ; afterwards when it is at $94\frac{3}{4}$ he sells the remainder. Find the alteration in his capital.

§10. Income.

In all examples given above, if will be seen that we did not take into account the rate of interest which any stock yielded for the question of *income* did not rise in any case. But in fact the question of income is the most important factor in all transactions on stocks or shares. Two persons might possess two different kinds of stocks by investing unequal amount of money but yet they might be of the same value to the holder in point of income. It will be easily noticed from the example solved below that all questions of income and transfer of stock from one kind to another belong to the theorem of 'Rule of Three' or 'Unitary Method.'

(*a*) Income derived from investment.

Example 1. What income is derived from investing Rs. 7560 in the 3 per cents. at $94\frac{3}{8}$, brokerage being $\frac{1}{8}$ p. c. ?

Sol. Here, investing Rs. $(94\frac{3}{4}+\frac{1}{8})$, *i.e.*, Rs. $94\frac{1}{2}$, a man could get a Rs. 100 stock, which will bring in an annual income of Rs. 3.

$\therefore$ Income on Rs. $94\frac{1}{2}$ each $=$ Rs. 3.
$\therefore$,, ,, Re. 1 ,, $=$ Rs. $3\times\frac{2}{189}$
$\therefore$,, ,, Rs. 7560 ,, $=$ Rs. $3\times\frac{2}{189}\times 7560$
$=$ Rs. 240. **Ans.**

EXERCISE 163.

1. What aunual income will be derived by investing Rs. 35190 in the $3\frac{1}{2}$ per cents. at 90 ?

2. *A* invests £ 1695, 16*s*. 8*d*. in the $3\frac{1}{2}$ per cents. at $101\frac{3}{4}$, *B* invests £ 1767, 3*s*. 9*d*. in the 3 per cents. at $94\frac{1}{4}$; what is the difference of their yearly incomes ?

3. What is the total income derived by investing Rs. 686 in the 3 per cents. at 98 and Rs. 765 in the $2\frac{1}{2}$ per cents. at 85 ?

4. What half-yearly dividend is obtained by an investment of £5000 in the 3 per cent. stock at $87\frac{3}{8}$ after deducting 7*d*. in the £ for income-tax ?

5. What amount of $2\frac{3}{4}$ per cent. consols will yield a net income of £ 152, 17*s*. 1*d*. when the income-tax is 8*d*. in the £ ?

(*b*) **Sum to be invested for a certain income.**

Example 2. How much must I invest in the $3\frac{1}{2}$ per cents. at 85 to have an annual income of £ 73, 10*s*.

Sol. To get £ $3\frac{1}{2}$, I shall have to invest £ 85 ;

$\therefore$,, ,, £ 1 ,, ,, ,, £ $85\times\frac{2}{7}$

$\therefore$,, ,, £ $73\frac{1}{2}$,, ,, ,, £ $85\times\frac{2}{7}\times\frac{147}{2}$.

$=$£ 1785. **Ans.**

EXERCISE 164.

1. How much must I invest in the 3 per cents. at $90\frac{1}{8}$ (brokerage as usual) to have an annual income of Rs. 1465?

2. How much must be invested in $3\frac{3}{4}$ per cent. stock at $112\frac{1}{2}$ to get an income of Rs. 300 ?

3. How much $3\frac{1}{2}$ per cent. Government Securities at $95\frac{1}{4}$ must be sold out in order to purchase enough 5 per cent. Municipal Debenture at $119\frac{3}{4}$ to produce an annual income of Rs. 665 ? (A brokerage of $\frac{1}{4}$ per cent. is charged on each transaction.)

4. What sum of money invested in the 4 per cents. at par would realise the same income as Rs. 10200 invested in the $4\frac{1}{2}$ per cents. at 102.

5. How much money must be invested in a 3 per cent. stock at $92\frac{1}{2}$ to produce the same income at £ 1710 invested in a $3\frac{1}{2}$ per cent. stock at 95 ?

6. A man invests one third of the capital in the $3\frac{1}{2}$ per cent. Government Securities at $96\frac{1}{2}$ and the remaining two-thirds in the $4\frac{1}{2}$ per cent. Debentures at $105\frac{1}{2}$. If the difference of the two annual incomes be Rs. 1997, find his capital.

(*c*) **Price of a stock to produce a certain income.**

Example 3. Find the price of 4 per cent. stock, so that an investment of Rs. 4680 may produce an annual income of Rs. 180, brokerage being as usual.

Sol. Income of Rs. 180 is on Rs. 4680.

,, ,, Re. 1 ,, Rs. $\frac{4680}{180}$.

,, ,, Rs. 4 ,, Rs. $\frac{4680}{180}\times4=$Rs. 104.

i.e. the holder of the stock altogether spent Rs. 104 for Rs. 100 stock and out of this, he paid Rs. $\frac{1}{8}$ to the broker, hence the price of the stock is Rs. $(104-\frac{1}{8})$, *i.e.*, $103\frac{7}{8}$. **Ans.**

EXERCISE 165.

1. What is the price of 4 per cent. stock, when an investment of £17430 produces an income of £830 ?

2. A person invests Rs. 4095 in the 4 per cent. stock and obtains an income of Rs. 173, 5*a*. 4*p*. What is the price of the stock ?

3. Find the price of $4\frac{1}{2}$ per cent. Government Promissory Notes when an investment of Rs. 59422, 8*a*. produces monthly income of Rs. 213, 12*a*.

4. A person sells £5000 of 3 per cent. stock and buys $3\frac{1}{2}$ per cent. stock at $87\frac{1}{2}$. If the increase in his income be £5, what is the price of the 3 per cent. stock ?

5. By selling £4500 of India 5 per cent. stock at $112\frac{1}{2}$ and investing the proceeds in China 7 per cent. stock, the income is increasd by £168, 15*s*. ; what is the price of the latter stock ?

6. Rs. 210,000 invested in the 4 per cent. stock produces annually Rs. 2166, 10*a*. 8*p*. more than if it had been invested in the $2\frac{3}{4}$ per cent. stock at 99. Find the price of the 4 per cent. stock.

(*d*) Change in income.

Example 4. A person invests Rs. 44100 in the $3\frac{1}{2}$ per cent. Government Securities at 98 and when they rise to $98\frac{1}{2}$ he sells out and invests the proceeds in the 5 per cent. Municipal Debentures at $110\frac{13}{16}$. Find the change in his income.

Sol Let us find what income he had at first

On Rs. 98 cash, gain = Rs $3\frac{1}{2}$.

$\therefore$,, Re. 1 ,, ,, = Rs. $\frac{7}{2}\times\frac{1}{98}$

$\therefore$,, Rs. 44100 ,, = Rs. $\frac{7}{2}\times\frac{1}{98}\times 44100$

= Rs. 1575.

Then let us see how much money he got by the sale of his Securities.

A bond bought at Rs. 98 was sold for Rs. $98\frac{1}{2}$;

$\therefore$,, ,, ,, Re. 1 ,, ,, Rs. $\frac{197}{2}\times\frac{1}{98}$

$\therefore$ A bond bought at 44100 was sold for

Rs. $\frac{197}{2}\times\frac{1}{98}\times 44100$ = Rs. 44325.

Now this money was invested in the debentures.

For every Rs. $110\frac{13}{16}$ invested he got an income=Rs. 5.

$\therefore$,, ,, Re. 1 ,, ,, ,, =Rs. $5\times\frac{16}{1773}$

$\therefore$,, ,, Rs. 44325 ,, ,, ,, =Rs. $5\times\frac{16}{1773}\times 44325$

=Rs. 2000.

Hence his income increased by Rs. 2000—Rs. 1575

i. e., Rs. 425. **Ans.**

EXERCISE 166.

1. A person transfers Rs. 11000 from the 4 per cents. at 92 to the 5 per cents. at 110 ; find the change in his income.

2. A person invested £9075 in the 3 per cents. at $90\frac{3}{4}$ and on the stock rising to 91 transferred it to the $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$. What increase does he produce in his yearly income ?

3. Find the change in income produced by shifting Rs. 11000 from the 3 per cents. at 101 $\frac{3}{8}$ to $4\frac{1}{2}$ per cent. stock at $137\frac{3}{8}$, the usual brokerage being charged on each transation.

4. Find the alteration in income caused by t ansferring £3200 stock from the 3 per cents. at $86\frac{3}{8}$ to the 4 per cents. at $114\frac{7}{8}$, the brokerage $\frac{1}{8}$ per cent. being in each transaction.

5. A person transfers Rs. 1000 stock from 4 per cent. stock at 90 to 3 per cent. stock at 72. How much of the latter stock will he hold and what will be the difference in his income ?

6. If I hold £10000 of a 3 per cent. stock and sell out at $94\frac{1}{2}$, investing the money obtained in the 4 per cent. railway debentures at 105, what change would I make in my income ?

7. I invest Rs. 12805 in the 4 per cents. at $98\frac{1}{4}$ and when they have risen to $102\frac{5}{8}$, I sell out and invest in the $4\frac{1}{2}$ per cents. at $105\frac{3}{8}$; what is the change in my income ? (Brokerage $\frac{1}{4}$ per cent. on all transactions).

8. A person invests Rs. 19700 in the $3\frac{1}{2}$ per cent. stock at $98\frac{1}{2}$ and when they rise to $101\frac{1}{2}$, he sells out and invests the proceeds in the $4\frac{1}{2}$ per cent. consols at $114\frac{3}{16}$. Find the change in his income.

9. The price of Govt. stock which pays $6\frac{1}{2}$ per cent. is $185\frac{3}{4}$; Rs. 6250 of this stock is transferred to a 3 per cent. stock at $92\frac{7}{8}$; find the loss in annual interest.

10. A person bought Rs. 14500, $3\frac{1}{2}$ per cent. stock at $72\frac{1}{2}$ and when the price fell to 68 he sold it ; with the proceeds of the sale he bought 4 per cent. stock at $75\frac{5}{9}$. Find the change in his income.

11. By selling out £3000 in the 4 per cents. at 90 and investing the proceeds in the India 5 per cent. stock at $106\frac{11}{19}$, what change in income has been effected ?

12. I invested Rs. 49700 in shares paying Rs. 7 per share, when the Rs. 100 share is at $122\frac{1}{2}$. If I had invested it in the $5\frac{1}{4}$ per cent. bonds at 105, what increase in my annual income would have been obtained ?

13. A man invests Rs. 42780 in the 5 per cents. at $114\frac{7}{8}$ but afterwards sells at $135\frac{1}{8}$ and invests the proceeds in the 4 per cents. at $92\frac{7}{8}$. What is the change in his income ? (Brokerage $\frac{1}{8}\%$ on each transaction.)

14. A man buys £3430 stock in the $3\frac{1}{2}$ per cents. When they are at $85\frac{1}{2}$, he sells out and invests his capital in the 4 per cents. at 98. Find the alteration in his income,

15. A man has Rs. 31000 in the 3 per cents. at 92 but sells out and with the proceeds purchases 5 per cent. Railway stock at 115. Find the change in his income.

16. A person invests £18150 in the 3 per cents. at $90\frac{1}{2}$, and on the stock rising to $91\frac{1}{4}$, transfers it to $3\frac{1}{2}$ per cents. at $97\frac{1}{4}$. Find the alteration in his income. (Brokerage $\frac{1}{4}$ per cent.)

17. A man transfers Rs. 3600 from the 4 per cent. stock at $95\frac{1}{8}$ to the $3\frac{1}{2}$ per cents. at $89\frac{7}{8}$. How much of the latter stock does he hold and what is the alteration in his income, allowing $\frac{1}{8}$ per cent. brokerage in each case ?

18. A person invested £1000 in the 3 per cents. at $90\frac{5}{8}$; but the price rising to $91\frac{1}{4}$, he sold out and invested the proceeds in the $3\frac{1}{2}$ per cents. at $97\frac{1}{8}$. Find the increase in his income.

19. A person invests Rs. 21390 in the 5 per cents. at 115 ; he afterwards sells it at 135 and invests the proceeds in the 4 per cents. at 93. Find the change in his income.

20. A man has £6680 stock in a 4 per cent. at 96. When the price has fallen 2 per cent. he transfers his capital to $3\frac{1}{2}$ per cent. stock at $83\frac{1}{2}$. Find the change in his income.

21. A person sells out of the $3\frac{1}{2}$ per cents. at $92\frac{3}{4}$ and realises Rs. 18550. If he invests $\frac{2}{5}$ of the proceeds in the 4 per

cents. at 96 and the remainder in the 3 per cents. at 90, find the alteration in his income.

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§11. Some Important Typical Examples.

Example 1. What rate of interest will a person receive if he invests in $6\frac{1}{2}$ per cents. at $129\frac{7}{8}$? (Brokerage $\frac{1}{8}$ per cent.)

Sol. Since $129\frac{7}{8}+\frac{1}{8}$, *i.e.*, 130 yields an interest of $6\frac{1}{2}$

$\therefore$ $130 : 100 :: 6\frac{1}{2}$: required rate.

$\therefore$ required rate $=\dfrac{100\times 13}{2\times 130}=5$.

i.e., the rate of interest is 5 per cent. **Ans.**

Example 2. Which stock is the better to invest : in the 4 per cents. at 102 or the $3\frac{1}{2}$ per cents. at 96 ?

Sol. (1) Income on Rs. 102 = Rs. 4

$\therefore$,, on Re. 1 = Re. $\frac{4}{102}$

(2) Income on Rs. 96 = Rs. $\frac{7}{2}$

$\therefore$,, on Re. 1 = Re. $\frac{7}{192}$

in $\frac{4}{102}$, $\frac{7}{192}$ first fraction, *i.e.*, $\frac{4}{102}$ is greater

$\therefore$ the 4 per cents. at 102 is the better investment. **Ans.**

Example 3. What is the price of a 3 per cent. stock, if after paying an income-tax of $5d.$ in the £ a man derives an income of $3\frac{1}{2}$ per cent. on his money ?

Sol. The tax being $5d.$ in the £, a man's net income in a £ is $235d.$, *i.e.*, £ $\frac{47}{48}$.

$\therefore$ £$\frac{47}{48}$: £ $\frac{7}{2}$ :: £1 : gross income.

$\therefore$ gross income = £ $\frac{7}{2}\times\frac{48}{47}$ = £ $\frac{168}{47}$.

$\therefore$ required price $=$ £$\dfrac{3\times 100\times 47}{168}$ = £ $\frac{1175}{14}$ = £$83\frac{13}{14}$:

i.e., the 3 per cent. stock is at $83\frac{13}{14}$. **Ans.**

Example 4. A person invests £ 14970 in the purchase of the 3 per cents. at 90 and the $3\frac{1}{4}$ per cents. at 97. His total income being £ 500, how much of each stock did he buy ?

Sol. Suppose he invests £ x in the 3 per cents. and

$\therefore$ £ $(14970-x)$ in the $3\frac{1}{4}$ per cents.

his income from the first $= £\frac{x}{30}$

and that from the second $= £\frac{13\,(14970-x)}{388}$;

$\therefore$ by the question, $\frac{x}{30}+\frac{13\,(14970-x)}{388}=500$;

which being solved gives $x=9150$, *i.e.*, he invested £9150 in the 3 per cents. and $\therefore$ £5820 in the $3\frac{1}{4}$ per cents.

Now let us find the amounts of stock he purchased,

£90 : £ 9150 : : £ 100 : 3 p. c. stock bought,

$\therefore$ 3 p. c. stock bought $=£\frac{915000}{9}=£\frac{305000}{3}$

$=£\ 10166\frac{2}{3}$. **Ans.**

and £ 97 : £ 5820 : : £ 100 : $3\frac{1}{4}$ p. c. stock bought,

$\therefore$ $3\frac{1}{4}$ p.c. stock bought $=£\ \frac{582000}{97}=£6000$. **Ans.**

Note. Questions of his type are easily solved, as shown above, by means of algebarical equations.

Aliter. Suppose he invested £ 14970 in the purchase of 3 per cents. at 90.

$\therefore$ income $= £\frac{3}{90}\times 14970=£499$

i.e., a decrease of £ 500—£ 499=£1.

Now suppose he invested £14970 in the purchase of $3\frac{1}{4}$ per cents. at 97.

$\therefore$ income $=£\ \frac{3\frac{1}{4}}{97}\times 14970 = £\frac{97305}{194}$

i.e., an increase of £ $\frac{97305}{194}$—£500=£ $\frac{305}{194}$

$\therefore$ he invested at 3 per cents, and $3\frac{1}{4}$ per cents. in the ratio of $\frac{305}{194}$: 1 respectively *i.e.*, 305 : 194.

$\therefore$ investment at 3 per cents. at 90 $=£\ \frac{305}{499}\times 14970=£9150$

and ,, ,, $3\frac{1}{4}$ per cents. at 97 $= £\frac{194}{499}\times 14970=£5820$

Now proceed further.

Example 5. I invest Rs. 163000 partly in a 4 per cent. stock at 108 and partly in a 5 per cent. stock at $109\frac{1}{2}$ so that my income is the same from each stock. How much do I invest in each stock?

Sol. To obtain an income of Rs. 5 from the first stock I must invest

Rs. 4 : Rs. : : 108 : required investment.

$\therefore$ investment=Rs. $\frac{5\times 108}{4}$=Rs. 135 ; but I get Rs. 5 from the second stock by investing only Rs. $109\frac{1}{2}$

Hence Rs. 163000 must be divided in the ratio of 135 : $109\frac{1}{2}$ *i.e.*, 270 : 219 or 90 : 73

$\therefore$ investment in the 1st stock=Rs. $163000\times\frac{90}{163}$=90000

and ,, the 2nd stock =163000−90000=Rs 73000

EXERCISE 167.

1. A man invests £ 4031 10*s*. in the 3 per cents. at $94\frac{3}{4}$; what will be his net income after an income-tax of 10*d*. in the pound has been deducted, $\frac{1}{8}$ p. c brokerage being allowed ?

2. A person investing in the 4 per cents. receives 5 per cent. on his money. What is the price of the stock ?

3. Which is the better investment, the 3 per cents. at $83\frac{1}{2}$ or the $3\frac{1}{2}$ per cents. at 3 per cent. discount ?

4. Which is the better investment: bank stock paying 10 per cent. at 319 or 3 per cent. consols at 96 ? What will be the cost of £1500, of 3 per cent. consols at $89\frac{3}{8}$, brokerage being $\frac{1}{8}$ per cent.? What rate of interest will such investment obtain ?

5. A man holds $15\frac{1}{4}$ shares of a bank and receives £ 19, 1*s*. 3*d*. per quarter. If the interest he receives be 5 per cent. per annum, find the value of a share.

6. Which is the better investment : $4\frac{3}{4}$ p. c. stock at 5 below par, or $5\frac{1}{4}$ p. c. stock at 5 premium ?

7. What is the price of a 4 per cent. stock if it yields 5 p. c. after paving an income-tax at 6*d*. in the £ ?

8. A man invests a certain sum in $4\frac{1}{2}$ per cent. Government paper at 104. The price falling to 101, he sells out and loses Rs. 600 by the transaction, inclusive of brokerage. Find the sum invested.

9. A person invests Rs. 44100 in the $3\frac{1}{2}$ per cent. Government Securities at 98 and when they rise to $98\frac{1}{2}$, he sells out and invests the proceeds in the 5 per cent. Municipal Debentures. His income thereby increases by Rs. 425. What is the price of the Debentures ?

10. A person invests Rs.19700 in the $3\frac{1}{2}$ per cent. Government Securities at $98\frac{1}{2}$ and when they rise to $101\frac{1}{2}$, he sells

out and invests the proceeds in Municipal Debentures at $114\frac{3\frac{3}{4}}{16}$. His income thereby increases by Rs. 100. What is the rate of dividend of the Debentures ?

11. Which is the better iuvestment—the $3\frac{1}{2}$ per cent. stock at $95\frac{3}{8}$, or the 4 per cent. Debentures at $101\frac{1}{2}$? What will be the difference in the annual income by investing Rs. 22127 in each of them ?

12. If by investing Rs. 59422, 8*a*. in Government stock at $104\frac{1}{4}$, I derive a monthly income of Rs. 213. 12*a*., what is the rate of interest of the stock ?

13. The difference between the income derived from investing a certain sum in 6 per cent. stock at 126, and in 9 per cent. stock at 210 is £22, 10*s*. Find the amount invested.

14. One third of a certain capital is invested in the $3\frac{1}{2}$ per cent. Government Securities at 105 one-fourth in the 3 per cent. Government Secutrities at $97\frac{1}{2}$, and the remainder in the $4\frac{1}{2}$ per cent. Calcutta Municipal Debentures at $112\frac{1}{2}$. If the total income is Rs. 830, what is his capital ?

15. The difference between the income derived from investing a certain sum in 5 per cent. stock at 127 and in $5\frac{1}{2}$ per cent. stock at 135 is £4, 14*s*. Find the amount invested and the income derived from each stock.

16. At what price must a person invest in the 4 per cent. Government Promissory Note, so that after paying income-tax at5 pies in the rupee, he may receive $4\frac{1}{2}$ per cent. on his mo ney ?

17. A man invested Rs. 5800 in the purchase of 5 per cent. Debenture stock at par. After he got the half yearly dividend he sold at $2\frac{1}{2}$ per cent. premium and with the whole money purchased 4 per cent. stock at Rs. 95, 2*a*. 6*p*. ; what change was produced in his annual income ?

18. A person had an annual income of Rs. 480 from a 4 per cent. stock. He sold it at Rs. 95, 14*a*. and with the proceeds of the sale bought 5 per cent. Railway Debentures at Rs. 119, 9*a*. Assuming that usual brokerage was charged on each transaction, find the change in his income.

19. £ 3000, which I held in the 4 per cents. was sold for me when they were at $82\frac{3}{4}$ by a broker; whose commission is $\frac{1}{4}\%$ and the proceeds were re-invested by him in the $4\frac{1}{2}$ per cents. at $98\frac{3}{4}$. What amount of the latter stock did he purchase?

20. A man invests a sum of money in a stock at 91 and an equal sum in another stock at 117. After some time the prices of the stocks have exchanged values, when he sells and makes a profit of Rs. 52. What sum did he invests?

21. A maninvested equal sums in two different stocks $3\frac{1}{2}$ p r c nt. Govern ment st ock at $103\frac{1}{2}$ and 4 per cent. consols at 105; his income from one was Rs. 93 more than from the other; what sum was invested in each stock?

22. By selling out £3000 in the 4 per cents. at 90 and investing the proceeds in the India 5 per cent. stock, a person finds his income increased by £6, 13*s*. 4*d*. What is the price of the India stock?

23. If I invest my money in shares paying Rs. 7 per share when the Rs. 100 share is at $122\frac{1}{2}$, I find that I get Rs. 355 a year more than if I invest it in the $5\frac{1}{4}$ per cent. bonds at 105. Find my capital.

24. A man invests £1980 in the $3\frac{1}{2}$ per cents. at 99 and £3220 in the $4\frac{1}{2}$ per cents. at 105. What is the average rate of interest on his whole investment?

25. What amount of stock must be purchased in English 5 per cents. at $111\frac{1}{8}$ to produce the same yearly return as 3 lacs of rupees (a rupee = 1*s*. $10\frac{1}{2}$*d*.) invested in the Government $4\frac{1}{2}$ per cents. at $101\frac{1}{4}$?

26. A person finds that if he invests a certain sum in Railway shares paying £6 per share when the £100 share is at 132 he will obtain £10, 16*s*. a year more for his money than if he invests in the 3 per cent. consols at 93. What sum has he to invest?

27. By investing a certain sum of money in the $3\frac{1}{2}$ per cents. at 72 a man gets Rs. 35 less income than if he would get by investing the same sum in the $4\frac{1}{2}$ per cents. at 90; find the sum invested.

28. A person invests a certain sum in the $3\frac{1}{2}$ per cent. Government Securities when they are at $97\frac{3}{8}$; and if he waited till they had fallen to $97\frac{1}{8}$, he would have had Rs. 400 more of Government Securities. How much money did he invest, $\frac{1}{8}$ per cent. being charged as brokerage in both cases?

29. What sum must a person invest in the 3 per cents. at 90, in order that by selling out Rs. 20000 stock, when they have risen to $93\frac{1}{2}$ and the remainder when they have fallen to

84¼ he may gain Rs. 125 by the transaction? If he invests the proceeds in the 4 per cent. at par what will be the difference in his income?

30. A person invests £ 6200 in the 3 per cents. at 89⅛ and pays income-tax of 10*d.* in the £; on the stock rising to 92 he sells out and invests the proceeds in £ 50 Railway shares at par which yield an annual dividend of 3½ per cent. clear of income-tax. Find the alteration in the income.

31. I invest Rs. 40000 partly in the 3 per cents. at 80 and partly in the 4 per cents. at 96; and then I find that on the whole I receive 4% interest on the sum invested. What sums have I invested in the two stocks?

32. A person invests £ 3459 in the 3 per cents. at 87. After receiving one year's dividend he sells out at 89. He then invests the whole Railway stock paying 5 p. c. at 115. What will be the difference in his income?

33. If 4% paper be at 110. what sum must I invest in order to secure a yearly income of Rs. 470, after paying an income-tax of 4*p.* in the rupee?

34. *A* invests £ 3500 in buying equal amounts of 3 per cent. stock at 78⅛ and of 6% stock at 109⅜. *B* invests the same sum, half in one stock and half in the other. Find (*i*) the difference in their incomes (*ii*) the ratio of their rates of interest.

35. Find what amount must be invested in the 2½ per cents. at 95¾ to produce a clear income of Rs. 1000 a year, brokerage being ⅛% and income-tax 4⅘ pies in the rupee.

36. A person bought consols at 94 and sold the same at 95⅝ thereby gaining Rs. 550. What cash did he receive for the stock, reckoning ⅛% brokerage on each transaction.

37. A man invests £1668, 15*s.* in the 3 per cents. at 89⅞. His first year's income he invests in the same stock but at a different price. His next year's income is £ 57, 10*s.* At what price did he invest in the second case? (Brokerage ⅛ in both cases.)

38. I buy £ 500 stock at 66 and afterwards £ 500 of the same stock at 69; I sell out the whole when the price has risen to 89. What is the increase in my capital?

39. A man prossesses £22400 of 2¾ per cents. consols. He sells them at 99⅜. He then invests one-half of the proceeds in

4% Railway debenture stock at $132\frac{1}{2}$, lends £7266, 18*s.* 4*d.* on mortgage at $4\frac{1}{2}$% and loses the rest. What alteration is produced in his annual income ?

40. A person bought Railway stock at 88 and after receiving the half year's dividend at $4\frac{1}{2}$% per annum sold out at $92\frac{3}{8}$, making a total profit of Rs. 1987, 8*a.* How much stock did he buy ?

41. *A* invests a certain sum in the 3 per cents. at 80and *B* invests half the sum in the 4 per cents. If *A*'s income is to *B*'s income as 8 to 5, find the price of the 4 per cent.

42. If money invested in the 3 per cent. consols gives exactly 3 per cent. after the payment of income-tax at the rate of 1*s.* per £, find the price of the consols allowing $\frac{1}{8}$ per cent. to a broker for purchase.

43. *I* invest $\frac{1}{3}$rd of my property in Bank Stock, $\frac{1}{6}$th in consols a nd the remainder in Railway shares; I sell out w'en I might make a profit of 5 per cent., 3 per cent. and 2 p. c. respectively on the investments and I altogether get £ 6190. What is the original amount of my property ?

44. A person after paying an income-tax of 7*d.* per £ has a clear income of £ 262,.2*s.* 6*d.* derived from stock in the $4\frac{1}{2}$ per ecnts,, he sells out $\frac{2}{3}$rd of his stock at $93\frac{3}{4}$ and invests the money in N. W. R. Stock at $112\frac{1}{2}$, which pays $5\frac{1}{4}$ per cent. per annum ; what is his clear income now after paying income-tax as before ?

45. A person invests Rs. 10000, partly in the 4 per cents. at 96 and partly in the $4\frac{1}{2}$ per cents. at 117. What amount does he invest in each stock, if he has the same income from each ?

46. What is the price of the 4 per cent. stock if after paying an income-tax of 4 pies in the rupee, a man's income may be $\frac{1}{21}$ of his capital.

47. A man had some 3 per cent. stock. He sold the same at 90 and with the proceeds purchased 4 per cent. stock at 95. His income increased by Rs. 243. How much 3 per cent. stock did he possess ?

48. 144 shares in a company are worth £ 856, 16*s.* when the dividend is at the rate of 7 per cent. How many shares ought to be worth £ 1193, 8*s.* when the dividend is at 9 per cent.

49. A person has 3 per cent. stock which yields Rs. 240 a year. He sells out $\frac{1}{4}$ of the stock at $87\frac{1}{4}$ and invests the proceeds in Railway stock at $174\frac{1}{2}$. What dividend ought the latter to pay that he may thereby increase his income by Rs. 40?

50. A person invests Rs. 36135 in the purchase of 4 per cent. stock at 63 and $3\frac{1}{2}$ per cents. at 90. His total income is Rs. 1428, 8*a*. How much money did he invest in each case?

51, I iuvest £ 28787, partly in the purchase of 5 per cent. stock at 115 and partly in the 3 per cent. stock at 90. My total income is £988, 10*s*.; How much do I invest in each stock?

52. A person has Rs. 24180 to invest in the $5\frac{1}{2}$ per cent. Government loan at 108 and in the 6 per cent. Calcutta Municipal loan at 102. How must he divide his capital between them so as to obtain the same income from each?

53. One year I receive a dividend of 6 per cent. on my stock and pay an income-tax of Re. 1, 10*a*. 8*p*. per Rs. 100. The next year I receive dividend of $6\frac{1}{2}$ per cent. and pay an income-tax at Re. 1, 4*p*. per Rs. 100 and find that my net income has increased by Rs. 2490. How much stock do I hold?

54. A man has £ 30043, 16*s*. $4\frac{1}{2}d$. Part of his money is invested by him in the purchase of 3 per cent. stock at 90 and the remainder in the 4 per cents. at $104\frac{3}{4}$. His total income is £ 1021, 8*s*. How much does he invest in the 3 per cent. stock?

55. If the 3 per cent. consols are at $81\frac{7}{8}$, what must be the price of the 5 per cents. that there may be no loss of income in selling out the former and investing the proceeds in the latter, allowing the usual brokerage on each transaction?

CHAPTER XXXI

APPLICATION OF ALGEBRA AND GEOMETRY TO ARITHMETIC.

1. Application of Algebra

§1. Formulae.

Some examples in Arithmetic can be easily solved by the application of Algebraical formulae. The following examples may be noted carefully :—

Example 1. Simplify $\cdot 121 \times \cdot 997 + \cdot 121 \times \cdot 003$.

Sol. Apply formula $ax+bx=x(a+b)$.

The expression $= \cdot 121\ (\cdot 997 + \cdot 003)$

$= \cdot 121 \times 1 = \cdot 121$. **Ans.**

Example 2. Simplify

$\cdot 526 \times \cdot 526 + 2 \times \cdot 526 \times \cdot 474 + \cdot 474 \times \cdot 474$.

Sol. Apply formula $a^2+b^2+2ab=(a+b)^2$

The expression $= (\cdot 526)^2 + (\cdot 474)^2 + 2(\cdot 526 \times \cdot 474)$

$= (\cdot 526 + \cdot 474)^2 = (1)^2 = 1$. **Ans.**

Example 3. Simplify

$\cdot 529 \times \cdot 529 + \cdot 379 \times \cdot 379 - \cdot 758 \times \cdot 529$

Sol. Apply formula $a^2+b^2-2ab=(a-b)^2$

The expression $= (\cdot 529)^2 + (\cdot 379)^2 - 2(\cdot 379 \times \cdot 529)$

$= (\cdot 529 - \cdot 379)^2 = (\cdot 15)^2$

$= \cdot 0225$. **Ans.**

Example 4. Simplify $\dfrac{(\cdot 821 \times \cdot 821) - (\cdot 179 \times \cdot 179)}{\cdot 821 - \cdot 179}$

Sol. Apply formula $a^2-b^2=(a+b)(a-b)$.

The expression $= \dfrac{(\cdot 821)^2 - (\cdot 179)^2}{\cdot 821 - \cdot 179} = \dfrac{(\cdot 821 + \cdot 179)(\cdot 821 - \cdot 179)}{\cdot 821 - \cdot 179}$

$= \cdot 821 + \cdot 179 = 1$. **Ans.**

Example 5. Simplify $\dfrac{85\frac{1}{4} \times 85\frac{1}{4} - 34\frac{5}{6} \times 34\frac{5}{6}}{120\frac{1}{12}}$

Sol. The expression $= \dfrac{(85\frac{1}{4})^2-(34\frac{5}{6})^2}{120\frac{1}{12}}$

$= \dfrac{85\frac{1}{4}\times 34\frac{5}{6})(85\frac{1}{4}-34\frac{5}{6})}{120\frac{1}{12}}$

$= \dfrac{120\frac{1}{12}\times 50\frac{5}{12}}{120\frac{1}{12}} = 50\frac{5}{12}$. **Ans.**

Example 6. Simplify $\dfrac{\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{5}\times\frac{1}{5}+\frac{3}{80}+\frac{3}{100}}{\frac{1}{16}+\frac{1}{10}+\frac{1}{25}}$

Sol. Apply formula $(a+b)^3 = a^3+b^3+3a^2b+3b^2a$

The expression $= \dfrac{(\frac{1}{4}+\frac{1}{5})^3}{(\frac{1}{4}+\frac{1}{5})^2}$

$= \frac{1}{4}+\frac{1}{5} = \frac{9}{20}$. **Ans.**

Example 7. Simplify

$$\frac{\cdot121\times\cdot121\times\cdot121-\cdot021\times\cdot021\times\cdot021-3\times\cdot121\times\cdot121\times\cdot021+3\times\cdot021\times\cdot021\times\cdot121}{\cdot121\times\cdot121-2\times\cdot121\times\cdot021+\cdot021\times\cdot021}$$

Sol. Apply formula $(a-b)^3 = a^3-b^3-3a^2b+3b^2a$.

The expression $= \dfrac{(\cdot121-\cdot021)^3}{(\cdot121-\cdot021)^2}$

$= \cdot121-\cdot021 = \cdot1$. **Ans.**

Example 8. Simplify $\dfrac{576\times576\times576+424\times424\times424}{576\times576-576\times424+424\times424}$

Sol. Apply formula $a^3+b^3 = (a+b)(a^2-ab+b^2)$

The expression $= \dfrac{(576)^3+(424)^3}{(576)^2-576\times424+(424)^2}$

$= 576+424 = 1000$. **Ans.**

Example. 9. Simplify $\dfrac{\frac{3}{4}\times\frac{3}{4}\times\frac{3}{3}-\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}}{\frac{9}{16}+\frac{3}{16}+\frac{1}{16}}$

Soi Apply formula $a^3-b^3 = (a-b)(a^2+ab+b^2)$.

The expression $= \dfrac{(\frac{3}{4})^3-(\frac{1}{4})^3}{(\frac{3}{4})^2+\frac{3}{4}\times\frac{1}{4}+(\frac{1}{4})^2}$

$= \frac{3}{4}-\frac{1}{4} = \frac{1}{2}$ **Ans.**

EXERCISE 168

Simplify :—

1. $5672 \times 937 + 5972 \times 1063$.
2. $1{\cdot}526 \times {\cdot}729 + 1{\cdot}526 \times {\cdot}271$.
3. ${\cdot}5625 \times 15{\cdot}221 + {\cdot}5625 \times {\cdot}779$.
4. $521 \times 521 + 379 \times 379 + 379 \times 1042$.
5. ${\cdot}561 \times {\cdot}561 + {\cdot}439 \times {\cdot}439 + 2 \times 561 \times {\cdot}439$.
6. ${\cdot}625 \times {\cdot}625 + {\cdot}375 \times {\cdot}375 - 2 \times {\cdot}625 \times {\cdot}375$.
7. $5\frac{1}{6} \times 5\frac{1}{6} + 4\frac{5}{6} \times 4\frac{5}{6} + 2 \times 4\frac{5}{6} \times 5\frac{1}{6}$.
8. $15\frac{7}{12} \times 15\frac{7}{12} + 11\frac{5}{12} \times 11\frac{5}{12} - 2 \times 15\frac{7}{12} \times 11\frac{5}{12}$.
9. $\dfrac{({\cdot}375)^2 + 2({\cdot}375 \times {\cdot}125) + ({\cdot}125)^2}{{\cdot}375 + {\cdot}125}$
10. $\dfrac{{\cdot}675 \times {\cdot}675 - {\cdot}325 \times {\cdot}325}{{\cdot}675 - {\cdot}325}$
11. $\dfrac{{\cdot}729 \times {\cdot}729 - {\cdot}529 \times {\cdot}529}{{\cdot}729 + {\cdot}529}$
12. $\dfrac{35\frac{2}{3} \times 35\frac{2}{3} - 21\frac{5}{6} \times 21\frac{5}{6}}{57\frac{1}{2}}$
13. $\dfrac{58\frac{3}{4} \times 58\frac{3}{4} - 41\frac{1}{4} \times 41\frac{3}{4}}{17\frac{1}{2}}$.
14. $\dfrac{({\cdot}79)^3 + ({\cdot}21)^3 + 3({\cdot}79)^2({\cdot}21) + 3({\cdot}21)^2({\cdot}79)}{{\cdot}79 \times {\cdot}79 + {\cdot}21 \times {\cdot}21 + 2 \times {\cdot}79 \times {\cdot}21}$
15. $\dfrac{({\cdot}179)^3 - 3({\cdot}179)^2({\cdot}079) + 3({\cdot}079)^2({\cdot}179) - ({\cdot}079)^3}{({\cdot}179)^2 - {\cdot}358 \times {\cdot}079 + ({\cdot}079)^2}$
16. $\dfrac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} - \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} - \frac{1}{4} + \frac{1}{6}}{\frac{1}{2} \times \frac{1}{2} - \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}}$
17. $\dfrac{\frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} + \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} + 3 \times \frac{1}{25} \times \frac{1}{6} + 3 \times \frac{1}{36} \times \frac{1}{5}}{\frac{1}{5} \times \frac{1}{5} + \frac{1}{15} + \frac{1}{6} \times \frac{1}{6}}$
18. $\dfrac{{\cdot}526 \times {\cdot}526 \times {\cdot}526 + {\cdot}474 \times {\cdot}474 \times {\cdot}474.}{{\cdot}526 \times {\cdot}526 - {\cdot}526 \times {\cdot}474 + {\cdot}474 \times {\cdot}474}$
19. $\dfrac{{\cdot}556 \times {\cdot}556 \times {\cdot}556 - {\cdot}496 \times {\cdot}496 \times {\cdot}496.}{{\cdot}556 \times {\cdot}556 + {\cdot}556 \times {\cdot}496 + {\cdot}496 \times {\cdot}496}$
20. $\dfrac{({\cdot}0347)^3 + ({\cdot}9653)^3}{({\cdot}0347)^2 - ({\cdot}347)({\cdot}09653) + ({\cdot}9653)^2}$

§2. Equations.

Example 1. What is the number from which if you take away 21 the remainder is $\frac{3}{4}$ of the original number ?

Sol. Since the number—21 $=\frac{3}{4}$ of the number
$\therefore$ the number—$\frac{3}{4}$ of the No.$=21$ [by transposition]
$\therefore$ $\frac{1}{4}$ of the No.$=21$
or the number$=84$. **Ans.**

Example 2. A boy loses $\frac{1}{5}$ of his money and then gains $8p.$; he then loses $\frac{1}{4}$ of what he has and then gains $6p.$; he afterwards loses $\frac{1}{3}$ of what he has and then find that he has $6a.$ $8p.$ left ; how much had he at first ?

Sol. $\frac{1}{5}$ of my money is lost $\therefore$ $\frac{4}{5}$ of it remains, $8p.$ is then gained.

$\therefore$ money now remaining$=\frac{4}{5}$ of original money $+8p.$; of this $\frac{1}{4}$ is lost.

$\therefore$ $\frac{3}{4}$ of ($\frac{4}{5}$ of original money$+8p.$) remains ;
$6p.$ is then gained.

$\therefore$ money now remaining$=\frac{3}{4}$ of ($\frac{4}{5}$ of original money$+8p.$) $+6p.$; of this amount $\frac{1}{3}$ is lost
$\therefore$ $\frac{2}{3}$ of [$\frac{3}{4}$ of ($\frac{4}{5}$ of original money$+8p.$)$+6p.$] remains
$=\frac{2}{3}$ of ($\frac{3}{5}$ of original money$+6p.$ $+6p.$)
or $\frac{2}{5}$ of original money$+8p.=6a.$ $8p.$
or $\frac{2}{5}$ of original money$=6a.$
$\therefore$ original money$=6a.\times\frac{5}{2}$ $=15a.$ **Ans.**

Example 3. The price of 3 cows and 5 oxen is Rs. 295 and that of 4 cows and 6 oxen is Rs. 370 ; find the price of a cow and an ox.

Sol. Price of 3 cows and 5 oxen$=$Rs. 295.....(1)
and price of 4 cows and 6 oxen$=$Rs. 370.....(2)
Multiply (1) by 4 and (2) by 3 we have,
price of 12 cows and 20 oxen$=$Rs. 1180.....(3)
price of 12 cows and 18 oxen$=$Rs. 1110.....(4)
Subtracting (4) from (3), we have
price of 2 oxen$=$Rs. 70.

$\therefore$ price of 1 ox $=$Rs. 35.

Subtracting the price of 5 oxen from Rs. 295, we have the price of 3 cows $=$Rs. 120.

$\therefore$ price of a cow $=$Rs. 40.

$\therefore$ cow cost Rs. 40 and ox Rs. 35. **Ans.**

Example 4. 5 men and 6 boys can do $\frac{5}{6}$th of a work in 2 days ; 2 men and 9 boys can do $\frac{7}{10}$th of the work in 2 days ; in what time can a boy do the whole work ?

Sol. In 2 days, 5 men and 6 boys can do $\frac{5}{6}$th of a work

$\therefore$,, 1 day, 5 men and 6 boys can do $\frac{5}{12}$th ,, ,, ,,

$\therefore$,, 1day, 10 men and 12 boys can do $\frac{5}{6}$th ,, ,, (1)

Again, in 2 days, 2 men and 9 boys can do $\frac{7}{10}$th of a work

$\therefore$,, 1 day, 2 men and 9 boys can do $\frac{7}{20}$th ,, ,,

$\therefore$,, 1 day, 10 men and 45 boys can do $\frac{7}{4}$,, ,, (2)

$\therefore$,, 1 day, 33 boys can do $(\frac{7}{4}-\frac{5}{6})$ of the work.

[Subtratcting (1) from (2).

i.e., 33 boys can do $\frac{11}{12}$th of the work in one day.

$\therefore$ 1 boy can do $\frac{11}{12}\times\frac{1}{33}$ or $\frac{1}{36}$ of the work in one day

$\therefore$ 1 boy can do the whole work in 36 days. **Ans.**

EXERCISE 169.

1. What is the number from which if you take away 17, the remainder is $\frac{5}{6}$ of the original number ?

2. What is the number in which if you add 20 the sum is $\frac{6}{5}$ of the original number ?

3. A boy loses $\frac{1}{3}$ of his money and then gains $4p.$; he then loses $\frac{1}{4}$ of what he has and then gains $5p.$; he afterwards loses $\frac{2}{3}$ of what he has and then finds that he has $3a.$ $2p.$ left. How much had he at first ?

4. If 5 sheep and 9 goats cost Rs. 107 and 9 sheep and 5 goats cost Rs. 103, how much will one sheep and one goat cost

5. If 3 horses and 5 cows cost Rs. 345 and 5 horses and 3 cows cost Rs. 335, find the price of a horse.

6. The cost of 9 chairs and 10 tables is Rs. 163, and that of 6 chairs and 15 tables Rs. 192 ; find the price of a chair and a table.

7. 2 men and 4 boys can do $\frac{7}{15}$th of a piece of work in 4 days, and 3 men and 5 boys can do $\frac{19}{24}$th of it in 5 days. In what time can a boy do the whole work ?

8. Two boys and one man can do a piece of work in 4 days, and one boy and two men can do it in 3 days. In what time can a boy do the whole work ?

9 5 men and 2 boys can do a piece of work in 4 days, 2 men and 4 boys can do $\frac{9}{10}$th of it in 5 days. In what time can a man do the work ?

10. If 3 men with 4 boys earn Rs. 51 in 6 days and 4 men with 5 boys earn Rs. 55 in 5 days. in what time will 6 men and 9 boys earn Rs. 144 ?

11. If 5 men and 6 women do a piece of work in 10 days, which 3 men and 10 childern can do in 12 days, find how long 19 men, 30 childern and 12 women working together will take to do it.

12. 12 men and 15 women can finsh a certain piece of work in 20 days ; if 10 more women are put on, the work is then finished 5 days earlier ; in what time can a man and a women together finish the work ?

13. If 2 boys and 1 man can do a piece of work in 4 hours and 2 men and 1 boy can do the same in 3 hours, find in what time a man, a boy separately and a man a boy together can do the same.

---

§ 3. **Surds.**

If we are asked to find the value of expressions which involve surds in their denominators *e.g.*,

$$\frac{1}{\sqrt{2}-1}; \quad \frac{2}{\sqrt{3}-\sqrt{2}}; \quad \frac{\sqrt{2}}{\sqrt{3}+1}; \quad \frac{\sqrt{2}+\sqrt{3}}{\sqrt{3}-\sqrt{2}}; \text{ etc.,}$$

we should remember the following algebraical facts :—

(*i*) $\sqrt{a}\times\sqrt{a} = a.$

(*ii*) $\sqrt{a}\times\sqrt{b} = \sqrt{a\times b}$

(*iii*) $\sqrt{a^2\times b^2} = \sqrt{ab}.$

(*iv*) $\sqrt{a^2\times b} = a\sqrt{b}.$

(*v*) $(\sqrt{a}+\sqrt{b})^2 = a+b+2\sqrt{ab}.$

(*vi*) $(\sqrt{a}-\sqrt{b})^2 = a+b-2\sqrt{ab}$

(*vii*) $(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b}) = a-b.$

(*viii*) $(a+b)(a-b) = a^2-b^2.$

Example 1. Find the value of $\frac{1}{\sqrt{2}-1}$

Sol. $\frac{1}{\sqrt{2}-1}=\frac{\sqrt{2}+1}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{\sqrt{2}+1}{2-1}$

$=\sqrt{2}+1$

If we now substitute the approximate value of $\sqrt{2}$ we get the required result to be 2·414. **Ans.**

Note. If we have substituted the value of $\sqrt{2}$ in the given expression we would have to divide the number 1 by a decimal fraction, a process which is more tedious than what is shown above.

Example 2. Find the value of $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$

Sol. The given expression $=\frac{(\sqrt{3}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2}}$

$=\frac{3+2+2\sqrt{6}}{3-2}=5+2\sqrt{6}$

in which the approximate value of $\sqrt{6}$ being substituted we will get the required result to be 9·898. **Ans.**

Note. In the following examples, the student will have to use the value of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, etc. etc. a record (1·414, 1·732, 2·236, 2·449 respectively) of which will, therefore, be very useful and convenient.

EXERCISE 170.

Find the value of the following to 3 places of decimals:—

1. $\frac{1}{\sqrt{5}+1}$ 2. $\frac{1}{\sqrt{3}-\sqrt{2}}$ 3. $\frac{2}{\sqrt{5}-\sqrt{3}}$

4. $\frac{4\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ 5. $\frac{2+\sqrt{3}}{\sqrt{3}+1}$ 6. $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$

7. $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{1}}$ 8. $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{11}-\sqrt{5}}$

9. $(\sqrt{6}+\sqrt{3}+\sqrt{2}+2)(\sqrt{6}-\sqrt{3}+\sqrt{2}-2)$.

10. $\frac{\sqrt{(\cdot05-\cdot005)}}{\sqrt{(\cdot05)-\cdot005}}$ 11. $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{}}$

Which is the greater quantity :—

12. $\sqrt{2}$ or $\sqrt[3]{3}$? 13. $\sqrt{3}$ or $\sqrt[4]{15}$?

14. Find the value of $\frac{15+\sqrt{\cdot 009}}{1-\sqrt{9}}$ correct to 3 places of decimals.

15. Simplify : $\left(\frac{\sqrt{11}+\sqrt{7}}{\sqrt{11}-\sqrt{7}}\right)^2-\left(\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}\right)^2$

II. Application of Geometry.

§4. We know that the square on the hypotenuse of a right angled triangle is equal to the sum of the squares *on* the other two sides. Hence the square *of* the measure of the side opposite to the right angle is equal to the sum of the square *of* the measures of the sides containing the right angle. Therefore we have the following rules :—

Rule (*i*) $(Hypo.)^2=(Perp.)^2+(Base)^2$
(*ii*) $(Perp.)^2=(Hypo.)^2-(Base)^2$
and (*iii*) $(Base)^2=(Hypo)^2-(Perp.)^2$, *i. e.*,

if any two sides of a right-angled triangle be given, we can find the third side.

Example 1. The side of a right-angled triangle are 16 feet and 63 feet : find its hypotenuse.

Sol. $\because\ 16^2+63^2=256+3969$
$=4225$
and $\sqrt{(4225)}=65$
$\therefore$ the hypotenuse is 65 feet. **Ans.**

Example 2. The diagonal of a rectangular courtyard is 355 yds., its length is 284 yds. ; find its breadth.

Sol. $\because\ (Base)^2=(Hypo.)^2-(Perp.)^2$
$\therefore\ (Breadth)^2=(355)^2-(284)^2$
$=(355+284)(355-284)$
$=639\times 71=71\times 9\times 71$

$\therefore$ the reqd. breadth is 71×3 or 213 yards. **Ans.**

EXERCISE 171.

1. The town A is 72 miles west of B and 135 miles south of C. What is the distance between B and C?

2. A ladder 25 ft. long has its foot placed in a street and its top resting against a wall on one side of the street at a height of 15 ft. from the ground. If the ladder be turned over to the other side, its top reaches to a point 20 ft. high on the opposite wall. Find the breadth of the street.

3. The tip of a bud of lotus was seen a span above the surface of water in a lake. Driven by wind, it gradually advanced and sank at a distance of 2 cubits. Tell me quickly, O mathematician, the depth of the water." (*Lilawati.*)

4. The tip of a bud of lotus was seen 16 inches above the surface of water in a lake. Driven by wind it gradually advanced and sank at a distance of 36 inches. Find the depth of the water.

CHAPTER XXXII

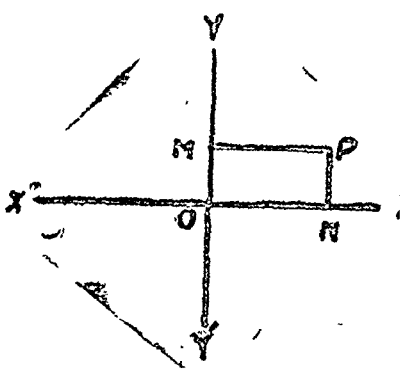
GRAPHS.

§ In all mathematical operations we find two different kinds of quantities— *constants* and *variable*. Quantities which have always the same value are called **constants,** *e.g.*, a rupee is constant quantity since its value never changes, it is always equal to 64 pice. Quantities which have different values at different places or times or whose values depend upon certain circumstances are known **variables,** *e.g.*, the temperature of a room, the speed of a train, the price of a certain article are variables as they change from time to time.

Again, variables are of two kinds, **dependent and independent,** *e.g.*, the earnings of a labourer depend upon the number of days he works. Here the number of working days as well as the earnings of the man are both variables, but since the latter depends upon the former, the latter is called the dependent and the former, the independent variables. Similarly the distance travelled by a man is dependent variable and the time required to cover the distance is the independent variable.

§2. If a train runs uniformly at the rate of 30 miles an hour, then we can easily find the distance it travells in a certain time, *i. e.*, there exists a certain relation detween the two variables, namely the time and distance travelled, The relation can be shown clearly by means of a diagram called a graph.

§3. Let the horizontal line XOX′ be cut at right angles by the vertical line YOY′ at the point O, thus dividing the place into four spaces called *quadrants, viz.*, XOY, YOX′, X′OY′ and Y′OX.



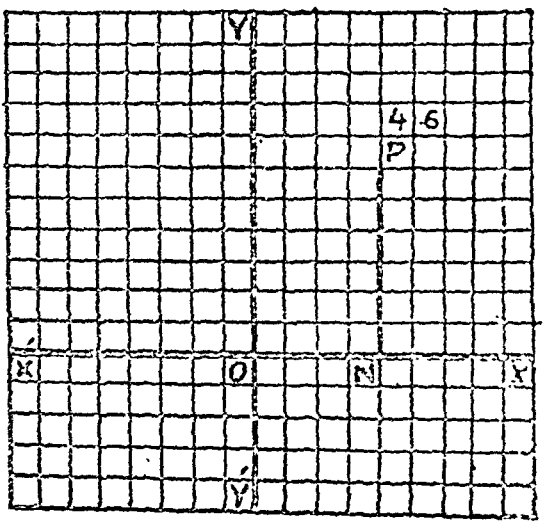
Take any point P in the first quadrant XOY and draw PM and PN perpendiculars on OY and OX respectively.

Let PM or ON $= x$;
and PN or OM $= y$.

x and y are called **co-ordinates** of P; the lines XX′ and YY′ are called the **axes of co-ordinates** or more briefly the axes and are respectively called the axes of X and Y. The point O is called the **origin**, x is called the **abscissa** and y the **ordinate** of P. A point is denoted by the ordinates, the abscissa being named first. Thus the point P is called the point (x,y).

The values of x are measured from O along the axis of X, the values being positive when taken to the right of O along OX and negative when drawn to the left of O along OX′. The values of y are positive when drawn above XX′ and negative when drawn below XX′. Thus the point (4,6) is plotted in the following way :—

Measure ON=4 units along the axis of X to the right of O and then by drawing a perpendicular at N above OX and measuring PN=6 units of length, we get the point P, which is called the point (4,6).



In the above figure the horizontal and vertical lines are drawn $\frac{1}{10}$th of an inch from one another. Such a paper is called a squared paper and questions on graphical method are very conveniently done on this paper.

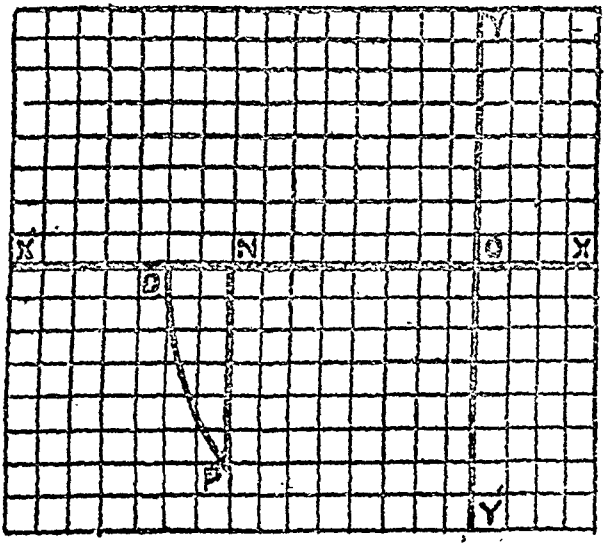
[For further description on the subject *see* our "Algebra Made Simple."

§4. the graphical *method* of solving examples will be clearly understood from the following examples :—

Example 1. A man walks westwards 8 miles and then turns south and walks 6 miles ; how far is he from the starting point ?

Sol. Let O be the starting point. Measure ON=8 units of length along OX'. Draw NP perpendicular and below the line XX' and cut off NP=6 units of length. then evidently the man came to P. To measure OP, draw a circle from O as centre and OP as radius cutting OX' at Q. Then OP=OQ and OQ=10 units *i. e.* he is 10 miles away from the starting place.

Observation. In this example, a mile has been denoted by $\frac{1}{10}$th of aninch which is the unit of measurement.



Example 2. Make a graph for converting inches to centimetres and *vice versa.*

Sol. 1 cm.=$\frac{2}{5}$ inch approximately, ∴ 5 cm.=2 in. approx.

Draw OX and OY at right angles and mark inches along OX and centimetres along OY.

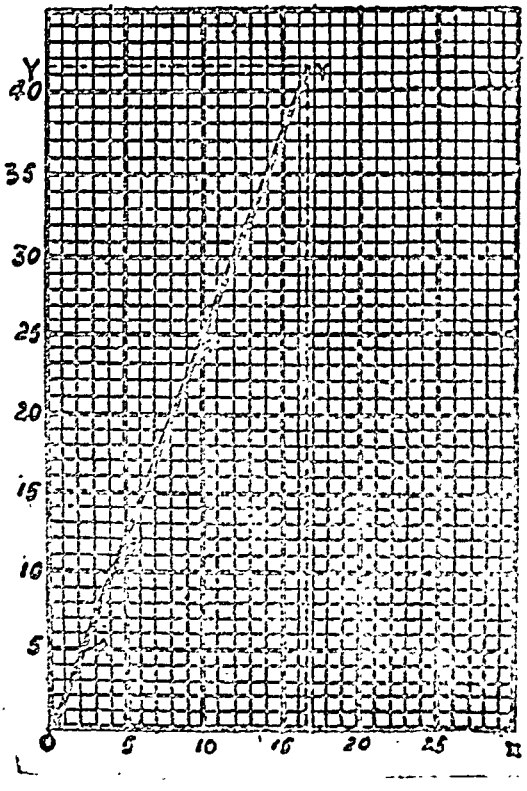
To draw the graph, find the point A where the lines from 5 cm. and 2 in. meet, and since 5 cm.=2 inche.

∴ the graph will pass through A. But to keep the line straight, take another point before drawing the line, say B where the 25 cm. and 10 in. lines meet. Then draw the graph OM.

Explanation. How to find the equivalent of 12 cm, dot the line from the number 12 on OY till it meets the graph at x; from x drop a perpendicular on OX and read as accurately as possible the number of inches along OX, $4\frac{4}{5}$ inches nearly. $\therefore$ 12 cm.$=4{\cdot}8$ in. approximately.

Again, to find the equivalent of $16\frac{1}{2}$ in. in centimetres, dot the line from $16\frac{1}{2}$ on OX to meet the graph at y. From y dot the line to OY and see where it falls between 41 and 42; it should be $41\frac{1}{4}$ if the drawing is very accurate.

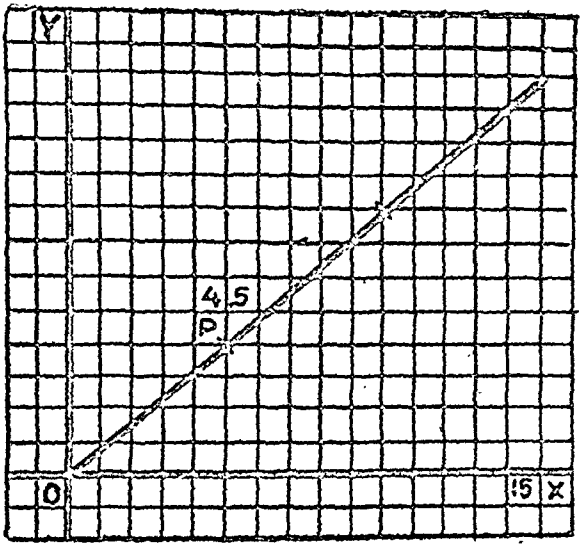
$\therefore$ $16\frac{1}{2}$ in.$=41{\cdot}25$ cm.



Thus a graph of this kind is very useful as it enables us to read off at once the equivalent of one table in terms of the other without calculation.

The student should practise several graphts of this kind, the chief point to be observed being the choice of a good scale when drawing the graph ; in the above example one division of the squared paper denotes 1 inch or one centimetre, but this may not always be convenient, since the figure may be very close thereby making it difficult to read from the graph.

Example 3. In a Reaumur Thermometer, the freezing point is 0° and the boiling point is 80°; in a Centigrade thermometer the freezing point is 0° and the boiling point is 100°. Draw a chart to convert Centigrade degree and *vice versa* and read off 15° C. in R.



Sol. If $x°$ in Centigrade is equivalent to $y°$ in Reaumur, then $\frac{x}{100}=\frac{y}{80}$

$\therefore\ y=\frac{4}{5}x$, therefore if $x=0$, then $y=0$, therefore the graph is a st. line passing through the origin O.

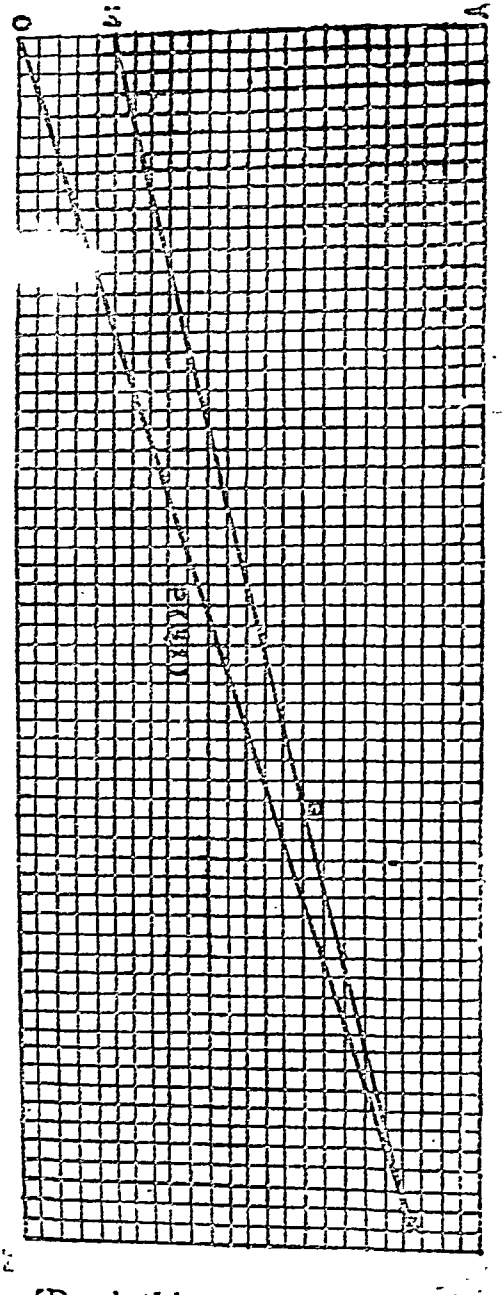
Measure the centigrade degrees along the axis of x and the Reaumur degrees along the axis of y.

$\therefore$ *if* $x=5$, then $y=4$, $\therefore$ the point P(5,4) is also on the graph.

Join OP and produce it. This is the required graph.

$\therefore$ corresponding to the abscissa 15, we have the ordinate 12 $\therefore$ 15°C.=12°R.

Example 4. *A* starts walking at the rate of 3 miles per our; 30 minutes after, *B* starts from the same place at the ıtɛ of 4 miles per hour. Find *graphically* when and where *B* ertakes *A*.



[Read this graph lengthwise]

Sol. Measure distance along OX to the scale of 10 divisions to 1 mile and time along OY to the scale of 10 divisions to 1 hour.

$\therefore$ in one hour A goes 3 miles, plot point P whose coordinates are, 3, 1);

$\therefore$ OP produced represent the graph of A's motion.

$\because$ B sraarts 30 minutes after $\therefore$ M denotes the starting point of B $\because$ in one hour B goes 4 miles, plot point O and. hence, as before, MQ represents the graph of B's motion.

Then the intersection of MQ and OP *viz.*, N will give the place and time when B overtakes A. From the figure, it is clear that B overtakes A two hours after A's starting and at a distance of 6 miles from the place where A started.

EXERCISE 172.

1. The reading of a Centigrade, thermometer in degrees and the corresponding readings on a Fahrenheit thermometer in degrees are given in the table below:—

C	5	10	15	20	30	50	80
F	41	50	59	68	86	122	176

Illustrate *graphically* the connection between the two scales. Express 104° Fahrenheit in Centigrade.

2. If a man walks 8 miles east and then 5 miles north, how far is he from the starting point ?

3. Represent *graphically* the increase of velocity of a train starting from rest as shown in the table below:—

Distance travelled.	50	200	800	1800	3200	5000	feet.
Velocity	5	10	20	30	40	50	miles per hour

4. Given that 25 francs =£1, find from a *graph* the value in francs, as nearly as you can, of 8*s*., £1, 5*s*. and find the English equivalent of 10 fr. 28 fr. and 45 fr.

5. A man bicycles from *A* to *B* at 10 miles an hour and returns from *B* to *A* at 15 miles an hour. If he takes 5 hours to go there and come back find the distance from *A* to *B*. Find also his average speed per hour.

6. *A* starts from Ambala to walk to Ludhiana, a distance of 68 miles at 3 miles an hour ; two hours later, *B* starts from Ludiana for Ambala at 5 miles per hour. When will *A* and *B* meet ? When will they be 20 miles apart?

7. Illustrate *graphically* the following variations in Indian Revenues in the years 1909 to 1914 from the following data.

Years.	1909	1910	1911	1912	1913	1914
Revenue in	225	229	227	321	307	351

8. A monkey climbing up a greased pole, ascends 5 ft. and slips down 2 ft. in alternate seconds, until he reaches the top of the pole. If the pole be 17 ft. high how long will it take him to reach the top ?

9. Two taps *A* and *B* will fill a cistern in 10 and 20 hours respectively. Find *graphically* in what time they will fill it together.

10. A stone falling from rest goes through the following distance in the time shown in the table below :—

No. of seconds	1	2	3	4	5	6
Distance in ft.	16	64	144	256	400	576

Draw a graph to illustrate the above relation and read the graph to find the distance fallen in 3·8 seconds.

CHAPTER XXXIII

CALENDAR

§1. The earth rotates round the sun in 365 days 5 hours 48 minutes and 48 seconds ; but an ordinary year is taken as having 365 days, *i.e.*, nearly $\frac{1}{4}$th day too short. 5 hours 48 minutes and 48 seconds in 4 years' duration become 23 hours 15 minutes and 12 seconds, so an extra day is added once in every four years and that year is called a **leap year.** Again there is a difference of 44 minutes and 48 seconds, *i.e.*, the leap year is taken 44 minutes 48 seconds too long. To ractify this error a century is considered an ordinary year. Thus we find that some hours are still to be taken into consideration ; so to make up this difference the year of the century divisible by 400 is considered a leap year. Briefly :

1. An ordinary year is of 365 days' duration.

2. Every fourth year is a leap year, *i.e.*, of 366 days.

3. Year of a century is an ordinary year, but year of a century divisible by 400 is a leap year.

OR. 1. In 100 years there are 24 leap years.

2. In 200 years there are 48 leap years.

3. In 300 years there are 72 leap years,

4. In 400 years there are 97 leap years.

§2. **To find the day of the year.**

In an ordinary year there are 365 days, *i.e.*, 52 weeks and one day. As the same day is repeated after a week, so there will be a difference of only one day in one year, that is to say, if January 1, 1929, was Tuesday, January 1, 1930, will be Wednesday. Similarly in 100 years there will be a difference of 100+24 days because there are 24 leap years in 100 years; 124 days make 17 weeks and 5 days so in one hundred years there will be a difference of 5 days only. If January 1, 1929 was Tuesday, January, 1, 2029, will be Sunday.

☞Now remember :—

1. In 100 yrs. there are 5 odd days.
2. In 200 yrs. there are 3 odd days (cancelling 1 week).
3. In 300 yrs. there is 1 odd day (,, 2 weeks).
4. In 400 yrs. there are $5\times4+1$ or no odd day (cancelling 3 weeks).

From this we conclude that there is no odd day in 800, 1200, 1600, 2000 years and so on.

Note. January I. A. D. was Monday. If there is no odd day in a given period, then the day of the week is Sunday. For one odd day Monday, for 2 odd days Tuesday and so on. Also the year in question should not be considered, e.g., 3rd February 1918 means 1917 complete years and 1 month and 3 days.

Example What day of the week was 18th Feb. 1925.

Sol. 1600 years give no odd day.
300 years give 1 odd day,
24 years give (24+6) or 2 odd days (cancelling 4 weeks)
January 1925 gives 31 or 3 odd days (cancelling 4 weeks)
18 days of Feb. give 4 odd days (cancelling 2 weeks)
Total=10 days or 3 odd days (cancelling 1 week).
∴ the day was Wednesday. **Ans.**

EXERCISE 173.

What day of the week was (1—6) :—

1. February 5, 1896.
2. March 15, 1921.
3. Jully 18, 1586
4. September 18, 1831.
5. October 26, 1925.
6. April 26, 1923.

Find the day of the week on which the following events took place :—

7. Death of King Edward VII, May 6, 1910.
8. Death of Queen Victoria, January 22, 1901.
9. The battle of Vaterloo, June 18, 1815.
10. The battle of Trafalgar, October 21, 1805.
11. Delhi Darbar, December 12, 1911.
12. The declaration of independence of America, July 4, 1776.
13. Opening of the Great English Railway, Sept. 15, 1830.
14. Reform Bill III, December 18, 1831.
15. Coronation of Aurangzeb, May 26, 1658.
16. Find the dates of March 1592 that fell on Monday.
17. I have a calendar for 1927. Will it serve for 1938 ?

MISCELLANEOUS EXERCISES III.

I

1. Simplify :—

$\frac{1664}{1408}$ of $(\frac{7}{10}$ of £ $3\frac{3}{14} + 6\frac{2}{3}$ of £3, 9*d*.$ - 4\frac{13}{31}$ of £3, 2*s*.)

2. Divide 2875963 by $5 \times 3 \times 11$ by short division and find the true remainder.

3. Multiply 3·720789426 by ·0086341532 to five places of decimals by the contracted method.

4. A person after paying 7*d*. in the £ for income-tax on his income, has £ 1632, 18*s*. 10*d*. What is his income ?

5. A certain number of men and women subscribe to a fund, the number of women being four times the number of men. Each man subscribes as many annas as there are men and each woman as many pies as there are women. The total amount subscribed is Rs. 756. Find the number of men and women.

6. If the income-tax be 6 pies in the rupee for the first half of the year and 3 per cent. in the second, what is the gross income of a gentleman, whose net annual receipts amount to Rs. 1454. 1*a*. ?

7. The manufacturer of an article makes a profit of 25 per cent., the wholesale dealer makes a profit of 20 per cent. and the retail dealer makes a profit of 28 per cent. What is the cost to the manufacturer of an article which is retailed for 16*s*.

8. A train, travelling at tha rate of $18\frac{1}{2}$ miles an hour started at 7 A.M. on a journey of 148 miles. A second train started from the same station ; its speed was to that of the former as 8 : 5 and arrived 15 minutes after the first time. When did the second train start ?

9. If 48 oxen would consume a field of uniformly growing grass in 8 days and 64 oxen in 5 days, in how many days will 40 oxen consume it ?

10. A person bought $2\frac{3}{4}$ per cent. stock at 95, sold it and with the proceeds bought $3\frac{1}{2}$ per cent. stock ; he got £900 less stock than before but the income remained the same. How much money did he originally invest ?

II

1. Find the G. C. M. and L. C. M. of 157 days 7 hours 4 min. 7 sec. and 243 days 2 hrs. 11 min. 49 sec.

2. The 15th of May, 1890, was Thursday. What day of the week was 27 April 1790 ?

3. Simplify $\frac{\frac{2}{3}-\frac{1}{5} \text{ of } \frac{7}{8}+\frac{8}{9}}{\frac{4}{9}\div\frac{8}{7}-\frac{1}{5}+\frac{2}{3}} - \frac{\frac{6}{11}-\frac{4}{9}}{1-\frac{5}{9}\div\frac{3}{5}}$.

4. (*a*) Find the value of $\cdot 0\dot{4}1\dot{6}$ of £ 33, 7*s*. 6*d*.$-\cdot 3\dot{4}\dot{5}$ of £ 32, 13*s*. $1\frac{1}{2}d$.

(*b*) Express Rs. 371, 2*a*. 6*p*. as the decimal of a lakh of rupees.

5. Find the value of $2\cdot 03+1\cdot 3\dot{4}\dot{5}+27\cdot 34+16\cdot\dot{2}31\dot{7}$ without reducing to fractions.

6. In the ten years from 1871 to 1881, the population of a country increased at the rate of 9·5 per cent. and in ten years from 1881 to 1891, the rate of increase was 10·5 per cent. If the population in 1891. was 31023759, find what it was in 1871?

7. A man bequeaths his property amounting to Rs.49166 in such a manner that $\frac{1}{3}$rd of his wife's share, $\frac{2}{5}$th of his elder son's, $\frac{3}{8}$th of his younger son's and $\frac{1}{2}$ of his daughter's share may be all equal. Find the share of the daughter.

8. Find the difference between the present value of £ 808 10 *s*. due 2 years hence, according as simple or compound interest is reckoned, money being worth 5 p. c.

9. *A* bought a horse which he afterwards sold to *B* and gained 5 per cent. on his outlay; *B* sold the horse to *C* and gained $16\frac{2}{3}$ per cent. *C* gave 140 guineas for the horse. What price did *A* give ?

10. What sum must a person invest in the 3 per. cents. at 90, in order that by selling ont £1000 stock when they rise to $93\frac{1}{2}$ and the remainder when they fall to $84\frac{1}{4}$ and then investing the whole proceeds in the 4 per cent. at par, he may increase his annual income by £ 9, 5s.

III

1. Divide the square root of 122·257249 by $\cdot 3685\dot{6}$ and multiply the quotient by the square root of ·000625.

2. Simplify:—

(*a*) $\frac{\frac{1}{2}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{2}-\frac{1}{4}+\frac{1}{12}} \text{ of } \frac{\frac{1}{2}-(\frac{1}{4}+\frac{1}{12})}{\frac{1}{2}-(\frac{1}{4}-\frac{1}{12})} \div \frac{\frac{1}{2}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{2}\times(\frac{1}{4}-\frac{1}{12})} \text{ of } \frac{\frac{1}{2}-\frac{1}{4}\times\frac{1}{12}}{(\frac{1}{2}-\frac{1}{4})\times\frac{1}{12}}$.

(*b*) Express $\frac{2}{3}$ of $\frac{6}{15}$ of £1, 10*s*.+$\frac{3}{4}$ of $\frac{5}{6}$ of 5*s*. 4*d*.−$8\frac{1}{2}$ of $\frac{1}{4\frac{1}{4}}$ of 5*s*. $3\frac{3}{4}$*d*. as the fraction of 2*s*. $1\frac{1}{2}$*d*.

3. (*a*) Divide Rs. 38340, 3*a*. 6*p*. by 441·75.

(*b*) Find the square root of $\frac{1\cdot 1\times 29\frac{4}{25}}{\cdot 000729}$.

4. A man died on Thursday, the 7th August 1890. He lived for 21000 days (excluding the day of his death). Find the day and date when he was born.

5. Find the greatest and the least number of 6 digits that have 251 for their common measure. What is their G.C.M. ?

6. A man used to pay Rs. 15 as income-tax when the rate was 8 pies in the Rupee. Under the new rules, the tax is reduced by Rs. 3, 12*a* ; what is the new rate ?

7. How much per cent. must be added to the cost price of an article, so that a profit of 20 per cent. may be made after allowing a discount of 10 per cent. from the labelled price ?

8. At an examination $\frac{1}{5}$ of a class gets $\frac{7}{8}$ of the maximum number of marks ; $\frac{1}{10}$ gets $\frac{3}{4}$; $\frac{2}{5}$ gets $\frac{1}{2}$; $\frac{1}{4}$ gets $\frac{1}{4}$ and the rest $\frac{1}{8}$. The average number of marks got by the whole class is 166. Find the maximum marks.

9. If the true discount on a bill of £14641 be £4641 at 10 per cent. compound interest, how many years has the bill to run ?

10. I find, on calculation that if I invest my money in the $3\frac{1}{4}$ per cents. at 91, my income will be £ 25 more than if I invest it in the 3 per cents. at 88. What is my capital ?

IV

1. Find the least sum of money that must be subtracted from £660, 7*s*. 4*d*. to make the remainder exctly divisible by 39.

2. (*a*) Convert $\frac{13}{20\times 8}$ into a decimal. Why is the result a terminating and not a recurring decimal.

(*b*) Subtract ·03 from $\cdot\dot{0}\dot{3}$ and divide the remainder by $\cdot 00\dot{7}$.

3. Simplify without reducing to vulgar fraction $(1\cdot 25)^3+2\cdot 25\times(1\cdot 25)^2-3\cdot 75\times(\cdot 75)^2+(\cdot 75)^3$.

4. The area of a rectangle is 2149908480 sq. in. Its length is 5 miles 7 fur. 5 po. 1 ft. 6 in.; find its breadth.

5. 20 men are employed to make a tank 40 ft. long, 20 ft broad and 6 ft. deep. They work for 30 days and just complete $\frac{1}{3}$ of the work, when it was decided to increase the length of the tank by 10 ft., the breadth by 4 ft. and the depth by 2 ft. How many additional men, must be employed in order that the tank may be completed in 30 days more.

6. Calculate correctly to 6 places of decimals the value of

$$2\frac{2}{5} + \frac{4}{5.10} + \frac{8}{5.10.15} + \frac{16}{5.10.15.20} + \ldots\ldots\ldots\ldots\ldots\ldots$$

7. Rs. 49 were divided amongst 150 children, each girl getting 8*a*. and each boy 4*a*. How many boys are there ?

8. *A* can beat *B* by 5 yds. in 150 yds and *B* can beat *C* by 10 yds. in 200 yds. By how much can *A* beat *C* in 600 yds. ?

9. Three apples are worth as much as 4 plums, 4 pears as much as 9 apples, 13 apricots as much as 7 pears and 7 apples sell for 3*d*. I wish to buy an equal number of each of the four fruits and to spend an exact number of pence. Find the least sum I must spend.

10. A person has 200 shares in a railway company for each of which he paid Rs. 1,000 when the dividend was at the rate of 2 per cent. He sells them at Rs. 460 per share and invests the proceeds in Government 3 per cents. at 92. Find the change in his income.

V

1. Simplify $\frac{2}{3}$ of $\frac{\frac{1}{7}-\frac{1}{2}\text{ of }\frac{1}{11}}{\frac{1}{9}-\frac{1}{5}\text{ of }\frac{1}{7}}$ of $\frac{1}{2}$ of $\frac{\frac{7}{8}-\frac{3}{5}\text{ of }\frac{17}{28}}{\frac{3}{8}\text{ of }\frac{3}{7}-\frac{1}{14}}$ $-$

$$\left\{\frac{2}{5}\left(\frac{3}{2}\times\frac{1}{6}+\frac{3}{8}\div\frac{2}{3}\right)-\frac{1\frac{2}{3}}{6\frac{2}{3}}\text{ of }\frac{13}{20}+\frac{3}{7}\text{ of }\frac{6\frac{5}{12}}{3\frac{2}{3}}\right\}$$

2. Find by practice, the cost of 24 cwt. 3 qr. 16 lb. 10 oz. at £2 0*s*. 8*d*. a cwt.

3. Find the value of

$$\frac{3\cdot\dot{3}}{6\cdot0625}\text{ of }\frac{9\,7}{2\cdot4\dot{2}}\div\frac{2\cdot5}{1\,09}\times(7\cdot25+2\cdot75)\times\frac{£3,\ 6s.\ 8d.}{£10,\ \ 13s.\ 8d.}$$

4. If a snail, on the average, creeps 2 ft. 3 in. up a pole during 12 hours in the night and slips down 1 ft. 4 in. during 12 hours in the day, how many hours will it take to get to the top of the pole, if its height be 25 ft. ?

5. A man buys wine at 4s. a gallon; he mixes it with water and by selling the mixture at 3s. a gallon gains 20 per cent. on his outlay. How much water did each gallon of the mixture contain?

6. A debt of Rs. 700 is cleared by a payment of Rs. 180 in cash and a bill of Rs. 533 due 6 months hence. At what rate is the discount calculated?

7. For two thirds of a the distance up a ghat the rise is 1 foot in 24 (measured along the road) and for the remaining third the rise is 1 ft. in 16. The top of the ghat is 14000 feet above the bottom; what is its length?

8. If the daily wages of the labourer rise from $4\frac{3}{4}a.$ to $6a.$ what percentage of the increase in the price of food and other necessities of life will make his position unaltered?

9. A merchant sells tea to a tradesman at a profit of 60 per cent. but the tradesman becoming a bankrupt pays only 2*s.* 6*d.* in the £. How much per cent. does the merchant gain or lose by the sale?

10. What must be the price of a £50 Railway share which pays a dividend of $2\frac{1}{2}$ per cent. if the purchaser is to make 4 per cent. on his outlay?

VI

1. Divide 1028·5 by ·0000017 and $\frac{23\frac{1}{5}}{3\frac{1}{4}}$ by $\cdot 00\dot{0}\dot{6}$ and multiply the difference of the quotients by ·00025.

2. Find the least and the greatest numbers of six digits which being divided by 240, 275, 320, and 400, the respective remainders will be 210, 245, 290 and 370.

3. Simplify $\left\{ \frac{2}{3-\frac{1}{1-\frac{1}{2}}} - \frac{1}{3} \text{ of } \left(5 - \frac{2}{\frac{3}{2}-\frac{1}{6}} \right) \right\} \div \frac{\frac{1}{2}+\frac{3}{4}}{1\frac{1}{2}}$

4. Find the value of $1\frac{8}{17}$ of $\cdot 0\dot{1}2\dot{3}\dot{6}$ of Rs. 5, 11*a.* 8*p.* and express the result as the decimal of one shiling taking a rupee to be equivalent to 1*s.* $4\frac{1}{2}d.$

5. A man buys eggs at a certain price per score and sells them at half that price per dozen; what is his gain or loss per cent.?

6. A room 20 ft. by 18 ft. has a carpet 18 ft by 16 ft. laid down so as to leave a margin of equal width all round.

Find the cost of covering this margin with cloth at 4*s*. 3*d*. a square yard.

7. A society has 3 classes of subscribers and each subscriber pays as many pies as there are subscribers in his class. The rates of subscription in the classes are in the proportion 81 : 64 : 100. The total collections are Rs. 510, 6*a*. 8*p*. Find the number of subscribers in each class.

8. When the income-tax was 7*d*. in a £, a person had to pay £ 63 more than when it was 4*d*. in a £, although his income had diminished in the meantime by £ 225. What was his income at first?

9. A crew, who can row 4 miles an hour in still water rows down a certain distance and up again in 1 hour 36, minutes. Find the distance in miles, if the velocity of the current be $1\frac{1}{2}$ miles.

10. A man has Rs. 41000 which he invests in the 3 per cents. at 87 and 5 per cents. at 104. What sums must he invest in each stock that he may make $3\frac{1}{2}$ per cent. on the whole?

VII

1. Divide Rs. 240 among 8 men, 12 women, and 16 boys so that each man receives Rs. 2 more than each woman and each woman Rs. 2 more than each boy.

2. Reduce $\frac{2\frac{1}{3}-1\frac{1}{4}}{2\frac{1}{3}+1\frac{1}{4}} \times 15\frac{17}{39} \div \frac{3\frac{1}{3}\times 3\frac{1}{3}\times 3\frac{1}{3}-1}{3\frac{1}{3}\times 3\frac{1}{3}+3\frac{1}{3}+1}$ of 1 cwt. 3qr. 7 lb. to the decimal of $2\frac{1}{2}$ tons.

3. What is the least number, which being divided by 48, 64, 72, 80, 120 and 140 leaves the remainders 38, 54, 62, 70 110 and 130 respectively?

4. A reduction of 5 p. c. in the price of tea enables a man to buy 2 lbs. of tea more for Rs. 23, 12*a*. Find the original price per lb.

5. If Rs. 450 amount to Rs. 540 in 4 years at simple interest, what sum will amount to Rs. 637, 8*a*. in 5 years at the same rate?

6. A tank can be filled by a pipe in 6 hours and empited by another in 4 hours. They are alternately opened and closed for $1\frac{1}{2}$ hours; find the time in which the tank when full can be emptied.

7. A person sells 40 horses at a gain of 10 per cent. and 50 horses at a gain of 20 per cent. If he had sold all of them

at a uniform profit of 15 per cent. he would have got Rs. 40 less. Find the cost price of each horse.

8. If 9 lbs. of rice costs as much as 4 lbs. of sugar, 14 lb. of sugar are worth as much as $1\frac{1}{2}$ lbs. of tea and 2 lbs. of tea are worth 5 lbs. of coffee, find the cost of 11 lbs. of coffee, if $2\frac{1}{2}$ lbs. of rice cost $6\frac{1}{4}d.$

9. If the manufacturer makes a profit of 20 per cent. the wholesale dealer a profit of 25 per cent. and the shop-keeper a profit of 40 per cent. What is the cost of the manufacturer of an artical bought from a shop at 17*s*. 6*d*. ?

10. If 3 per cent. stock be at such a price as to give $3\frac{1}{3}$ per cent. interest, what rate of interest will it pay when the price of £ 100 stock has increased by £ 6 ?

VIII

1. I give away to each of my 4 children $\frac{1}{12}$, $\frac{7}{30}$, $\frac{5}{18}$ and $\frac{7}{18}$ of a basket of apple. I had only just enough to be able to do this without cutting an apple. How many had I ?

2 The difference between simple and compound interests on a certain sum of money for 2 years at 4 per cent. is Rs. 20. What is the sum ?

3. Express the difference between $\cdot 3\dot{3}78$ of 13*s*. $10\frac{1}{2}d.$ and $\cdot 3\dot{7}8$ of 16*s*. 6*d*. as a fraction of

$$\cdot 42\dot{6} \text{ of } \frac{3\frac{1}{3}}{\cdot 08} \text{of } \frac{\cdot 3}{\cdot \dot{7}\dot{3}\dot{5}} \text{ of } \frac{\cdot 147 \times 4\frac{4}{9}}{11\frac{1}{10}} \text{of £1, } 17\tfrac{1}{2}s.$$

4. Simplify:—

$$[(\tfrac{2}{19}+\tfrac{1}{3})\div 2\tfrac{2}{3}] \text{ of } \cdot 5\dot{3}\div[1\tfrac{1}{17} \text{ of } (1-\tfrac{64}{81})+\tfrac{8}{11} \text{ of } \cdot 1\dot{6} \text{ of } (\tfrac{1}{2}+\tfrac{5}{12})]$$

5. Find the product of ·18988 and 8·08, divide it by ·0235 and from the quotient thus obtained subtract the square of 8·08

6. What sum will amount to Rs. 15916, 9*a*. 5·28*p*. in 3 years at C. I. the rate of interest being 3, 2 and 1 p. c. for the 1st, 2nd and 3rd years respectivly.

7. A person pays an income-tax of 4*d*. in the £ during the first half of the year and of 3*d*. in the £ during the second half. He finds that owing to an increase in his income, he pays the same amount of tax for both the half years. If his gross income for the year is £ 700, find his net income.

8. I bought goods at 23*s*. 9*d*. with 4 months credit and sold them forthwith at 25*s*. 6*d*. with such allowance of credit as made my gain $6\frac{2}{3}$ per cent. How long credit did I give, nterest being reckoned at 4% ?

9. In a library $\frac{1}{3}$ of the books are literature, $\frac{1}{4}$ mathematics, $\frac{1}{5}$ history, $\frac{1}{6}$ philosophy and the rest novels. What is the least number of novels that the library can have ?

10. A man calculates that if he invests his whole capital in the 4 per cent. stock at 92, his income will be less by £21 than if he invests it in the $4\frac{1}{2}$ per cent. at par. What is the capital ?

IX

1. Find the value of :—

$$\frac{6{\cdot}27\times0{\cdot}5}{(\frac{1}{2}\text{ of }\frac{3}{4})\times8{\cdot}36}\div\frac{(\frac{1}{4}\text{ of }{\cdot}1)\times{\cdot}75\text{ of }21{\cdot}\dot{3}}{(\cdot\dot{6}\text{ of }\frac{5}{6})+1{\cdot}\dot{4}}$$

2. *A*'s income is 150 per cent. more than *B*'s. How much per cent. is *B*'s income less than *A*'s.

3. Two clocks begin to strike 8 together, one of them finishes in 14 seconds and the other in $10\frac{1}{2}$. What is the interval between their fifth strokes ?

4. A woman bought eggs at the rate of 8 for 5*d*. and sold them so as to gain $1\frac{1}{2}d$. on a dozen. What did she charge for each and what was her gain per cent. ?

5. A postage stamp, measures $\frac{8}{9}$ in. by $\frac{3}{4}$ in. How many such stamps will be required to cover the walls of a room whose height is 10 ft. 6 in., length 15 ft. and breadth 12 ft. 2 in., allowing a space of 50 sq. ft. for the doors and windows ?

6. A whole sale dealer sells to a retailer at 10 per cent. profit, the retailer sells to the consumer at 50 per cent. profit. What proportion of the price is paid by the consumer as profit ?

7. A man borrows two equal sums of money at the same time at 5 per cent. and $3\frac{3}{4}$ per cent. respectively. He finds that if he repays the former sum with interest just a year before the latter, he will have to pay in each case the same amount, *viz.*, Rs. 736. Find the amount borrowed.

8. A person, who pays income-tax 5*d*. in the £, finds that a rise of interest from 6 to $6\frac{1}{2}$ per cent. increases his net income by £23. 10s. What is his capital ?

9. By selling 4 dozen mangoes for 13 rupee, it was found that $\frac{3}{10}$ of the outlay was gained ;what ought the retail price per mangoe to have been in order to have gained 60 per cent.

10. I invest equal sums in a 4 per cent. stock and in a 3 per cent. stock and get 5 per cent. for money; the 4 per cents. are at 90. What is the price of the 3 per cents. ?

X

1. Simplify :—

$$\frac{4\frac{3}{4}+\frac{7}{12}}{8{\cdot}16-3{\cdot}7\div\frac{5}{8}} \text{ of } \frac{\frac{7}{8}\div 1\frac{4}{5}}{1\frac{5}{6}+3\frac{11}{18}} - \frac{0{\cdot}07+\frac{1}{105}-\frac{1}{91}}{0{\cdot}5\mathscr{A}} \text{ of } \frac{1\frac{687}{2253}}{1\frac{1}{4}+0{\cdot}2225}$$

and reduce £ 3, 15*s*. 4*d*. to the decimal of Rs. 100. (£=Rs. 15.

2. What are the times between 3 and 4 when the hands of a watch are equidistant from Fig. III on its dial ?

3. In the centre of a room 21 ft. square, there is a square carpet, the rest of the floor is covered with oil-cloth. The carpet and oil-cloth cost respectively Rs. 8, 4*a*. and Rs. 4, 4*a*. per sq. yd. and the total cost of both is Rs. 352, 4*a*. ; find the width of the oil-cloth broder.

4. If Rs. 31250 put out at compound interest amount in 3 years to Rs. 43904, what is the rate per cent. ?

5. *A* and *B* start at the same time from two places $3\frac{1}{2}$ miles apart and walk towards each other. The lengths of their steps are as 15 : 14, but while *A* takes 10 steps *B* takes 11. Find who will reach the midway first and how many yards off the other will then be?

6. In a half-mile race, *A* gives *B* 10 yards, start and wins by $10\frac{1}{2}$ sec., but if *A* gives *B* 15 sec. start *B* wins by 22 yds. Find the time each takes to run the half-mile.

7. There are 3 pendulums. The first makes 35 beats in 36 sec. the second 36 beats in 37 sec., and the third 37 beats in 38 sec. Supposing they begin together, find how many times they will beat simultaneously in 24 hours.

8. The average of 25 returns is 43. The first 9 average 52, the next 12 average 37. Find the average of the last 4.

9. A person buys two kinds of tea at 5*s*. a lb and 6*s*. a lb respectively. He mixes them and gains 17 per cent. by selling the mixture at 6*s* 6*d*. a lb. In what ratio does he mix them ?

10. A person invests a certain sum in the 8 per cents. when they are at $96\frac{3}{4}$; had he waited till they had fallen to $96\frac{1}{8}$ he would have got £16 more of stock. How much money did he invest, brokerage $\frac{1}{8}$ per cent. being charged in both transactions.

UNIVERSITY EXAMINATION PAPERS

PUNJAB UNIVERSITY

1942

1. (*a*) Simplify and express the result as a decimal :—

$$\frac{13\left(\frac{6}{13}-\frac{25}{78}\right)-21\left(\frac{1}{3}-\frac{1}{4}\right)}{13\left(\frac{1}{3}+\frac{1}{7}\right)+5\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{6}+\frac{1}{7}\right)}.$$

(*b*) The G. C. M. of two numbers is 11, and their L.C.M. is 693. If one of the numbers is 77, find the other.

2. Find by practice the cost of 35 bags of rice each containing 2 mds. 15 sr. 8 ch. at Rs. 7, 8*a*. per maund.

3. Find the difference of simple and compound interests on Rs. 250 at 6% for $2\frac{1}{2}$ years.

4. A man sells out £ 1, 600 three per cent. stock at 105 and invests the proceeds in a 4% stock at 120 ; find the change in his income.

5. The cost of matting a floor of a room 16 ft. broad 15 ft. high at 1*s*. 3*d*. per square yard is £2, 13*s*. 4*d*. What will be the cost of papering the walls at one penny per square foot, allowing for 8 doors each 4 ft. by $7\frac{1}{2}$ ft. ?

6. What day of the week will the 1st of March 1914 be ?

7. A man's net income after paying income-tax at the rate of 8 pies in the rupee is Rs. 950, 10*a*. 8*p*. What will be his net income when the tax is reduced by 2 pies in the rupee ?

8. Divide Rs. 5,625 among *A*, *B*, *C*, so that *A* may receive $\frac{1}{2}$ as much as *B* and *C* together and *B* $\frac{1}{4}$ of what *A* and *C* together receive.

1943

(1. (*a*) Express 6*d*. as a decimal of £1().

(*b*) Simplify the following giving the answer c rrect to two places of decimals :—

$$\frac{(7{\cdot}95)^2-(3{\cdot}45)^2}{8{\cdot}86}\times\frac{3{\cdot}2\dot{1}\dot{2}}{95{\cdot}4}$$

2. Find by practice the cost of 24 tons, 18 cwt., 3 qr. of cotton at Rs. 17, 5*a*. 4*p*. per ton.

3. A cube of metal, each edge of which measures $\frac{5}{8}$th of an inch, weighs ·625 lb. What is the length of each edge of a cube of the same metal which weighs 40 lb. ?

4. A milk seller purchases milk at the rate of 4*a*. per seer and adds one-fourth of water to it. Find what per cent. profit he makes by selling the mixture at 5*a*. per seer.

5. The banker's discount on Rs. 450 at 4% per annum is the same as the true discount on Rs. 459 for the same time and at the same rate. Find the time.

6. *A* can do a piece of work in 6 days, *B* in 8◦ days, and *C* in 12 days. All begin together ; but *A* leaves the work after 2 days and *B* leaves 3 days before the work is finished. How long did the work last ?

7. The inside circumference of a circular running track is 440 yds. long and the track is every where 10 yards wide. Find the cost of levelling the track at the rate of 7 pies per sq. yds.

8. When the total income of a person exceeds Rs. 2000 the rates of income-tax and surcharge per rupee are as follws .-

	Income-tax	*Surcharge*
1. On first Rs. 1500	nil	nil
2. On next Rs. 3500	9 pies	6 pies
3. On next Rs. 5000	15 pies	9 pies

Find the total income-tax including surcharge for an income of Rs. 7200.

1944

1. (*a*) Find the greatest number that will divide 398, 436 and 542 leaving the numbers 7, 11 and 15 respectively.

(*b*) Simplify $\dfrac{1\frac{1}{7}-\frac{2}{3}+\frac{\frac{2}{5}}{1-\frac{1}{2\frac{1}{2}}}}{1-\frac{1}{7}\text{ of }\left(\frac{1}{3}+\frac{\frac{2}{5}}{1-\frac{3}{5}}\right)}$

2. Find by practice the cost of 25 bags of wheat at Rs. 11, 8*a*. per maund, each bag containing 2 md. 18 sr. 12 ch.

3. A man spent Rs. 1250 in making a certain number of tables which he sold at Rs. 6, 4*a*. each thus making a profit of $12\frac{1}{2}\%$. Find how many tables he made.

4. Find the amount of stock obtained by investing Rs. 1,097, 4*a*. in the $3\frac{1}{2}\%$. War Loan at $104\frac{1}{2}\%$. Find also the annual income yielded by the stock.

5. The length of a hall is three times the breadth. The cost of white washing the ceiling at $5\frac{1}{3}d$. per square yard is £4, 12*s*. $7\frac{1}{9}d$. and the cost of papering the walls at 1*s*. 9*d*. per square yard is £35. Find the height of the hall.

6. Find the compound interest on Rs. 2,250 for three years at 3% per annum, correct to the nearest pie.

7. The average salary per head of the entire staff of an office, including the officer and the clerks, is Rs. 60. The average salary per head of the officers is Rs. 400 and that of the clerks Rs. 56. Given that the number of officers is twelve, find the number of clerks in the office.

8. What must be the depth of a tank whose base is a square on a side of 1 yd., if it holds as much water as a second tank whose dimensions are 4 ft. 6 in. by 2 ft. 3 in., by 1 ft. 4 in.

9. What day of the week was on the 8th of March, 1942 ?

1945

1. (*a*) Multiply 3164 by 1575 in two lines.

(*b*) Find the greatest number of four digits which when divided by 12, 18, 28, and 35 leaves in each case a remainder 5.

2. Simplify : $\dfrac{\frac{2}{3}\text{of }\frac{4}{9}-3\times\frac{3}{5}\text{ of }\frac{4}{9}+3\times\frac{9}{25}\text{ of }\frac{2}{3}-\frac{3}{5}\text{ of }\frac{9}{25}}{\frac{4}{9}-2\times\frac{3}{5}\text{ of }\frac{2}{3}+\frac{9}{25}}$

3. Find by practice the cost of 33 miles 5 furlongs and 170 yds. of wire at Rs. 34, 6*a*. per mile.

4. The price of sugar having risen 60%, how much per cent. must a house-holder reduce his consumption of sugar so as not to increase his expenditure ?

5. A person sells two horses for Rs. 308 each. On one he gains 12% and on the other he loses 12%. Find his total gain or loss.

6. *A* began business with £ 4500 and was joind afterwards by *B* with £ 5400. When did *B* join, if the profits at the end of the year were divided in the ratio of 2 : 1 ?

7. *A* can do a piece of work in 40 days. He works at it for 5 days and then *B* finishs it in 21 days. In what time can *A* and *B* together do it ?

8. The compound interest on a certain sum of money for 2 years at 5% is Rs. 328. What is the simple interest ?

Or,

A person invests Rs. 5528, 4*a*. in the 4 per cents. at 102⅜. He afterwards sells out at 105 and re-invests in the 5 per cents. at 126. Find the change in income.

9. Find the cost of plastering the inside of a well 21 ft. 4 in. deep with 7 ft. 10½ in. diameter at Re. 1. 6*a*. 6*p*. per sq. yd.

10. Germany invaded Poland on September 1, 1939. What day of the week was it ?

1946

1. (*a*) Simplify :— $\frac{2+2\times2}{2\div2\times2} \cdot \frac{\frac{1}{2}\div\frac{1}{2} \text{ of } \frac{1}{2}}{\frac{1}{2}+\frac{1}{2} \text{ of } \frac{1}{2}}$

(*b*) Expess 3 annas as a decimal of Rs. 2.

2. (*a*) Find by practice the value of 563 mds. of sugar at Rs. 31, 6*a*. 8*p*. per maund.

(*b*) By selling oranges at 32 for a rupee a man loses 40%. How many for a rupee should he sell in order to gain 20% ?

3. (*a*) True discount on a bill for Rs. 540 is Rs. 90. Find Banker's Discount and also Banker's gain.

(*b*) Divide Rs. 525 among *A*, *B* and *C* so that *B*'s share may be two third of *C*'s share, and *A* may get half as much as *B* and *C* togethar.

4. (*a*) If Rs. 450 amount to Rs. 504 in 3 years at simple interest, what will Rs. 650 amount to in 2 years and 6 months, the rate being the same in both the cases.

(*b*) The cost of cultivating a square field at the rate of Re. 1, 8*a*. per 100 sq. yd. is Rs. 1,350. Find the cost of putting a fence round it at the rate of 6 annas per yard.

5. (*a*) How much of 6% stock at 125 must be purchased to secure an annual income of Rs. 1,440? What would be the change in income if later on this stock is, transferred to 5 per cent. at 120?

(*b*) The average weight of a class of 41 boys is 1 maund 2 seers, $3\frac{1}{2}$ chhataks, but when a new boy joins the class the average increases by $1\frac{1}{4}$ chhataks. Find the weight of the new boy.

6. (*a*) The vertical height of a conical tent is 14 ft. and the diameter of its base is 6 yards. How many persons can it accomodate if each person is to be allowed 108 cubic feet of space?

(*b*) A train moving at a speed of 20 miles per hour takes 5 seconds to pass a man walking at the rate of 4 miles per hour in the same direction. Find the length of the train.

1947

1. (*a*) Find the difference between $1\frac{2}{5}$ of 4*s*. 7*d*. and $\frac{3}{7}$ of $5\frac{3}{5}$ of 2*s*. 1*d*.

(*b*) A man spends $\frac{1}{5}$ of his salary to meet pocket expenses and $\frac{4}{5}$ of the remainder to meet other expenses. If his monthly saving amounts to Rs. 12-8-0, find his monthly salary.

2. (*a*) I lost 10% by selling a watch for Rs. 56-8-0. What percentage will I gain or lose by selling it for Rs. 81-6-0?

(*b*) In a joint business the capitals of *A*, *B* and *C* were Rs. 4,500, Rs. 3,000 and Rs. 2,250. *A* withdrew half of his capital after four months. If there was a total profit of Rs. 2,840 after nine months more; how much should each get?

3. (*a*) When the income-tax is 3*d.* in the £, a person's net income is £ 2,370. What will it be when income-tax is raised to 7*d.*

(*b*) I borrowed a cersain sum of money for two years at 6% p a. S.I. and lent it out immediately for the same time and at the same rate of C.I. gaining thereby Rs. 10-2-0. What sum did I borrow ?

4. (*a*) The area of the four walls of a room is 660 sq. ft. and the length is twice the breadth, the height being 11 ft. Find the area of the ceiling.

(*b*) Find, by practice, the price of 63 bales of cotton at Rs. 18-8 0 per maund, if each bale contains 8 maunds 26 seers and 4 chattaks.

5. (*a*) A bill for Rs. 1,700, drawn on January 30, 1946, for six months, is discounted on March 9, 1946 at 5%. What does the banker gain by the difference between true and commercial discount ?

(*b*) How many planks 17 ft. long, 8 in. broad, and $3\frac{1}{2}$ in. deep can be stored in a place 51 yds. long by 6 yds. broad and 14 ft. deep.

6. (*a*) What day of the week was March 4, 1894 ?

(*b*) Which is the better investment : 4% consols at 120 *or* $4\frac{1}{2}$% Railway Stock at 122 ?

What equal sums must be invested in the two stocks to get a total annual Income of Rs. 2,570 ?

1948.

1. (*a*) Simplify :—

$$\frac{\frac{\frac{1}{3}+\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}}+\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}}{\frac{\frac{1}{3}+\frac{1}{2}}{\frac{1}{2}-\frac{1}{3}}-\frac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}}} \div \frac{12}{13} \text{ of } \frac{1}{2}$$

(*b*) Which is greater, $\frac{455}{400}$ or $\frac{\sqrt{9}}{\sqrt{7}}$ and by how much ?

2. (*a*) Find by practice the cost of 31 bags of sugar when two bags contain 5 mds. 25 seers 10 chks. at the rate of Rs. 5–10–8 per maund.

(*b*) Define the G. C. M. and L. C. M. of fractions, and state clearly how to find them. Find the G. C. M. and L. C. M. of :—

$$\frac{40}{34},\ \frac{60}{189},\ \frac{55}{56}.$$

3. (*a*) *A*, *B*, *C* are partners in a business. Their shares are in the proportion of $\frac{1}{3} : \frac{1}{4} : \frac{1}{5}$. *A* withdraws half his capital at the end of 15 months, and after 15 months more a profit of Rs. 4340 is divided. Find the share of each.

(*b*) A sum of money put at compound interest amounts in two years to £ 3380 and in three years to £ 3515. 4*s*. Find the sum and the rate per cent.

4. (*a*) Apples are bought at the rate of 20 for a rupee 3nd sold at the rate of 5 for $4\frac{1}{2}$ annas. Find the gain or loss per cent.

(*b*) *A* can mow a field in 80 days. He works at it for 10 days and then *B* finishes the mowing in 42 days more. In how many days could *A* and *B* finish the mowing by working together from the start.

5. (*a*) If incomes below Rs. 2500 are subject to an income tax of 8 pies in the rupee and those above Rs. 2500 to an income tax of 12 pies in the rupee, find who is poorer, and by how much: a man with an annual income of Rs. 2550 or a man with an annual income of Rs. 2499.

(*b*) What do you understand by Banker's discount ? A bill for Rs. 14600 drawn on 27th November 1947 at seven months was discounted on 1st Feb. 1948 at 5%. How much did the banker give for the bill. (Take 365 days=1 year)

6. (*a*) An open box whose external length is 5 ft. 3 in., breadth 4 ft. 4 in. and height 2 ft. 4 in. is made of wood one inch thick. Find the cost of painting its inside at Rs. 1-11-0 per sq. yd.

(*b*) Which is the better investment ; $3\frac{1}{2}$% stock at 120 or 4% stock at 135 ? What equal investments should be made in the two stocks so that the income from the better stock may be greater than that from the other by Rs. 5 ?

RAJPUTANA BOARD

1940

1. (*a*) At an election, *A* expected to beat his only rival *B* by 300 votes; but 50 of his supporters did not vote at all and an equal number changed over to the other side. If *A* still wins by a margin of 15% of the votes finally cast in his favour, find the votes of his rival.

(*b*) A batsman had a certain average of runs for 11 innings. In the 12th inning he maks a score of 90 runs, thereby increasing his average by 5. what is his average after the 12th inning ?

2. (*a*) A merchant marks prices on his articles, so that after allowing 10% commisson to the customer, on marked prices, he may still have a profit of 10% on his outlay. what is the cost price of an article marked Rs. 10 ?

(*b*) A grocer buys two kinds of tea at 3s. 9*d*. and 3s. per lb. respectively. In what proportion does he mix them, if by selling the mixture at 4s. a lb. he gains 20% ?

3. (*a*) From a distance, I observe a railway train entering a bridge, a mile in length. If the train is half of the bridge in length, and clears it in 5 min. find the speed of the train.

(*b*) From my place I walked to my school at 3 miles an hour and reached 20 minutes late. Next day, starting at the same hour I walked at 8 miles an hour and arrived 5 min. before school time. How far is my school from my place ?

4. (*a*) The cost of whitewashing the walls and the ceiling of a room at 3*p*. per square foot is Rs.19, 12*a*. If its length, breath, and height be proportional to 5, 3, 4, find the area of its floor.

(*b*) Owing to scarcity of water, the municipality wants to reduce the supply of water by half, by replacing one inch diameter pipes by narrow ones. Calculate correct to two decimal places, the diameter of the narrower pipes.

5. (*a*) A man has Rs. 400 in the post office on January 1. He deposits Rs. 100 on the first of every subsequent month except on August 1, when he withdraws Rs. 500. What will be his final balance at the end of the year, if he stops deposits after the withdrawal? Post office pays 3% p. a. simple interest added to the principal at the end of June and December.

(*b*) A weekly offers a prize of 5000 down or Rs 2000 down plus two instalments of Rs. 1800 each payable at intervals of six months. If the market interest be 10% p. a, and interest be payable every six months, by how much is the one offer better than the other?

6. (*a*) The true discount on a certain bill is five sixth of the trade discount and the rate is 4%. Find the time?

(*b*) A man sells £ 6000 of $3\frac{1}{5}$% stock and buys 3% stock at 75, thereby increasing his annual income by £ 18. At what price did he sell his $3\frac{1}{5}$ % stock?

1941

1. (*a*) I have travelled between 770 and 790 miles; had I travelled 5 miles less, I could have completed my journey in a train which goes at the rate of 30 miles an hour, or in a carriage which goes at the rate of 12 miles an hour, or on foot at the rate of 5 miles an hour, in an exact number of hours in every case. Find the distance I have travelled.

(*b*) Find a vulgar fraction equal to $\frac{13}{23}$ and having the denominator 170 more than the numerator.

2. (*a*) A foot-rule, by which the sides of a rectangle are measured, is too short by ·04 of an inch. Find by what decimal of the calculated area the true area will be less than the calculated area.

(*b*) A school of boys and girls consists of 453 children; the number representing the boys is ·52 of the number representing the girls. How many boys are there?

3. (*a*) How long will two examiners, working 8 hours a day, take to examine a number of answer-books, if four examinars, working 5 hours a day, can do it in 8 days?

(*b*) A tradesman, who gains 21% on his capital allows 1*d.* in the shilling discount off the price marked on each article. What per cent. above the cost price does he mark the articles for sale.

4. (*a*) A dealer bought goods at Rs. 612 with four months credit and sold them forthwith at Rs. 689, with such allowance of credit as made his immediate gains 8¾%. For what period did he allow credit, reckoning interest at 6 p. c. per annum?

(*b*) How much 3% stock must be sold at 89½% in order that the owners' income may be £12 more by investing the proceeds in 4% stock at 91⅞, the brokerage on each sale being ⅛%?

5. (*a*) A person bought 30 animals, consisting of cows and oxen for Rs 690. If he had bought as many oxen as he had bought cows, and as many cows as he had bought oxen, the whole lot would have cost him Rs. 660. If the cost of a cow be Rs. 15, find the cost of an ox and the number of each kind of animal bought.

(*b*) I invested part of Rs. 500 in a speculation by which I gained 10% but by investing the remainder in another speculation I lost 10% thereby reducing my gain on the whole to 2%. What amount did I invest in each speculation?

6 (*a*) A watch was 5 minutes fast at 9 A. M. on Monday and 12 minutes slow at 12 Noon on the following Wednesday. Find when it was exactly right, assuming that it lost time uniformly

(*b*) Two persons *A* and *B* start simultaneously from two places, *a* miles apart, and walk in the same direction. *A* travels at the rate of *p* miles an hour, and *B* at the rate of *q* miles an hour. How far will *A* have walked before he overtakes *B*?

1942

1 (*a*, Find the value of ;—

$$\frac{(3{\cdot}47)^2-(2{\cdot}53)^2}{1-06}$$ of £1, 5s.

(*b*) The mean temprature of a place on Sunday, Monday and Tuesday was 69° and on Monday, Tuesday and Wednesday it was 72°. What was the temperature on Sunday if it was 74° on Wednesday ?

.2 (*a*) A person bought 15 mds. of wheat at Rs. 5 a md. and mixed 5 mds. of barley with it. By selling the mixture at Rs. 5 per md. he gains $6\frac{2}{3}\%$. Find the cost of barley per md.

(*b*) The subscription to War Fund from students of a certain school amounts to Rs. 1,728, and each student subscribes as many pies as there are students in the school. Find the number of students in the school and their individual subscription.

3. (*a*) Find the cost of carpeting a room 33 ft. 4 in. long and 12 ft. 6 in. wide with carpet 18 in. at 4*a*. 6 *p*. per yard.

(*b*) It is required to raise a Territorial Force battalion of 1,008 men. What is the least number of men that must present themselves for enrolment if 20% are rejected on offering, 16% of the remainder after a strict medical examination.

4. (*b*) A tradesman marks his goods 15% above what they cost him. If he allows 5% discount, what is his gain per cent.?

(*b*) *A* gives *B* a bill for Rs. 10,351, 8*a*. due 8 months hence in exchange of a bill for Rs. 10,759 due 16 months hence. If the rate of interest is $4\frac{1}{2}\%$ per annum determine which of the parties is the gainer and by how much.?

5. (*a*) A man invests Rs. 10,240 in a company paying 5% per annum when its Rs. 100 share can be bought for Rs. 160. Find (*i*) his annual dividend, (*ii*) his percentage on his investment.

(b) Rs. 11,000 of $2\frac{3}{4}\%$ consols are sold at $108\frac{5}{8}$ and the proceeds are invested in the 3% at $96\frac{3}{4}$. Find (*i*) how much of the second stock is bought, (*ii*) the change in income, (brokerage $\frac{1}{8}\%$ in each case).

6. (*a*) *A* can do $\frac{2}{5}$ of a work in 12 days and *B* can do $\frac{3}{4}$ of it in 18 days. In how many days will they do the whole work together ?

(*b*) *A* and *B* run a 4 mile race; and when *A* has run $2\frac{1}{2}$ miles, *B* has run $2\frac{3}{8}$ miles. In what ratio must *B* increase his speed from the point, in order to win by 300 yards ?

7. (*a*) If the compound interest on a certain sum for 2 years at 3 % be £57, 1 *s.* $10\frac{1}{2}$*d.*, what would be the simple interest ?

(*b*) A market woman sells 1,000 oranges, some at a gain of 25 %, and the rest at a gain of 15%, and thereby gains 18% on the whole. How many of each sort does she sell ?

1943.

1. (*a*) Find the difference between the local values of 3 in 437355.

(*b*) Find the value of $\frac{\frac{3}{4} \text{ of } \frac{3}{4} \div \frac{3}{4}}{\frac{3}{4} \div \frac{3}{4} \text{ of } \frac{3}{4}}$ of Rs. $\frac{16}{9}$.

(*c*) Evaluate $\frac{1}{8^1} + \frac{1}{8^2} + \frac{1}{8^3} + \ldots\ldots$correct to three places of decimals.

2. (*a*) Find the least number of square marble slabs required to pave the floor 276 ft. long and 204 ft. wide.

(*b*) A batsman has a certain average of runs for 16 innings. In the 17th inning he makes a score of 85 runs, thereby increasing his average by 3 runs. What is his average after the 17th inning ?

3. (*a*) A room is 36 ft. long and the cost of paving it is Rs. 78. If the breadth of the room were diminished by 7 ft. the cost would be Rs. 57. Find the breadth of the room.

(*b*) A garrison of 1,200 men is provisioned for 50 days allowing 10 oz. per man per day. If it is reinforced by 300 men, to what extent must the daily allowance be reduced that the provision may last the increased number of men for 60 days ?

4. (*a*) A commission agent allows 25% discount on his advertised prices, and then makes a profit of 20% on his outlay. What is the advertised price of a machine on which he gains Rs. 3 ?

(*b*) The true discount on a sum of money, due 6 months hence, is £400, and the banker's discount on the same sum for the same time and at the same rate is £104. Find the sum and the rate per cent.

5. (*a*) A bill for £505 drawn on 7th of March at 4 months is discounted on the 28th of April at 5%. How much does the holder of the bill receive ?

(*b*) A train whose length is 176 ft. moving at the rate of 45 miles an hour. How long will it take to pass a telegraph post ?

1944

1. What annual payment will discharge a debt of Rs. 225 due in 4 years, the rate of interest being 5 p. c. per annum.

2. The population of a town is 32,000 and its annual increase is 50 per thousand. What will be the number of its inhabitants after three years.

3. (*a*) The length of a courtyard is $1\frac{1}{2}$ times its breadth. The cost of cementing it at 5 annas per square yard is Rs. 1,470. Find the cost of fencing it at Rs. 4/-per yard.

(*b*) How deep will a piece of ground. $13\frac{1}{2}$ ft. long, 6 ft. 3 in. broad, have to be excavated in order to build a wall 16 ft. by $2\frac{1}{2}$ ft. by 8 ft. with a window 5 ft. by 4 ft. ?

4. (*a*) A certain number of men can do a piece of work in 28 days. If there were 10 men more it could be finished in 8 days less. How many men are there ?

(*b*) In a mile race *A* can beat *B* by 40 yards, and *B* can beat *C* also by 40 yards. By how many yards can *A* beat *C* ?

5. (*a*) A shopkeeper marks his goods 20% above the cost but allows 8% discount for cash. What per cent. profit does he really make?

(*b*) The difference between the interest and discount on a sum of money for 2 years at 5% is Rs. 50. Find the sum.

ANSWERS

EXERCISE I.

1. Three thousand four hundred and sixty.
2. Fifteen thousand and seventy.
3. Two hundred thousand nine hundred and three.
4. Ten million five thousand and eight.
5. Five hundred and six million seven hundred and twenty thousand eight hundred and ninety one.
6. Four hundred and seven million eighty thousand two hundred and ninety- three.

7. 300300. 8. 3005006. 9. 406007005.
10. 400000000021. 11. 60002000027000500009.
12. 4000, 7000, 10, 2.
13. 1000000, 200000, 10000, 3000, 400, 50, 6.
14. 4000000, 400000, 40000, 6000, 600, 7.
15. 30000000000, 7000000000, 900000000, 60000000, 10000, 2000, 400, 50, 6.
16. XXXVII ; LXVII ; LXXIX ; LXXXIV ; XCIX.
17. CVII ; CCLXXXIX ; DCCII ; DCCCLXVIII.
18. MCMXX ; MMLXX ; $\bar{I}$VDCLIX Or MMMMDCLIX ; $\bar{V}\bar{I}\bar{I}$CC.
19. CCXVIII ; $\bar{M}\bar{X}$I. 20. 661032. 21. 1863.
22. 1880. 23. 1766. 24. 2002610.
25. 560227. 26. 99999 ; 10000.
27. 98520 ; 20589. 28. 548, 584, 485, 458, 854, 845.
29. 5000 instead of 5. 30. 79993. 31. 899992.
32. 9543210 ; 1023459.

EXERCISE 2.

1.	136.	2.	187.	3.	226.	4.	288.
5.	3165.	6.	1285.	7.	2845.	8.	2490.
9.	17496.	10.	30152.	11.	310074.	12.	1231393.
13.	1793091.	14.	1688786.	15.	1933161.	16.	11030576.
17.	90272260.	18.	5194290.	19.	144182.	20.	10983542
21.	1200.	22.	1935 A.D.	23.	1212	24.	271.
25.	366.	26.	1070.	27.	9577550.	28.	9706702.
29.	Rs. 1188.	30.	3001.				

EXERCISE 3.

1. 7221.	2. 5432.	3. 4311.	4. 6424.
5. 36236.	6. 13857.	7. 19172.	8. 2647.
9. 11967.	10. 211402.	11. 22639.	12. 400152.
13. 75759.	14. 80002.	15. 180104.	16. 604534.
17. 1075.	18. 461	19. 8798.	20. 379.
21. 5132 3762 8894.	22. 35256 29475 64731	23. 83256 92639 175895.	24. 19516 9237 28753.
25. 7895 3327 4568.	26. 2835 1396 1439.	27. 25329 9098 16231.	28 10000 9935 65.
29. 962615.	30. 788461.	31. Rs. 596.	32 293440.
33. 8608717.	34. 470.	35. Rs. 298.	33. 1875 A.D.
37. 57 yrs.	33. 8630098.	39. Rs. 449 loss.	
40. 1936 A. D. ; 61 yrs.		41. 4548 ft.	42. Rs. 46.
43. 5538.	44. Rs. 91790.		

EXERCISE 4.

1. 6244.	2. 44824.	3. 63624.	4. 65299.
5. 99246.	6. 134580.	7. 100380.	8. 151725.
9. 221670.	10. 56050.	11. 126500.	12. 732100.
13. 3164500.	14. 53610000.	15. 172208000	

EXERCISE 5

1. 879060.	2. 607068.	3. 1000467.	4. 6477056.
5. 7530640.	6. 2895420.	7. 18720.	8. 50400.
9. 78204	10. 39375.	11. 51072.	12. 160875.
13. 16387150.	14 879668400.		15. 1512000000.
16 2331000000.			17. 1250475045.
18. 7340483975.			19. 7956117875.
20. 80407124282.	21. 2025, 5041, 14641, 15625, 65536.		
22. 1331, 9261, 15625, 46656, 64000.			23. 179.
24 121.	25. 251.		26. 71808.
27. 2170671.	28 Rs 88592.		29. 6095604.
30. Rs. 33114.	31. 3744 gallons.		32. 130800.
33. 924 miles.	34. 83328000 miles.		

EXERCISE 6.

1. 4696.	2. 2024.	3. 5621.	4. 536.
5. Q=203, R=32.		6. Q=856, R=1.	
7. Q=346, R=255.		8. Q=1250, R=25.	

9. Q=748, R=414. 10. Q=517, R=1081.
11. Q=22, R=5853. 12. Q=85, R=3784.
13. Q=5653569, R=8; Q=5653356, R=98.
Q=56535, R=698; Q=56536, R=5698.
14. Q=8930002, R=2; Q=893090, R=22.
Q=89300, R=22; Q=8930, R=22.
15. Q=483009, R=87; Q=48300, R=987.
Q=4830, R=987. Q=483, R=987.
16. Q=10858, R=5 17. Q=16669, R=263.
18. Q=220153, R=113 19. Q=28714, R=41.
20. Q=26810, R=198 21. Q=57960, R=76.
22. 163. 23. 187. 24. Q=137.

EXERCISE 7.

1. 26. 2. 11. 3. 5. 4. 100. 5. 18.
6. 54. 7. 0. 8. 3. 9. 664. 10. 10.
11. 11. 12. 53. 13. 49. 14. 638. 15. 16.

EXERCISE 8.

1. 59634000, 59634000. 2. 2268720, 8318640.
3. 42544502, 4250625O2. 4. 1319455.
5. 3958425. 6. 9238450. 7. 13263285.
8. 37376925. 9. 45196875. 10. 27419625.
11. 174739375. 12. 2246168750. 13. 56120892.
14. 378622998. 15. 45295569999. 16. 3783121650.
17. 6297585. 18. 959395162 19. 1633771936.
20. 4699309950. 21. 3503991036. 22. 4813348118.
23. 26404176. 24. 269267592. 25. 23619424S.
26. 376548496. 27. 17803164S0. 28. 419651592O.
29. 102539974-44. 30. 3138896016. 31. 59785888525.
32. 12131251596. 33. 1737683904. 34. 1843953930.
35. 92154273576. 36. 60185781103S.
37. 8211942220350. 38. 2804618541324S.
39. 101645725658288. 40. 6338036301611O4.

EXERCISE 9.

1. Q=30791, R=1. 2. Q=179126, R=4.
3. Q=383755, R=23. 4. Q=87825, R=14.
5. Q=592906, R=6. 6. Q=592923, R=9.
7. Q=62733, R=28. 8. Q=180647, R=25.
9. Q=1108894, R=26. 10. Q=491318. R=12.
11. Q=4552022. R=113. 12. Q=3114855, R=6.

13. Q=7517193, R=27. 14. Q=2370724, R=66,
15. Q=15416003, R=181. 16. Q=10212209, R=686.
17. Q=5706476, R=382. 18. Q=5182291, R=697.
19. Q=78580930, R=765. 20. Q=319112473, R=4527.

EXERCISE 10.

1. 2. 2. 78. 3. 28. 4. 11 5. 255.
6. 490. 7. 417. 8. 364 9. 563. 10. 118.
11. 5 12. 34. 13. 104. 14. 40. 15. 15.
16. 1808. 17. 230.

EXERCISE 11.

1. 112. 2. 96. 3. 81. 4. 11.
5. 7. 6. 239. 7. 99905. 8. 999810.
9. 100008. 10. 10400. 11. 22188. 12. 26560.
13. 2655. 14. 119. 15. 123934, 229, 45.
16. 359348, 1119, 149. 17. 526, 328. 18. 826, 721.
19. 541. 20. 1626. 21. 4, 0, 3. 22. 1, 5, 4.
23. 6. 24. 356393. 25. 5177. 26. 3440.
27. Multipliers are (*a*) 556, 543.
28. 1911036, 8972. 29. 494424, 324.
30. *A* 10 ; *B* 22 ; *C* 15. 31. Rs. 55, 75, 95.
32. Rs. 42, 31, 25. 33. Rs. 35, 28, 23.
34. Rs. 740. 35. Rs. 60. 36. Rs. 50.
37. Rs. 2500, 2100, 1800. 38. Rs. 1244, 1460, 1796.
39. 2220. 40. 52 yrs.
41. *A* Rs. 400, *B* Rs. 600, and *C* Rs. 800.
42. *A* Rs. 820, *B* Rs. 1804, *C* Rs. 2624.
43. Rs. 320-. 44, 25.
45. Rs. 22500. 46. 7222 and 8456.
47. (*i*) 119, (*ii*) 2268. 48. 1062. 49. 8654.
50. 5367. 51. Gain Rs. 3.
52. *C* is 23 years younger than *D*.
53. Rs. 3395, 3250, 3120, 3000.
54. Rs. 8240. 55. After 9 hours.

EXERCISE 12.

1. 247. 2. 340. 3. 598. 4. 569.
5. 767. 6. 890. 7. 4096. 8. 4497.
9. 6801. 10. 5536. 11. 5061. 12. 5446.
13. 2022. 14. 2035. 15. 3301. 16. 1835.
17. 1845. 18. 2687. 19. 429. 20. 975.

21. 638. 22. 916. 23. 1139. 24. 746.
25. 6666. 26. 23949. 27. 5155. 28. 10959.
29. 7618. 30. 33076. 31. 28937. 32. 30879
33. 22650. 34. 55002. 35. 57311. 36. 50492.
37. 846. 38. 951. 39. 1815. 40. 2107.
41. 9500. 42. 8223. 43. 1401. 44. 1145.
45. 1676. 46. 3047. 47. 3167. 48 784
49. 443135. 50. 2311720. 51. 1888530. 52. 4105655.
53. 3968325. 54. 2227536. 55. 17437. 56. 51394.
57. 51163. 58. 37251. 59. 79798. 60. 50814.
61. Rs. 150, 14*a*. 4*p*.
62. Rs. 122, 12*a*. 7*p*.
63. Rs. 71, 12*a*. 11*p*.
64. Rs 1970, 9*a*. 8*p*.
65. Rs. 1995, 4*a* 4*p*.
66. Rs. 2518, 9*a*. 5*p*.
67. Rs. 5992, 13*a*. 6*p*.
68. Rs 4973, 7*a*. 6*p*.
69. Rs. 5614, 14*a*.
70. Rs. 1718 12*a* 3*p*.
71. £ 897, 6*s* 4*d*.
72. £ 1459, 3*s*. 8*d*.
73. £ 3723, 6*s*. 4*d*.
74. £ 3466, 15*s*. 9*d*.
75. £ 1295, 15*s*. 3*d*.
76. £ 3705, 12*s*. 10*d*.
77. £129. 4*d*. 3*q*.
78. £ 8773, 16*s*.
79. £ 5916, 15*s*.
80. £ 402.
81. £ 1438, 10*s*.
82. £ 182, 6*s*.
83. 442 mds. 11 sr. 3 chk.
84. 547 mds. 13 chk.
85. 747 mds. 14 sr. 5 chk.
86. 838 mds. 23 sr. 13 chk.
87. 971 mds. 10 sr. 10 chk.
88. 1493 mds. 36 sr. 5 chk. 1 tola.
89. 56 mds. 36 sr. 1 chk. 8 mashas.
90. 124 mds. 19 sr. 7 chk. 2 tola 7 mashas.
91. 10449 tons 3 cwt. 3 qr. 92. 1689 tons 2 cwt. 10 lb.
93. 819 tons 10 cwt. 14 lb. 94. 1697 tons 11 cwt. 1 qr. 24 lb.
95. 937 tons 10 cwt. 3qr. 9lb. 96. 2099 tons 5 cwt. 1 qr. 8 lb.
97. 88593 weeks 2 days 8 hours 42 min.
98. 44663 weeks 3 days 16 hours 34 min.
99. 720 weeks 2 days 1 hour 59 min. 2 sec.
100. 802 weeks 2 days 16 hours 59 min. 51 sec.
101. 8886 miles 1110 yds. 1 ft. 3 in.
102. 13516 miles 183 yds. 6 in.
103. 92 1 miles 1126 yds. 2 ft. 5 in.
104. 6474 miles. 369 yds. 11 in.
105. 158 right angles 3 deg 48 min.
106. 39 right angles 73 deg. 48 min.
107. 1 right angle 41 deg. 33 min. 10 sec.
108. 3 right angles 5 deg. 4 min. 19 sec.

109. 1 ac. 1222 sq. yds. 4 sq. ft. 9 sq. in.
110. 1 ac. 2876 sq. yds. 63 sq. in.
111. 1 ac. 1161 sq. yds. 3 sq. ft. 49 sq. in.
112. 13 ac. 4209 sq. yds. 5 sq ft. 97 sq. in.

EXERCISE 13.

1. Rs. 123, 10*a*. 10*p*.
2. Rs. 153, 12*a*. 4*p*.
3. Rs. 122, 12*a*.
4. Rs. 913, 8*a*. 5*p*.
5. Rs. 449, 12*a*. 10*p*.
6. Rs. 1183, 13*a*. 5*p*.
7. £ 7844, 3*s*. 8*d*.
8. £ 10993, 18*s*. 8*d*.
9. £ 3163, 8*s*. 6*d*.
10. £ 9655, 9*s*. 2*d*.
11. £ 2600, 1*s*. 6*d*.
12. £ 2284, 19*s*. 9*d*.
13. 9188 mds. 26 sr. 5 ch.
14. 8470 mds. 18 sr. 14 ch.
15. 5873 mds. 5 ch.
16. 2055 yds. 1 ft. 9 in.
17. 627 yds 1 ft.
18. 951 yds. 1 ft. 7 in.
19. 147 tolas 11 mashas 7 ratis.
20. 362 tolas 2 mashas 2 ratis.
21. 265 tolas 8 mashas 3 ratis.
22. 171 miles 3 fur. 18 p.
23. 379 miles 7 fur. 21 p.
24. 479 miles 7 fur. 34 p.
25. 99 hrs. 44 min. 51 sec.
26. 190 hrs. 36 min 38 sec.
27. 276 hrs. 26 min. 42 sec.
28. 263 sq. yds. 3 sq. ft. 53 sq. in.
29. 493 sq. yds. 3 sq. ft. 74 sq. in.
30. 516 sq. yds. 7 sq. ft. 120 sq. in.
31. 216 ac. 3 rood 6 sq. p.
32. 424 ac. 2 rood 30 sq. p.
33. 339 ac. 2 rood 7 sq. p.
34. 56 wks. 5 days 9 hrs.
35. 52 weeks 6 days 16 hrs.
36. 138 wks. 2 days 9 hours.
37. 93 deg. 11 min. 20 sec.
38. 138 deg. 8 min. 26 sec.
39. 128 rt. ang. 54 deg. 1 min.

EXERCISE 14.

1. Rs. 4, 1*a*. 4*p*.
2. Rs. 5, 1*a*. 2*p*.
3. Rs. 26, 13*a*. 6*p*.
4. Rs. 5, 11*a*. 8*p*.
5. Rs. 38, 14*a*. 11*p*.
6. Rs. 58, 10*a*. 8*p*.
7. £ 70, 10*s*. 11*d*.
8. £ 8, 7*s* 8*d*.
9. £ 51, 7*s*. 5*d*.
10. 17*s*. 9*d*.
11. £ 3, 13*s*. 10*d*.
12. 10*s*. 8*d*.
13. 2 tons 16 cwt. 3 qr.
14. 67 tons 5 cwt. 3 qr.
15. 31 tons 6 cwt. 1 qr.
16. 15 mds. 29 sr. 15 chk.
17. 15 mds. 17 sr. 14 chk.
18. 37 mds. 20 sr. 13 chk.
19. 6 yds. 5 in.
20. 3 yds. 1 ft. 11 in.
21. 1 ft. 8 in.
22. Rs. 1326, 11*a*. 7*p*.
23. Rs. 1399, 14*a*. 7*p*.
24. £ 550, 18*s*. 8*d*.

25. £ 2381, 14*s*. 9*d*. 26. 110 mds. 32 sr. 13 chk.
27. 189 mds. 38 sr. 13 chk. 28. 35 yards 6 in.
29. 64 tons 14 cwt. 3 qr. 30. 30 tons 7 cwt. 3 qr.
31. Rs. 27, 13*a*. 5*p*. 32. £ 31, 4*s*. 9*d*.
33. Rs. 5, 6*a*. 11*p*. 34 £ 14, 3*s*. 2*d*.
35. 25 mds. 33 sr. 7 chk.

EXERCISE 15.

1. Rs. 100. 2. Rs. 325. 3. Rs. 219 4. Rs. 241, 5*a*. 4*p*.
5. Rs. 448, 3*a*. 6*p*. 6. Rs. 657, 4*a*. 8*p*.
7. £ 424. 8. £ 463, 10*s*. 9 £995, 6*s*. 8*d*. 10. £ 713, 13*s*.
11. £ 2398, 16*s*. 12. £ 3752, 10*s*. 8*d*.
13. £ 20865, 9*s*. 8*d*. 14. £ 20707, 10*s*. 6*d*.
15. 1864 mds. 27 sr. 8 chk. and 1896 mds. 22 sr. 8 chk.
16. 2780 mds. and 2454 mds. 8 sr. 12 chk.
17. 1646 tons 7 cwt. 16 lb. ; 1472 tons 4 cwt. 1 qr. 24 lb.
18. 6162 tons 6 cwt. 2 qr. 7 lb. ; 6801 tons 7 cwt. 2 qr. 21 lb.
19. 4353 sq. yd. 3 sq. ft. 68 sq. in. and 4641 sq. yds. 1 sq. ft. 132 sq. in. 20. 4571 sq. yds. 8 sq. ft. 48 sq. in. and 5198 sq. yds. 6 sq. ft. 72 sq. in.
21. Rs. 3658, 3*a*. 8*p*. 22. Rs. 6042, 9*a*. 23. Rs. 6538, 10*a*.
24. Rs. 1682, 5*a*. 6*p*. 25. £ 7115, 13*s*. 9*d*.

EXERCISE 16.

1. Rs. 47, 13*a*. 8*p*. 2. Rs. 56, 13*a*. 1*p*.
3. Rs. 80, 11*a*. 2*p*. 4. Rs. 85, 15*a*. 10*p*.
5. £ 135, 4*s*. 10*d*. 6. £ 121, 11*s* 11*d*.
7. £ 175, 7*s*. 7*d*. 8. £ 140, 2*s*. 1*d*.
9. Rs.1 57, 10*a*. 2*p*. 10. Rs 217, 13*a*. 5*p*.
11. £ 175, 6*s*. 8*d*. 12. £ 215, 12*s*. 10*d*.
13. Rs. 143, 12*a*. 3*p*. 14. Rs. 322, 4*a*. 7*p*.
15. £ 226, 16*s* 5*d*. 16. £ 635, 8*s*. 3*d*.
17. Rs. 38, 9*a*. 1*p*. 18. Rs. 51, 9*a*. 7*p*.
19. £ 48, 8*s*. 20. £ 51, 8*s*.
21. £ 15, 7*s*. 10*d*. 22. £ 49, 11*s*. 2*d*.
23. Rs. 38, 8*a*. 6*p*, ; rem. 17*p*.
24. Rs. 123, 13*a*. 8*p*. ; rem. 57*p*.
25. Rs. 52, 8*a*. 10*p*. ; rem. 19 *p*.
26. Rs 44, 10*p*. ; rem. 112*p*.
27. £ 38, 1*s* 5*d*. ; rem. 47*d*. 28. £ 48, 6*s*., rem. 153*d*.
29. £ 30, 12*s*. 6*d*. ; rem. 86*d*. 30. £ 36, 17*s*. rem, 34*d*.
31. £ 65, 14*s*. 11*d*., rem. 63*d*.

32. £ 27, 2*s*. 2*d*. ; rem. 11*d*. 33. Rs. 774, 9*a*. 3*p*.
34. Rs. 721, 5*a*. 4*p*. 35. £ 351, 3*s*. 8*d*.
36. £ 301, 14*s*. 3*d*. 37. 417 mds. 21 sr. 7 chk.
38. 375 mds, 39 sr. 5 chk. 39. 96 tons 3 cwt. 24 lb.
40. 360 tons 17 cwt. 3 qr. 13 lb.
41. 15. 42. 16. 43. 21. 44. 45.
45. 25. 46. 41. 47. 5. 48. 8.
49. Rs. 5, 5*a*. 8*p*. 50. 13. 51. 14. 52. 21.
53. 12222. 54. 10 miles 2 fur. 12 pole. 55. 55.

EXERCISE 17.

1. Rs. 501, 4*a*. 2. Rs. 495, 9*a*.
3. *A* Rs. 356, 3*a*. ; *B* Rs. 428, 2*a*. ; *C* Rs. 512, 9*a*.
4. *A* Rs. 728, 12*a*. 9*p*. ; *B* Rs. 536, 8*a*. 6*p*. ; *C* Rs. 321, 7*a*. 3*p*.
5. *A* Rs. 39, 12*a*. , *B* Rs. 52, 8*a*. ; *C* Rs. 24, 12*a*.
6. *A* Rs. 49, 8*a*. ; *B* Rs. 58, 8*a*. ; *C* Rs. 75, 6*a*.
7. 37 yards. 8. 40 sheep. 9. 240 times.
10. Carriage Rs. 64, 4*a*. ; Horse Rs. 257.
11. Furniture Rs. 432, 7*a*. ; Horse Rs. 3459, 8*a*.
12. *A* Rs. 33 : *B* Rs. 16, 8*a*. *C* Rs. 8, 4*a*.
13. *A* Rs. 25, 4*a*. ; *B* Rs. 101 ; *C* Rs. 404.
14. 11 pies. 15. 1*a*. 9*p*. 16. 1651¾ times.
17. 957 18. Rs. 9, 6*a*. 3*p*. 19. Rs. 23, 11*a*. 10*p*.
20. Rs. 35, 4*a*. 21. Rs. 8, 8*a*. 6*p*. 22. 15 days.
23. 20 days. 24. 7 days. 25. 20 days.
26. 10 sov., 20 cr., 50 half cr., 80*s*., 120 six pences.
27. 80. 28. 45. 29. 30.
30. 1*s*. 6*d*. 31. Rs. 12, 11*a*. 8*p*.
32. 4 srs. 33. 11 gallons.
34. Rs. 728, 9*a*. ; Rs. 684, 7*a*. ; Rs. 576, 8*a*.
35. Rs. 832, 7*a*. ; Rs. 356, 9*a*. ; 348, 11*a*.
36. Man £ 5, 7*s*. 6*d* ; wom. £ 3, 11*s*. 8*d*. ; child. £ 1, 15*s*. 10*d*.
37. 10. 38. 10. 39. Rs. 56, 14*a*. 40. Rs. 62, 8*a*.
41. Rs. 2322. 42. Rs. 100000.

EXERCISE 18.

1. 5×3. 2. 5×5. 3. 13×3. 4. 7×6.
5. 11×5. 6. 13×5. 7. 13×6. 8. 17×5.
9. 7×7. 10. 35×2 or 14×5 or 10×7.
11. No. 12. No. 13. Yes. 14. Yes. 15. No.
16. No. 17. Yes. 18. No. 19. No. 20. Yes.
21. No. 22. Yes.

EXERCISE 19.

1.	Yes.	2.	Yes.	3.	No.	4.	No.	5.	No.
6.	Yes.	7.	Yes.	8.	No.	9.	Yes.	10.	No.
11.	Yes.	12.	Yes.	13.	No.	14.	No.	15.	No.
16.	Yes.	17.	Yes.	18.	Yes.	19.	No.	20.	No.
21.	Yes.	22.	No.	23.	Yes.	24.	No.	25.	No.
26.	Yes.	27.	Yes.	28.	No.	29.	Yes.	30.	Yes.
31.	No.	32.	Yes.	33.	Yes.	34.	No.	35.	Yes.
36.	Yes.	37.	No.	38.	No.	39.	Yes.	40.	Yes.
41.	No.	42.	Yes.	43.	Yes.	44.	No.	45.	Yes.
46.	Yes.	47.	4.	48.	6.	49.	2.	50.	3 or 7
51.	1, 5or 9	52.	2 or 6.	53.	2	54.	4.	55.	2.
56.	4.	57.	6.	58.	8.	59.	3.	60.	6.
61.	1.	62.	2.	63.	2.	64.	8.	65.	3, 3.
66.	5, 2.	67.	5, 8.	68.	6, 8.	69.	4, 9.	70.	3, 9.

EXERCISE 20.

1. $2\times2\times2\times7\times13$. 2. $2\times2\times2\times67$. 3. 3×109.
4. $3\times5\times7\times7\times7$. 5. $2\times2\times7\times139$.
6. $3\times3\times4\times11\times13$. 7. $3\times3\times5\times11\times11$.
8. $2\times2\times3\times3\times151$. 9. $2\times11\times307$. 10. $2\times3\times5\times293$.
11. $3\times3\times11\times59$. 12. $5\times11\times11\times13$.
14. $2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times5$. 14. $2\times3\times5\times281$.
15. $5\times7\times11\times19$. 16. $2\times2\times2\times3\times3\times5\times7\times11$.
17. $3\times3\times3\times7\times11\times13\times37$.
18. $2\times2\times2\times2\times2\times2\times2\times2\times2\times3\times3\times3\times3\times5\times7$.
19. $3\times3\times7\times5\times11\times13\times37$.
20. $2\times2\times2\times3\times5\times7\times7\times11\times37$.
21. $3\times3\times3\times3\times3\times7\times7\times7\times11\times13$.
22. 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79,
23. $x=2, y=2$. 24. $x=0, y=4$.
25. $x=9, y=3$. 26. 909090909090909090909.
27. 3, 5, 7, 11. 28. 3, 5, 7, 11, 19. 29. 9, 11, 17, 19.
31. 29. 32. 165825. 33. 71. 34. $a=9, b=5$.

EXERCISE 21.

1.	5.	2.	8.	3.	5.	4.	3.	5.	7.
6.	10.	7.	16.	8.	13.	9.	28.	10.	8.
11.	6.	12.	4.	13.	4.	14.	5.	15.	17.
16.	7.	17.	12.	18.	16.	19.	24.	20.	25.
21.	144.	22.	50.	23.	42.	24.	74.	25.	160.
26.	20.								

EXERCISE 22.

1. 18. 2. 16. 3. 25. 4. 4. 5. 3. 6. 13.
7. 13. 8. 2. 9. 25. 10. 15. 11. 125. 12. 25.
13. 4. 14. 7. 15. 45. 16. 180. 17. 180. 18. 25.
19. 24. 20. 5. 21. 61. 22. 20. 23. 51. 24. 30.
25. 30.

EXERCISE 23.

1. 223. 2. $4a$. 3. $5a.4p$. 4. 5 tolas.
5. 25 tolas. 6. Rs. 1215. 7. 14. 8. 21.
9. 15. 10. 27. 11. 18. 12. 11.
13. No. 14. No. 15. Yes. 16. 12.
17. 21. 18. 15, 165 and 75, 105.
19. 24; 168 and 72, 120.
20. 18, 144 and 36, 126 and 72, 90.
21. 15,180 and 45,60. 22. 24, 360 and 72, 120.
23. 45. 24. 71. 25. 35. 28. 9971, 10140.
29. 9936, 10080. 30. 99960, 10030, 170.
31. 4032. 32. 179, 96. 33. 6660, 5696.
34. 143, 199. 35. 132, 428.36. 945, 2925.
37. 1625, 16725. 38. 13080, 12765; 1, 10.
39. 999984, 100149 ;G.C.M.=753. 42. No.
43. 33 ; 8 oranges and 21 mangoes.

EXERCISE 24.

1. 12. 2. 18. 3 36. 4. 60. 5. 48. 6. 80.
7. 60. 8. 30. 9. 36. 10. 40. 11. 20. 12. 24.

EXERCISE 25.

1. 144. 2. 216. 3. 375. 4. 1815. 5. 2816.
6. 450. 7. 6400. 8. 672. 9. 666. 10. 2500.
11. 1584. 12. 15730 13. 1344. 14. 2520.15. 2760.
16. 5460. 17. 3720. 18. 4875.

EXERCISE 26.

1. 36. 2. 48. 3. 189. 4. 180. 5. 72.
6. 756. 7. 540. 8. 1260. 9. 1080. 10. 1815.
11. 2040. 12. 900. 13. 1134. 14. 360. 15. 1344.
16. 18876. 17. 1232. 18. 2376. 19. 1512. 20. 2016.
21. 504.

EXERCISE 27.

1. 36. 2. 120. 3. 60. 4. 300. 5. 600.
6. 96. 7. 120. 8. 60. 9. 60. 10. 630.
11. 1440. 12. 720. 13. 2520. 14. 3150. 15. 600.

16. 2520. 17. 1680. 18. 45045. 19. 4200. 20. 1008.
21. 360. 22. 2940. 23. 17160. 24. 1008. 25. 51480

EXERCISE 28.

1. 144. 2. 840 3. After 6 min. 4. 115.
5. 1893.. 6. 1885. 7. 1263. 8. 2518. 9. 1796.
10. 933. 11 10080. 12. 1075. 13. 999846. 14. 99988.
15. 214. 16. 1012. 17. 119. 18. 5882. 19. 30.
20. 99960. 21. 67054. 22. 875. 23. 842. 24. 60 min.
25. 22 ft. 26. 2 mds. 10 srs. 27. 420, 480, 540.
28. 342, 390. 29. 533, 713. 30. 1999,998. 32. 480.
33. 904. 34. 175,225. 35. 2880. 36. 7.
37. 10080. 38. 90090. 39. 100801.
40. 60, 120, 180, 240. 41. 85, 170, 255, 340, 425, 510.
42. (15, 1260);(45, 420);(60, 315);(105, 180)
43. (21, 2520);(63, 840);(105, 504);(168, 315)
44. (1125, 2100)or(1500, 1575). 45. 11940, 16915.
46. 288, 432, 576. 47. 24, 36. 48. 90, 126.
49. (12, 144);(36, 48) 50. (14, 168) ; (42, 56).
51. 1155, 385, 231.

MISCELLANEOUS EXERCISES 1

I

1. 651000218400. 2. 127 3. 11*a*. 8*p*.
4. 5 P.M. on the 10th day after leaving New york.
5. He copied 8 instead of 3. 6. 10080.
7. 42 half crowns, 126 florins. 168 shillings 8. Rs. 900.

II

1. 998130. 2. Rs. 937, 8*a*. 3. 121.
4. Rs.97, 3*a*.; Rs. 48, 9*a*. 6*p*. 5. 262.
6. 600. 7. 160472299564 80.
8. 10 sovereigns, 40 shillings, 60 four-penny pieces.

III

1. 5040. 2. 4266833663232. 3. 89356 ; 36.
4. 288. 5. A Rs. 80 ;B Rs. 90 ; C. Rs. 120.
6. Horse Rs. 1093. 12*a*., Saddle Rs. 156, 4*a*.
7. 3, 4. 3, 8. (*a*) 9920. 10044 (*b*) Seventy-seven billion six hundered and sixty-five thou-and five hundred and forty-four million three hundered and thirty-two thousand two hundered and eleven.

IV

1. 94. 2. No.=999984, 100149 ; G.C.M.=753. 8. £980, 2*s*
4. 71955. 5. Dividend=153296, Q.=652, R=76.
6. *A* Rs. 2636, *B*. Rs. 329, 8*a*.; *C*. Rs. 988, 8*a*.
7. £ 76, 17*s*. 3*d*. 8. 5.

V

1. 713. 2. £ 168, 7*s*. 6*d*. 3. 325.
4. $3\times13\times7\times101$. 5. 6124. 6. Rs. 325.
7. 12. 8. 24720 and 4155.

VI

1. 25. 2. 4*s*. 2*d*. gain. 3. Rs. 210. 4. 5, 3, 3. 5. 2153.
6. Rs. 156, 7*a*. 7. 1353. 8. (*a*) 231 (*b*) 124.

VII

1. 214072620945792. 2. 2. 3. 15. 4. 6. 5. *A* £21, 10*s*., *B* £ 18, 3*s*. 4*d*. ; *C*. £ 17. 6. 5. 5, 4. 7. 15. 8. $99900,1020_0$.

VIII

1. 180, 2. 105 persons and one pie will be left.
3. *A* Rs. 220, 10*a*. ; *B*, Rs. 150, 8*a*. : *C* Rs. 310, 4*a*.
4. 301. 5. 6 minutes. 6. 12, 72. 7. Rs. 52. 8*a*. 8. 15.

IX

1. (*a*) 75349394307 (*b*) Q=263779, R=9472.
2. 13897879672655844. 3. Rs. 50. 4. £ 5 ; 25*s*. ; 40*d*.
5. 704. 6. (5, 140) or (20, 35).
7. Dividend=2439883, R.=19. 8. 54 gallons.

X

1. Rs. 1600. 2. 450. 3, £ 249, 4*s*. 4*d*.
4. (15, 120) and (30, 105) and (60, 75). 5. Rs. 66, 9*a*.
6. Rs. 15, 8*a*. 7. Multipliers are (*a*) 615 (*b*) 423.
8. (*a*) 731)52781(72. (*b*) 1301.

5117
1611
1462
149.

EXERCISE 30

1. $\frac{5}{7}, \frac{3}{4}$. 2. $\frac{1}{3}, \frac{3}{8}$. 3. $\frac{3}{5}, \frac{4}{5}$. 4. $\frac{5}{6}, \frac{3}{5}$. 5. $\frac{7}{10}, \frac{5}{11}$. 6. $\frac{3}{5}, \frac{7}{9}$.
7. $\frac{11}{15}, \frac{9}{15}$. 8. $\frac{13}{21}, \frac{16}{21}$. 9. $\frac{12}{15}, \frac{13}{15}$. 10. $\frac{19}{21}, \frac{5}{7}$. 11. $\frac{24}{35}, \frac{8}{9}$. 12. $\frac{10}{11}, \frac{4}{7}$.
13. $\frac{8}{11}, \frac{5}{6}$. 14. $\frac{7}{9}, \frac{3}{10}$. 15. $\frac{11}{15}, \frac{5}{7}$. 16. $\frac{3}{4}, \frac{9}{15}$. 17. $\frac{16}{25}, \frac{15}{17}$. 18. $\frac{18}{19}, \frac{3}{4}$.
19. $\frac{8}{10}, \frac{11}{31}$. 20. $\frac{50}{63}, \frac{5}{17}$. 21. $\frac{11}{14}, \frac{25}{28}$. 22. $\frac{21}{31}, \frac{13}{85}$. 23. $\frac{15}{16}, \frac{25}{29}$. 24. $\frac{7}{15}, \frac{3}{4}$.
25. $\frac{761}{983}$. 25. $\frac{15}{41}$. 27 $\frac{29}{85}$. 28. $\frac{2}{15}$. 29. $\frac{7}{12}$. 30. $\frac{1}{6}$.
31. $\frac{1}{36}$. 32. $\frac{40}{189}$.

EXERCISE 31.

1. $\frac{7}{2}$. 2. $\frac{13}{4}$. 3. $\frac{17}{5}$. 4. $\frac{57}{8}$. 5. $\frac{32}{4}$.
6. $\frac{103}{8}$. 7. $\frac{71}{6}$. 8. $\frac{101}{6}$. 9. $\frac{135}{8}$. 10. $\frac{240}{11}$.
11. $\frac{303}{16}$. 12. $\frac{231}{8}$. 13. $\frac{806}{5}$. 14. $\frac{292}{5}$. 15. $\frac{6482}{20}$.
16. $\frac{6257}{12}$. 17. $\frac{20605}{21}$. 18. $\frac{18552}{21}$. 19. $\frac{2078324}{35}$. 20. $\frac{14046}{25}$.
21. $1\frac{1}{7}$. 22. $3\frac{3}{4}$. 23. $4\frac{1}{5}$. 24. $5\frac{3}{5}$. 25. $8\frac{5}{9}$.
26. $9\frac{2}{11}$. 27. $9\frac{7}{13}$. 28. $9\frac{2}{15}$. 29. $20\frac{2}{15}$. 30. $58\frac{3}{16}$.
31. $68\frac{1}{18}$. 32. $67\frac{17}{24}$. 33. $248\frac{28}{29}$. 34. $245\frac{30}{31}$. 35. $366\frac{13}{32}$.
36. $342\frac{25}{49}$. 37. $407\frac{13}{45}$. 38. $552\frac{1}{35}$. 39. $384\frac{13}{43}$. 40. $426\frac{4}{51}$.

EXERCISE 32.

1. 3. 2. 8. 3. 12. 4. 6. 5. 15. 6. 8.
7. 8. 8. 25 9. 35. 10. 25. 11. 24. 12. 208.
13. 12. 14. 36. 15. 72. 16. 65. 17. 80. 18. 100.
19. 125. 20. 216. 21. 126. 22 $\frac{156}{195}, \frac{105}{195}, \frac{208}{195}, \frac{260}{195}$.
23. $\frac{220}{220}, \frac{480}{220}, \frac{500}{220}, \frac{900}{220}$. 24. $\frac{64}{96}, \frac{80}{96}, \frac{84}{96}, \frac{54}{96}$.
25. $\frac{80}{128}, \frac{56}{128}, \frac{56}{128}, \frac{84}{128}$. 26. $\frac{9}{12}, \frac{10}{12}$.
27. $\frac{20}{48}, \frac{21}{48}$. 28. $\frac{30}{36}, \frac{21}{36}$. 29. $\frac{21}{24}, \frac{22}{24}$.
30. $\frac{15}{42}, \frac{22}{42}$. 31. $\frac{12}{42}, \frac{27}{42}$. 32. $\frac{16}{24}, \frac{20}{24}, \frac{21}{24}$.
33. $\frac{18}{30}, \frac{27}{30}, \frac{14}{30}$ 34. $\frac{25}{30}, \frac{16}{30}, \frac{7}{30}$. 35. $\frac{18}{180}, \frac{12}{180}, \frac{75}{180}, \frac{150}{180}$.
36. $\frac{60}{228}, \frac{18}{228}, \frac{16}{228}, \frac{15}{228}$. 37. $\frac{150}{420}, \frac{135}{420}, \frac{72}{420}, \frac{90}{420}$.
38. $\frac{40}{96}, \frac{42}{96}, \frac{27}{96}, \frac{68}{96}$. 39. $\frac{162}{378}, \frac{546}{378}, \frac{105}{378}, \frac{112}{378}$.
40. $\frac{80}{120}, \frac{168}{120}, \frac{105}{120}, \frac{248}{120}$.

EXERCISE 33.

1. $\frac{9}{22}$. 2. $\frac{5}{12}$. 3. $\frac{11}{12}$. 4. $\frac{3}{22}$. 5. $\frac{4}{13}$.
6. $\frac{3}{7}$. 7. $\frac{9}{10}, \frac{7}{8}$. 8. $\frac{9}{16}, \frac{5}{21}$. 9. $\frac{3}{11}, \frac{5}{26}$.
10. $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}$. 11. $\frac{11}{12}, \frac{7}{8}, \frac{5}{6}$. 12. $\frac{7}{15}, \frac{13}{30}, \frac{5}{12}$.
13. $\frac{6}{11}, \frac{16}{33}, \frac{10}{22}$. 14. $\frac{8}{9}, \frac{7}{12}, \frac{7}{16}$. 15. $\frac{5}{8}, \frac{11}{18}, \frac{9}{16}$.
16. $\frac{5}{72}, \frac{10}{27}, \frac{7}{18}, \frac{15}{36}$. 17. $\frac{1}{7}, \frac{9}{14}, \frac{17}{21}, \frac{5}{6}$. 18. $\frac{3}{7}, \frac{16}{35}, \frac{13}{28}, \frac{13}{21}$.
19. $\frac{15}{76}, \frac{9}{38}, \frac{15}{57}, \frac{7}{19}$. 20. $\frac{71}{84}$. 21. $\frac{289}{720}$.

EXERCISE 34.

1. $1\frac{7}{8}$. 2. $1\frac{11}{12}$. 3. $2\frac{11}{24}$. 4. $1\frac{13}{42}$. 5. $\frac{68}{75}$.
6. $1\frac{29}{48}$. 7. $1\frac{59}{225}$. 8. $1\frac{7}{12}$. 9. $1\frac{13}{66}$. 10. $1\frac{11}{72}$.
11. $10\frac{3}{4}$. 12. $17\frac{1}{4}$. 13. $26\frac{232}{360}$. 14. $9\frac{87}{140}$. 15. $\frac{22}{63}$.
16. $\frac{7}{8}$. 17. $24\frac{23}{48}$. 18. $26\frac{7}{27}$. 19. $18\frac{7}{18}$. 20. $39\frac{7}{4}$.

EXERCISE 35.

1. $\frac{1}{2}$. 2. $\frac{1}{5}$. 3. $\frac{1}{3}$. 4. $\frac{1}{8}$. 5. $\frac{3}{20}$. 6. $\frac{13}{58}$.
7. $4\frac{1}{24}$. 8. $2\frac{1}{8}$. 9. $2\frac{11}{75}$. 10. $3\frac{59}{64}$. 11. $8\frac{1}{8}$. 12 $11\frac{5}{12}$.
13. $10\frac{11}{72}$. 14. $1\frac{97}{100}$. 15. $\frac{1}{4}$. 16. $1\frac{7}{12}$. 17. $16\frac{2}{3}$. 18. $\frac{419}{430}$.
19. $1\frac{3}{4}$. 20. $2\frac{181}{225}$. 21. $\frac{15}{78}$. 22. $\frac{2}{3}$. 23. $3\frac{31}{56}$.

EXERCISE 36.

1. 48. 2. 399. 3. $192\frac{1}{2}$. 4. $238\frac{1}{2}$. 5. $603\frac{1}{3}$.
6. 762. 7. $1119\frac{101}{111}$. 8. $9999\frac{899}{999}$. 9. $\frac{3}{10}$. 10. $\frac{49}{90}$.
11. $7\frac{83}{395}$. 12. $49\frac{2}{27}$. 13. $23\frac{5}{13}$. 14. $93\frac{1}{3}$. 15. 120.
16. $765\frac{1}{3}$. 17. 82. 18. 1. 19. $5\frac{1}{7}$. 20. $247\frac{1}{3}$.

EXERCISE 37.

1. $\frac{1}{24}$. 2. $\frac{3}{80}$. 3. $\frac{1}{4}$. 4. $\frac{6}{11}$. 5. $\frac{29}{72}$. 6. $\frac{101}{259}$.
7. $2\frac{7}{10}$. 8. $4\frac{7}{12}$. 9. 14. 10. $1\frac{1}{2}$. 11. $2\frac{1}{8}$. 12. $\frac{9}{49}$.
13. $\frac{11}{25}$. 14. $\frac{5}{9}$. 15. $1\frac{5}{16}$. 16. $\frac{91}{99}$. 17. $1\frac{111}{225}$. 18. $1\frac{2}{9}$.
19. 8. 20. Rs. 7, 8*a*. 21. $1\frac{5}{36}$. 22. $\frac{2}{11}\frac{5}{6}$.

EXERCISE 38

1. 5. 2. 7. 3. $11\frac{1}{3}$. 4. $1\frac{7}{11}$. 5. $2\frac{5}{8}$.
6. $16\frac{4}{5}$. 7. $\frac{1}{4}$. 8. $\frac{3}{5}$. 9. $5\frac{1}{2}$. 10. $7\frac{5}{8}$.
11. $1\frac{1}{4}$. 12. $4\frac{1}{4}$. 13. $7\frac{1}{2}$. 14. $3\frac{3}{4}$. 15. 1.
16. $1\frac{1}{4}$. 17. $7\frac{1}{2}$. 18. $\frac{3}{5}$. 19. 1. 20. 1.
21. $1\frac{1}{6}$. 22. $2\frac{1}{35}$. 23. $4\frac{11}{12}$. 24. $3\frac{3}{16}$. 25. $\frac{1}{6}$.

EXERCISE 39.

1. $\frac{1}{2}$. 2. $\frac{5}{7}$. 3. 2. 4. $1\frac{1}{2}$. 5. $\frac{1}{2}$. 6. $\frac{5}{16}$.
7. 1. 8. $\frac{6}{11}$. 9. 1 10. 24. 11. $11\frac{16}{67}$.
12. 4. 13. 5. 14. 3. 15. 5. 16. 1. 17. $\frac{1}{6}$.
18. 1. 19. $\frac{1}{2}$. 20. $\frac{4}{5}$. 21. $\frac{7}{8}$. 22. 2. 23. 2.
24. $49\frac{11}{16}$. 25. 2. 26. $\frac{1}{7}$. 27. $1\frac{1}{2}$. 28. 1. 29. $\frac{1}{4}$.
30. 1. 31. 1. 32. $\frac{1}{2}$. 33. 1. 34. $\frac{3}{7}$. 35. $\frac{41}{61}$.
36 $\frac{1}{4}$. 37. $\frac{3}{4}$ 38. 9. 39. 3. 40. 1. 41. 14.

EXERCISE 40.

1. $2\frac{3}{17}$. 2. $\frac{41}{640}$. 3. $\frac{3}{4}$. 4. $\frac{68}{157}$. 5. $1\frac{71}{170}$.
6. $9\frac{13}{19}$. 7. $\frac{1}{3}$. 8. $\frac{19}{193}$. 9. $\frac{33}{76}$. 10. $17\frac{2}{7}$.
11. $1\frac{4}{101}$. 12. $\frac{132}{355}$. 13. $\frac{1}{2}$. 14. $7\frac{3}{4}$. 15. 3.
16. (*i*) $43\frac{3}{5}$. (*ii*) $1\frac{37}{68}$. 17. $\cfrac{1}{7+\cfrac{1}{3+\frac{1}{3}}}$

EXERCISE 41.

1. £49, 13*s*. $\frac{11}{24}d$.
2. £51, 17*s*. $1\frac{11}{16}d$.
3. Rs. 37, 9*a*. $2\frac{29}{32}p$.
4. 113 mds. $9\frac{13}{24}$ ch.
5. 123 tons 10 cwt. $1\frac{1}{4}\frac{1}{8}$ qr.
6. 75 mi. 6 fur. $5\frac{8}{15}\frac{3}{5}$ po.
7. £28, 18*s*. $10\frac{1}{2}d$.
8. Rs. 5, 13*a*. $7\frac{1}{16}p$.
9. 18 yds. 1 ft. $9\frac{17}{36}$ in.
10. 11 mds. 11 sr. $10\frac{13}{16}$ ch.
11. Rs. 3, 13*a*. 10*p*.
12. Rs. 8, 9*a*. 8*p*.
13. Rs. 26, 12*a*.
14. Rs. 31, 7*a*. 6*p*.
15. 17 mds. 33 sr. 9 ch.
16. 21 mds. 28 sr. $3\frac{1}{2}$ ch.
17. 7 tons 19 cwt. $3\frac{8}{9}$ qr.
18. 23 tons 9 cwt. $2\frac{3}{4}$ qr.
19. Rs. 17, 11*a*. $7\frac{7}{12}p$.
20. Rs. 87, 13*a*. $1\frac{1}{4}p$.

21. 152 mds. 7 sr. $13\frac{1}{3}$ chk. 22. 155 tons 6 cwt. $3\frac{1}{3}$ qr.
23. 157 tons 2 cwt. $2\frac{7}{24}$ qr. 24. 52 yds. 1 ft. $7\frac{2}{3}$ in.
25. 55 yds. $7\frac{1}{2}$ in. 26. Rs. 1766, 12*a*. 11*p*.
27. Rs, 25, 12*a*. 6*p*. 28. Rs. 8, 5*a*. 4*p*.
29. 7 mds. 5 sr. $7\frac{1}{2}$ chk. 30. 2 mds. 12 sr. 13 $\frac{3}{35}$ chk.
31. 3 tons 10 cwt. $3\frac{1}{3}$ qr. 32. 1 ton 5 cwt. 2 qr. $8\frac{1}{4}$ lb.
33. Rs. 67, 9*a*. $8\frac{2}{3}$*p*. 34. Rs. 56, 4*a*.
35. £ 50. 36. 6 mds. 18 sr.
37. Rs. 5, 10*a*. 7*p*. 38. £ 2, 8*s*. $7\frac{1}{2}$*d*.
39. £ 2, 4*s*. 5*d*. 40. Rs. 217, 15*a*. 6*p*.

EXERCISE 42.

1. $\frac{1}{3}$. 2. $\frac{1}{6}$. 3. $\frac{1}{5}$. 4. $\frac{2}{3}$. 5. $\frac{1}{4}$. 6. $\frac{5}{14}$.
7. $\frac{2}{3}$. 8. $\frac{5}{8}$. 9. $\frac{5}{8}$. 10. $\frac{1}{4}$. 11. $\frac{1}{6}$. 12. $\frac{241}{645}$.
13. $\frac{73}{225}$. 14. $\frac{77}{118}$. 15. $\frac{2}{27}$. 16. $\frac{2}{13}$. 17. $\frac{1}{48}$. 18. $\frac{1}{30}$.
19. $\frac{17}{28}$. 20. $\frac{2}{3}$. 21. $\frac{8}{15}$. 22. $\frac{1}{3}$. 23. $\frac{5}{9}$. 24. $\frac{12}{49}$.
25. 1. 26. $1\frac{11}{16}$. 27. $\frac{13}{320}$. 28. $\frac{13}{56}$. 29. 7 cwt. 11 lb.
30. $\frac{2}{5}$. 31. $\frac{5}{18}$. 32. $\frac{49}{400}$.

EXERCISE 43.

1. $\frac{1}{12}$, $8\frac{3}{4}$. 2. $\frac{7}{40}$, $1\frac{1}{20}$. 3. $\frac{3}{38}$, $8\frac{11}{38}$. 4. $\frac{1}{72}$, $2\frac{1}{2}$.
5. $\frac{1}{120}$, $8\frac{2}{5}$. 6. $\frac{1}{252}$, 15. 7. $\frac{1}{72}$, $178\frac{3}{4}$. 8. $\frac{1}{201}$, $80\frac{5}{17}$.
9. $\frac{1}{33}$, 60. 10. 57. 11. 14*s*. 8*d*. 12. 54.
13. $1\frac{1}{2}$ hours. 14. 6336 pounds.

EXERCISE 44.

1. Rs. 2672. 2. Rs. 139, 4*a*. $\frac{3}{5}$*p*. 3. Rs. 2528.
4. Rs. 33, 10*a*. 4*p*. 5. Rs. 6, 4*a*.
6. Rs 3, 13*a*. $10\frac{2}{15}$*p*. 7. 45.
8. 5*a*. ; Re. 1. 9. $857\frac{1}{7}$. 10. Rs. 24000.
11. 100. 12. 50. 13. Rs. 25000.
14. Rs. 429, 8*a*. 15. $39\frac{3}{5}$. 16. Re. $\frac{3}{4}$
17. 7 in. 18. 630 sec. 19. Rs. 6750.
20. 400 in. 21. Re.1, 5*a*. $5\frac{1}{7}$*p*. 22. 1 furlong.
23. Rs. 10 ; $\frac{375}{26}$. 24. 5. 25. 240.
26. 945. 27. Rs. 563, 5*a*. 4*p*.

EXERCISE 45.

13. $7+\frac{1}{10}+\frac{3}{100}+\frac{5}{1000}$. 14. $8+\frac{3}{10}+\frac{4}{100}$.
15. $12+\frac{3}{1000}$. 16. $16+\frac{2}{10}+\frac{3}{100}$.
17. $20+\frac{3}{100}+\frac{5}{10000}$. 18. $15+\frac{1}{10}+\frac{3}{100}+\frac{3}{1000}+\frac{4}{10000}$.
19. $7+\frac{1}{100}+\frac{5}{10000}$. 20. $21+\frac{1}{10}+\frac{3}{100}+\frac{4}{10000}$.

21. ·6. 22. ·06. 23. ·006 24. ·0006.
25. 5·3. 26. 15·906. 27. ·35. 28. ·357.
29. ·0959. 30. ·8079. 31. 5·0387. 32. 8·0097.
33. 15·3905. 34. 21·0046. 35. 15·0709. 36. 15·8007.

EXERCISE 46.

1. $\frac{1}{2}$. 2. $\frac{1}{4}$. 3. $\frac{3}{4}$. 4. $\frac{1}{8}$.
5. $\frac{3}{8}$. 6. $\frac{5}{8}$. 7. $\frac{1}{20}$. 8. $\frac{1}{40}$.
9. $\frac{1}{80}$. 10. $\frac{11}{250}$. 11. $\frac{7}{125}$. 12. $\frac{21}{125}$.
13. $\frac{63}{125}$. 14. $\frac{27}{500}$. 15. $\frac{63}{1250}$. 16. $\frac{51}{10}$.
17. $\frac{5023}{1000}$. 18. $\frac{45}{8}$. 19 $\frac{53}{4}$. 20. $\frac{35}{4}$.
21. $\frac{171}{8}$. 22. $\frac{181}{8}$. 23. $\frac{1801}{20}$. 24. $\frac{18011}{20}$.
25. ·7. 26. ·17. 27. 1·77. 28. ·75.
29. ·185. 30. ·1825. 31. ·9. 32. ·09.
33. ·009. 34. ·375. 35. ·075. 36. ·005.
37. 5·07. 38. 8·009. 39. 15·0175. 40. ·05.
41. ·004. 42. ·054. 43. 789·35. 44. 758·353.
45. 7·5001. 46. ·015. 47. ·0007. 48. ·00101.

EXERCISE 47.

1. 1. 2. ·1. 3 ·01. 4. ·001.
5. 10·1. 6. 10·01. 7. 10·001. 8. 25.
9. 25·5. 10. 252·5. 11. 250·02. 12. 1·5.
13. 1·15. 14. 1·015. 15. ·61. 16. ·825.
17. 1·0123. 18. 1·53. 19. 1·503 20. 1·2156.
21. ·0153. 22. ·01057. 23. ·01425. 24. ·001.
25. ·0001. 26. ·00001. 27. 100·1 28. 10·01.
9. 1·001. 30. 1602·3. 31. 1512·5. 32. 2115·07.

33. 15425.	34. 12310.	35. 15802.	36. 10300·2
37. 10300·02.	38. 8·2813.	39. 12·2602.	40. 12·02009
41. 120 209.	42. ·001.	43. ·0001.	44. ·00001.
45. ·000001.	46. ·02001.	47. ·20001.	48. 2·0001.
49. 20.	50. 10.	51. 1.	52. 3.
53. 4.	54. 504.	55. 4030.	56. 15003.
57. 21007.	58. 100·2.	59. 1·2.	60. ·25.
61. ·000003	62. ·000516.	63. ·004123.	64. ·016123.
65. 1·880116.	66. 56·21002.	67. 60·23105.	68. ·21231.
69. ·72345.	70. 5·5605.		

EXERCISE 48.

1. 18·0022.	2. 18·9255.	3. 198·111.
4. 136·301.	5. 242·243.	6. 154·2185.
7. 86·404.	8. 5510·6207.	9. 557·45001.
10. 317·16035.		

EXERCISE 49.

1. ·25.	2. ·06.	3. ·075.	4. ·123.
5. 108·9.	6. ·1114.	7. 13·86965.	8. ·88698.
9 10·9765.	10. ·0644.	11. ·99765.	12. 9·019235.
13. 14·886.	14. 22·01995.	15. 3·09955.	

EXERCISE 50

1. 21	2. 244.	3. 450·65.	4. 420·46.
5. 1816·78.	6. 1427·081.	7. 30·625.	8. 42·93315.
9. 290·602852.	10. 21·05300125·	11. 6·4016001.	
12. 375·145008.	13. 141·4333.	14. 5248·2136.	
15. 1432·3113.	16. 4277·5295.	17. ·0000000001.	
18. ·000016	19. ·000004.	20. 655·36.	

EXERCISE 51.

1. 1·6.	2. ·54.	3. 22·43.	4. 59·514.
5. 9·919.	6. 4·578.	7. ·763.	8. ·00118.
9. ·000554.	10. ·00006925	11. 5·12.	

12. 2·417. 13. 17945. 14. 281500. 15. ·24172.
16. ·0000746. 17. 10. 18. 10·01. 19. 110000.
20. 281·315. 21. ·039. 22. 1·092. 23. ·1092.
24. 1092 25. ·0001092. 26. ·0032. 27. 602400.
28. 1·644. 29. 1·353. 30. ·005. 31. 5·570.
32. 2·772. 33. ·001. 34. ·2. 35. ·00007.
36. 250. 37. 000025. 38. ·05. 39. 4.

EXERCISE 52.

1. ·25. 2. ·125. 3. ·04. 4. ·625.
5. ·35. 6. ·275. 7. ·8. 8. ·375.
9. ·28. 10. ·256. 11. ·1953125. 12. ·128.
13. ·0512. 14. ·084. 15. ·128125. 16. ·484375.
17. ·6875. 18. ·1171875. 19. 3·94615. 20. 5·20714.
21. 1·62179. 22. 56·89873. 23. 59·85333. 24. 56·23003.
25. $\frac{3}{4}, \frac{7}{16}, \frac{5}{12}$ 26. $\frac{7}{8}, \frac{5}{6}, \frac{4}{5}, \frac{3}{4}$. 27. 8·75. 28. 2·59375.
29. ·6. 30. 1·37. 31. 1·5. 32. ·625.
33. ·0029. 34. greatest $\frac{3}{25}$, least $\frac{13}{100}$.
35. greatest $\frac{36}{75}$, least $\frac{16}{36}$.

EXERCISE 53.

1. ·07, 1·68. 2. ·11, 1·32. 3. ·15, 2·25. 4. ·13, 1·56.
5. ·14, 1·68. 6. ·005, ·5. 7. ·0014, 1·4. 8. ·009, 8·1.
9. ·24, 10·8 10. ·00011, 1·21.

EXERCISE 54.

1. 4·75. 2. 1. 3. 241. 4. ·85.
5. 267. 6. 2·925. 7. 1. 8. 10·9975.
9. 3. 10. 1. 11. 1001. 12. 112.
13. 1. 14. 2. 15. 2. 16. 9·45.

EXERCISE 55.

1. No. 2. No. 3. Yes. 4. No. 5. Yes.
6. Yes. 7. No. 8. No. 9. No. 10. No.
11. $\cdot\dot{8}\dot{3}$. 12. $\cdot\dot{4}$. 13. $\cdot\dot{9}0$. 14. $\cdot\dot{8}57142$.
15. $\cdot\dot{3}84615$. 16. $\cdot0\dot{7}$. 17. $\cdot1\dot{9}\dot{4}$. 18. $\cdot0\dot{7}\dot{4}$.
19. $\cdot160\dot{7}14285$. 20. $\cdot0\dot{8}\dot{1}$. 21. $5\cdot\dot{5}71428$ 22. $3\cdot\dot{1}53846$.
23. $9\cdot\dot{2}\dot{7}$. 24. $5\cdot\dot{2}96$. 25. $14\cdot136$. 26. $41\cdot\dot{6}$
27. $29\cdot629$. 28. $\cdot\dot{8}46153$. 29. $5\cdot\dot{7}14285$. 30. $\cdot\dot{6}$.

EXERCISE 56.

1. $\frac{4}{9}$. 2. $\frac{25}{99}$. 3. $\frac{3}{11}$. 4. $\frac{5}{11}$. 5. $\frac{3}{37}$.
6. $\frac{13}{111}$. 7. $\frac{1}{3}$. 8. $\frac{64}{111}$. 9. $2\frac{2}{111}$. 10. $5\frac{4}{9}$.
11. $7\frac{4}{27}$. 12. $10\frac{8}{27}$. 13. 8. 14. 13. 15. $8\frac{709}{3333}$.
16. $15\frac{8}{11}$. 17. $\frac{8}{13}$. 18. $\frac{6}{7}$. 19. $\frac{12}{13}$. 20. $\frac{5}{7}$.
21. $\frac{73}{990}$. 22. $\frac{7}{55}$. 23. $\frac{127}{495}$. 24. $\frac{4}{15}$. 25. $\frac{1}{2}$.
26. 6. 27. $\frac{1}{10}$. 28. $\frac{21}{50}$. 29. $25\frac{502}{55}$. 30. $21\frac{203}{360}$.
31. $2\frac{10712}{41625}$. 32. ·035, ·531, ·026, ·0023, ·0141.

EXERCISE 57.

1. ·523423. 2. ·54686. 3. ·56756.
4. ·21565. 5. ·235035. 6. ·25438854.
7. ·2564256. 8. ·123567123.
9. ·2444444, ·576576, ·575757. 10. ·255, ·565.
11. ·3566, ·7878. 12. ·3456, ·3783. 13. ·0232, ·5678.
14. ·555555, ·0050505, ·0005005.
15. ·06565656, ·08555555, ·08508508.
16. ·2252252252, ·0225452525, ·0022555555.
17. ·576576576, ·0035656565. ·0004325325.
18. 3·125612561256125, 5·012356356356350.
2·1235235235235235.
19. ·2343434343434 ; ·2345345345345 ; ·234523452345.

EXERCISE 58.

1. 1·85. 2. 25·523401674. 3. 8·5689868804.
4. 40·18250. 5. 23·7310393912. 6. 40·92643.
7. 40·463536697. 8. 54.529016. 9. 29·318401219.
10. 92·8502. 11. 6·32608699. 12. 16·48014924
13. 6·60788. 14. 16·00631722. 15. 6·99999898908.

EXERCISE 59.

1. 3·072. 2. 5·030. 3. ·92.
4. 26·590. 5. 91·4189. 6. 1361·3405.
7. ·5893217. 8. ·26304713 8. 9. ·4100.
10. 1·0001. 11. ·1105. 12. ·03.

13. $\cdot(0\dot{9})$. 14. $\cdot\dot{1}$. 15. $\cdot0\dot{2}$.
16. $\cdot\cdot1\dot{0}6$. 17. $1\cdot\dot{3}$. 18. $\cdot285714\dot{2}$.
19. $14\cdot\dot{7}$. 20. $\cdot\dot{2}30769$, 21. $3\cdot3125$.
22. $47\cdot\dot{3}$. 23. $\cdot\dot{0}76923$. 24. $\cdot68921875$.
25. $\cdot5$. 26. $\cdot\dot{1}4285\dot{7}$. 27. $1\cdot5$.
28. $\cdot\dot{5}71428$. 29. $92\cdot5$. 30. $\cdot4$. 31. 65.

EXERCISE 60

1. 72. 2. 2916. 3. 259·2. 4. 37·5,
5. 509·4. 6. 13716. 7. 234000. 8. 1384.
9. 16906. 10. 9234·4. 11. 26·25. 12. 2232.
13. ·0078125. 14. ·609375. 15. ·734375. 16. 51·3375.
17. 2·1875. 18. ·06875. 19. 24·406875. 20. 6·875.

EXERCISE 61.

1. Rs. 12, 2*a*. ·84*p*. 2. Rs. 16, 3*a*. 7·92*p*.
3. 16 mds. 11 sr. 13·6 chk. 4. 24 mds. 36 sr. 8 chk.
5. £ 16, 3*s*. 3*d*. 6. £ 21, 9*s*. 9*d*. 7. 12 yds. 6·741 in.
8. 12 tons 3 cwt. 1 qr. 15·12 lb. 9. Rs. 15·53125.
10. Rs. 21·859375. 11. £ 18·6375. 12. £ 12·875.
13. 3·778125 tons. 14. 16·81875 tons.
15. 12·75 yards. 16. 12·70625 mds.
17. Rs. 3, 3*a*. 5·25*p*. 18. Re. 1, 3*a*. 2·4*p*.
19. Rs. 2, 11*a*. ·75*p*. 20. Rs. 6, 5*a*.
21. 1 yd. 1 ft. 8·25 in. 22. £ 4. 15*s*. 1·875*d*.
23. £ 1, 9*s*. 1·47*d*. 24. 9*s*. 3·248*d*. 25 $5\frac{1}{2}d$.
26. £1. 15s. 3·37$\dot{0}9d$. 27. 1*a*. 5·76*p*.
28. Rs. 6, 1*a*. 2*p*. 29. 19*s*. $1\frac{1}{2}d$.
30. 19 mds. 15 sr. 31. £ 1, 18*s*. 7*d*.
32. Rs. 3, 15*a*. 4·8*p*. 33. £ 14, 10*s*. 8·5*d*.
34. £ 4, 0*s*. 9*d*.

EXERCISE 62.

1. ·5. 2. ·25. 3. ·2. 4. ·25. 5. ·125.
6. ·26. 7. ·5. 8. ·2. 9. ·25. 10. ·25.
11. ·125. 12. ·25. 13. ·125. 14. ·25. 15. ·25.
16. ·001$\dot{3}6$. 17. ·0$\dot{8}3$ 18. $\frac{1}{25}$; 0$\dot{3}6$.
19. 1*a*. 4*p*.; ·114$58\dot{3}$. 20. ·0203125.

EXERCISE 63.

1. $\cdot 00\dot{9}$. 2. $\cdot\dot{1}42857$. 3. $\cdot 58\dot{3}$. 4. 25.
5. $1\cdot 125$. 6. $\cdot 12$. 7. 20. 8. 49.
9. $\cdot 5$. 10. $2\frac{1}{3}\frac{1}{6}\frac{1}{3}$. 11. Rs. 516. 12. 1.
13. $\cdot 034\dot{5}$. 14. 5·90625. 15. 9.
16. $6\cdot\dot{3}$. 17. $350\cdot\dot{6}$. 18. 16*s*.
19. Re. 1, 8*a*. 20. $1\cdot 91\dot{6}$. 21. $14\frac{14}{17}$.
22. $\frac{1}{7}$. 23 Rs. 7, 14*a*. 1*p*. 24. 3.
25. 1. 26. $\cdot\dot{1}$. 27. 814 ; Rem. ·003.

EXERCISE 64.

1. ·641. 2. ·488. 3. 9·068. 4. ·001.
5. 8·482. 6. ·008. 7. 2500. 8. 215.
9. 330. 10. 1·04 11. 860
12. 45·8195338. 13. 194·049. 14. ·000906.
15. 2·14. 16. 1293·47. 17. ·33. 18. 575·56
19. ·086. 20. 1·911. 21. ·524. 22. ·003.
23. ·95424. 24. 69·6. 25. ·000408. 26. ·00534.
27. ·07. 28. 1·58. 29. ·79. 30. ·3183.
31. ·146. 32. ·017 33. 1 0298212.
34. 1·4106861. 35. ·202733. 36 ·333. 37. 1·297.
38. ·1667: 39. ·23077. 40. ·0684931. 41. 2 98.
42. ·04 43. 2·9522 44. ·11157. 45. ·2554.

EXERCISE 65.

1. Rs. 4. 2. Rs. 13, 5*a*. 4*p*. 3. Rs. 18, 12*a*.
4. Rs. 40, 10*a*. 5. Rs. 62, 8*a*. 6. Rs. 63, 12*a*.
7. Rs. 287, 6*a*. 8. Rs. 573, 8*a*. 9. Rs. 398,7*a*.
10. Rs. 674, 11*a*. 11. Rs. 1078, 7*a*. 12. Rs. 1176, 3*a*.
13. Rs. 1484, 5*a*. 14. Rs. 1768.
15. Rs. 2679, 11*a*. 16. Rs. 3776, 1*a*.
17. Rs. 4021, 14*a*. 18. Rs. 6660, 15*a*.
19. Rs. 2015. 20. Rs. 1078.
21. Rs. 1445, 8*a*. 22. Rs. 443, 10*a*. 8*p*.
23. Rs. 1417, 15*a*. 6*p*. 24. Rs. 2701. 13*a*. 4*p*.
25. Rs. 1537, 8*a*. 26. Rs 2114, 1*a*.
27. Rs. 5467. 28. Rs. 7115, 1*a*.
29. Rs. 5923, 14*a*. 6*p*. 30. Rs. 10142, 13*a*. 8*p*.
31. Rs. 5435, 14*a*. $2\frac{2}{3}p$. 32. Rs. 17060, 1*a*. 3*p*.
33. Rs. 4859, 2*a*. 8*p*. 34. Rs. 12733, 9*a*. $10\frac{2}{3}p$.
35. Rs. 8847, 2*a*. 2*p*. 36. Rs. 11272, 4*a*. $6\frac{3}{4}p$.

37. £38718, 12*s.* 6*d.*
38. £19316, 13*s.* 4*d.*
39. £66021, 7*s.* $9\frac{1}{3}$*d.*
40. £105268, 0*s.* 5*d.*
41. £419, 13*s.* $1\frac{1}{2}$*d.*
42. £1838, 3*s.* 1·86*d.*
43. Rs. 546, 14*a.*
44. Rs. 6, 15*a.* $1\frac{1}{2}$*p.*
45. Rs. 3, 10*a.* $1\frac{1}{5}$*p.*
46. Rs. 7299, 11*a.* $6\frac{2}{3}$*p.*
47. Rs. 80606, 15*a.* $1\frac{1}{3}$*p.*

EXERCISE 66.

1. Rs. 10, 13*a.* 4*p.*
2. Rs. 24, 12*a.* 8*p.*
3. Rs. 32, 0*a.* 4*p.*
4. Rs. 50, 3*a.* $7\frac{1}{8}$*p.*
5. Rs. 92, 0*a.* 11*p.*
6. Rs. 53, 5*a.* $6\frac{1}{5}$*p.*
7. Rs. 55, 3*a.* 10*p.*
8. Rs. 1116, 13*a.* 10*p.*
9. Rs. 1212, 14*a.* 3*p.*
10. Rs. 39, 14*a.* $10\frac{3}{4}$*p.*
11. Rs. 50, 14*a.* 5*p.*
12. Rs. 352, 7*a.* $3\frac{9}{20}$*p.*
13. Rs. 93, 12*a.*
14. Rs. 95, 0*a.* $11\frac{5}{8}$*p.*
15. £87, 3*s.* 8*d.*
16. £32, 17*s.* 1*d.*
17. Rs. 110, 9*a.* 11*p.*
18. Rs. 253, 6*a.* $9\frac{7}{9}$*p.*
19. Rs. 401, 0*a.*3 $\frac{5}{6}$*p.*
20. Rs. 746, 0*a.* $4\frac{1}{3}$*p.*
21. Rs. 628, 13*a.* $5\frac{1}{2}$*p.*
22. £2400, 15*s.* $8\frac{3}{64}$*d.*
23. £96, 16*s.* $9\frac{7}{8}$*d.*
24. £934, 18*s.* 2*d.*
25. Rs. 5267, 11*a.* $11\frac{1}{4}$*p.*
26. Rs. 3955, 3*a.* 11*p.*
27. Rs. 268, 12*a.*
28. Rs. 1635, 15*a.* 10*p.*
29. Rs. 4781, 4*a.*
30. Rs. 765, 15*a.* 10*p.*
31. £240, 10*s.* $9\frac{3}{4}$*d.*
32. Rs. 12137, 15*a.* $9\frac{3}{10}$*p.*
33. £513, 6*s.* $6\frac{1}{3}$*d.*
34. £36, 17*s.* 6*d.*
35. £17, 12*s.* $2\frac{1}{4}$*d.*
36. £1034, 14*s.* $4\frac{13}{56}$*d.*
37. £5247, 2*s.* $6\frac{39}{64}$*d.*
38. Rs. 1968, 7*a.* $7\frac{1}{2}$*p.*
39. Rs. 606, 11*a.* $9\frac{3}{4}$*p.*
40. £77, 7*s.* $7\frac{1}{4}$*d.*
41. £97, 11*s.* $5\frac{3}{4}$*d.*
42. £1455, 19*s.* $4\frac{1}{2}$*d.*
43. Rs. 409, 3*a.* 4*p.*
44. £57, 19*s.* 11*d.* nearly.

EXERCISE 67.

1. Rs. 91, 13*a.*
2. Rs. 142, 6*a.*
3. Rs. 392, 4*a.*
4. Rs. 85, 4*a.*
5. Rs. 108, 10*a.*
6. £4, 19*s.* $8\frac{1}{4}$*d.*
7. Rs. 46, 15*a.* 6*p.*
8. Rs. 95, 1*a.* 6*p.*
9. Rs. 1265, 7*a.* 3*p.*
10. £6, 12*s.* $8\frac{1}{2}$*d.*
11. Rs. 259, 6*a.*
12. Rs. 408.

EXERCISE 68.

1. 1936.
2. 3249.
3. 1296.
4. 2304
5. 2401.
6. 2116.
7. 3481.
8. 7744
9. 2809.
10. 2209.
11. 9801.
12. 3136.

13. 16384. 14. 24336. 15. 27556. 16. 11236.
17. 4225. 18. 5625. 19. 21025. 20. 24025.
21. 27225. 22. 42025. 23. 60025. 24. 93025.
25. 390625. 26. 863041. 27. 1755625. 28. 4515625.

EXERCISE 69.

1. 20. 2. 21. 3. 66. 4. 130. 5. 35.
6. 45. 7. 96. 8. 132. 9. 140. 10. 144.
11. 252. 12. 286. 13. 3, 5, 91, 30030. 14. 810000.

EXERCISE 70.

1. 160. 2. 170. 3. 210. 4. 225.
5. 271. 6. 293. 7. 916. 8. 105.
9. 989. 10. 1516. 11. 2125. 12. 3105.
13. 8972. 14. 8513. 15. 9009. 16. 15631.
17. 15321. 18. 21556. 19. 162550. 20. 921986.
21. 1500025. 22. 7000125. 23. 56 24. 115.
25. 125. 26. 232. 27. 526. 28. 501.
29. 136, 41. 30. 126. 31. 14400.

EXERCISE 71.

1. ·3. 2. ·08. 3. ·09. 4. 2·7.
5. 4·1. 6. 5·9 7. 3·05. 8. 9·21.
9. 6·09. 10. 12·25. 11. 21·85. 12. 15·02.
13. 7·256. 14. ·125. 15. ·902.. 16. 501·95.
17. 15·093. 18. ·3162. 19. ·0031. 20. ·0707.
21. 3·8746. 22. 11·0165 23. 16·0032. 24. 3·31662.
25. 4·35889. 26. 5·56776. 27. 10·04987. 28. 22 38302.
29. 24·59674. 30. 10·81665. 31. ·073. 32. ·0231.
33. ·0007008. 34. 200·001. 35. ·125, ·625. 36. 2·1,3·4,5 6.

EXERCISE 72.

1. $\frac{8}{9}$. 2. $\frac{11}{12}$. 3. $\frac{16}{19}$. 4. $\frac{16}{25}$. 5. $2\frac{1}{2}$.
6. $2\frac{2}{3}$. 7. $2\frac{1}{4}$. 8. $2\frac{2}{5}$. 9. $5\frac{6}{7}$. 10. $9\frac{2}{11}$.
11. $8\frac{3}{7}$. 12. $5\frac{3}{7}$. 13. $3\frac{4}{15}$. 14. $4\frac{8}{13}$. 15. $\frac{12}{13}$.
16. $\frac{13}{25}$. 17. ·559. 18. ·692. 19. ·440. 20. ·553.
21. 1·452. 22. 2·397 23. 1·820. 24. 2·291.
25. ·912. 26. ·935. 27. ·645. 28. 1·443.
29. 16·959. 30. 19·164. 31. ·845. 32. 5·606.
33. $6\frac{2}{3}$, $7\frac{1}{2}$. 34. $2\frac{1}{3}$, $3\frac{1}{4}$, $5\frac{1}{6}$.

EXERCISE 63.

1. $\cdot\dot{1}\dot{6}$. 2. $5\cdot\dot{3}$. 3. $68\cdot8\dot{3}\ldots$ 4. $\cdot3\dot{6}$. 5. $\cdot\dot{2}$.
6. 1·5367. 7. ·2659. 8. 2·0074. 9. ·0750.

EXERCISE 74.

1. 11. 2. 28. 3. 127. 4. 267.
5. 473. 6. 8888. 7. 23·4. 8. 4·7.
9. ·08. 10. ·0231. 11. ·957 12. $\frac{8}{9}$
13. $7\frac{3}{4}$ 14. 1·275. 15. 2·65. 16. ·101.
17. $\cdot\dot{3}$. 18. $11\cdot\dot{6}$. 19. ·893 20. 2·648.
21. ·144. 22. ·464. 23. 1·959. 24. 1·759.
25. 111111111.

EXERCISE 75.

1. 36. 2. 6·3. 3. 9. 4. 2·6.
5. 11. 6. 12. 7. 5. 8. 8.

EXERCISE 76.

1. 1 dm. 7 cm. 9 mm. 2. 1 Dm. 2m. 1dm. 5cm.
3. 16 m. 7 dm. 9 cm. 4. 1500 mm.
5. 15500000 mm. 6. 5 m. 7cm. 5mm.
7. 9 Km. 7 Hm. 1 m. 8 dm. 8. 5 m. 1 dm. 5 cm. 4 mm.
9. 8 m. 9 dm. 7 cm. 2 mm. 10. 195 m. 8 dm. 5 cm. 5 mm.
11. 7 m. 3 cm. 3 mm. 12. 33 m. 9 dm. 4cm. 9 mm.
13. 3·45 mm. 14. 3 m. 2 dm. 8 cm. 7·5 mm.
15. 166812000 mm. 16. ·12104.
17. 1 hr. 46 min. 40 sec.

EXERCISE 77.

1. 3700 kilos. 2. 127 3. 270 grammes 4. 7·92.
5. 1·452. 6. (1) 6c. (2) 1s. $2\frac{9}{5}d$. nearly 7. $\frac{3}{17}$
8. 1024 9. 42 min 40 sec. 10. 6·45148.

EXERCISE 78.

1. 180 sq. ft. 2. 133 sq. ft. 3. 143 sq. ft.
4. 26 sq. yds. 6 sq. ft. 48 sq. in.
5. 36 sq. yds. 6 sq. ft. 112 sq. in.
6. 144 sq. yds. 8 sq. ft. 48 sq. in.
7. 20 acres. 8. $62\frac{1}{2}$ acres. 9. 25 acres.
10. $91\frac{2}{3}$ sq. yds. 11. 12 ft. 12. 24 ft. 4 in.
13. 440 yds. 14. 220 yds. 15. 239·197...sq. yds.
16. 440 yds. 17. 220 sq. ft. 72 sq. in.
18. 18 ft. 19. 50 min. 20. 2288 yds.

EXERCISE 79.

1. 30 yds. 2. 42 yds. 3. 37 yds. 4. $38\frac{1}{4}$ yds
5. 14. 6. 10. 7. 375. 8. 666.
9. 800. 10. Rs. 54, 10*a*. 11. Rs. 5, 8*a*.
12. Rs. 5, 9*a*. 5*p*. 13. Rs. 22. 14. Rs. 38.
15. Rs. 352. 16. 30 ft. 17. 40 ft. 18. 25 ft.
19. 36, 12 ft. 20. 45, 30 ft. 21. 50, 40 ft.
22. 2 ft. 1 in. 23. $4\frac{1}{2}a$. 24. Rs. 32, 8*a*.
25. Rs. 32. 26. 15 in. square; 744 titles.
27. 18 in. square: 525 tiles.
28. Rs. 15326, 10*a*. 8*p*., Rs. 7340. 29. £ 39, 3*s*. 9*d*.
30. 10 yds. ; 22 yds. 31. 16 ft. 32. Rs. 24.
33. Rs. 51, 9*a*. 34. Rs. 31. 4*a*. 35. 25, 16 ft.

EXERCISE 80.

1. 2624 sq. ft.; Rs. 95, 10*a*. 8*p*. 2. Rs. 136, 7*a*. 4*p*.
3. Rs. 4. 1*a*. 4. Rs. 595. 5. Rs. 610.
6. Rs. 34, 15*a*. 7. Rs. 124, 7*a*. $1\frac{1}{3}p$. 8. 5256 sq. ft.
9. 110400. 10. 210, 150 yds. 11. 5 ft. 12. £491, 8*s*.
13. 4 ft. 14. 5 ft. 15. 4 ft.
16. 30yds.; $2\frac{1}{2}$ft. 17. 11 ac. 2929 sq. yds.; Rs. 877, 10*a*, 3*p*.
18. Rs. 65. 19. Rs. 666, 12*a*. 20. Rs. 312, 12*a*.
21. Rs. 458.

EXERCISE 81.

1. 792 sq. ft. 2. 1170 sq.ft. 3. 1170 sq. ft.
4. 972 sq. ft. 5. 120 yards. 6. 136 yds.
7. Rs. 52, 12*a*, $5\frac{1}{3}p$. 8. Rs, 10, 6*a*. 8*p*.
9. £55, 10*s*. $3\frac{17}{2}d$. 10. Rs. 4, 8*a*. 4*p*.
11. 29 in. 12. 8*a*. per yd. 13. 12 ft.
14. 15 ft. 15. 702 sq. ft. 16. Rs. 17, 2*a*.
17. Width $18\frac{1}{2}$ ft. ; height $14\frac{1}{2}$ ft. 18. Rs. 20, 4*a*.
19. 25, 20, 16 ft. 20. 40, 32, 24 ft.

EXERCISE 82.

1. Rs. 5, 7*a*. $1\frac{1}{3}p$. 2. Rs. 10, 15*a*. 6*p*.
3. Rs. 19, 12*a*. $9\frac{3}{5}p$. 4. Rs. 17, 8*a*. 5. 16 ft.
6. 19 ft. 7. 24 ft. 8. 18 ft.
9. 24, 16 ft. 10. 20 ft. 11. 3024.
12. 32 ft. 13. 1066 yds. 2 ft. 14. 4 ft.
15. 25 ft., 20 ft. 16. 25 ft., 20 ft. 17. 15 ft.
18. 20 ft. 19. 15 ft. 20. Rs. 24.

21. Rs. 54. 22. Rs. 81, 4*a*. 23. Rs. 20.
24. 20 ft. 25. Rs. 6, 4*a*. 26. Rs. 20, 13*a*.
27. Rs. 237, 8*a*. 28. 8000 sq. ft.; Rs. 673, 12*a*.
29. 301 ac. 2424 sq. yds. 30. Rs. 66, 10*a*. 8*p*.
31. Rs. 15326, 10*a*. 8*p*.; Rs. 7340.
32. 12 ft. $7\frac{1}{2}$ in. 33. 16 ft.; 10 ft. 34. 5 ft.
35. 10 ft. 36. £3, 5*s*. $10\frac{3}{4}$*d*. 37. 14 ft.
38. Rs. 1120. 39. $12\frac{1}{2}$ ft. 40. £3 3*s*. 9*d*.

EXERCISE 83.

1. 24 sq. yds. 2. $10\frac{1}{2}$ sq yds. 3. 75 sq. ft.
4. 44 sq. yds. 5. 90 sq. ft. 6. $13\frac{1}{2}$ sq. ft.
7. 28 ft. 8. $24\frac{1}{2}$ ft. 9. 154 sq. ft.
10. 6·928 sq. ft. 11. 10 ft. 8 in. 12. 11 ft.
13. 96 ft. 14. 187·083 sq. ft. 15. 217 sq. ft.
16. 40 ft. 17. $22\frac{2}{5}$,; $12\frac{14}{39}$; 30 ft.
18. 1082·5 sq. ft. 19. 25 ft. 20. 10 ft. 9·9 in.
21. $52\frac{1}{2}$ sq. ft. 22. 70 sq. ft. 23. 11 ft. 3 in.
24. 160 sq. ft. 25. 15 ft. 4 in. 26. $17\frac{1}{2}$ sq. ft.
27. 15 ft. 28. 33 ft. 29. 44 ft.
30. 14 ft. 31. 14 ft. 32. $38\frac{1}{2}$ sq. ft.
33. 3850 sq. ft. 34. 5 ft. 35. 60, 80 ft.
36. 32 ft. 37. $18\frac{6}{7}$ sq. ft. 38. 3·55 : 4.
39. 3 sq. ft. 63 sq. in. 40. 210 sq. lks.

EXERCISE 84.

1. 18 cu. ft. 2. 6 cu. ft. 3. 108 cu. in.
4. 75 cu. in. 5. $11\frac{1}{4}$ cu. ft. 6. 240 cu. cm.
7. 1 cu. ft. 8. 1 cu yd. 9. 10 cu. yds.
10. 48 cu. ft. 11. (*a*) 64 cu. in. (*b*) $\frac{1}{8}$ cu. ft.
(*c*) 27 cu. yds. 12. 27·13 ft.
13. 15·588 ft. 14. 746 sq. in. 15. $41\frac{1}{2}$ sq. ft.
16. $28\frac{1}{6}$ sq. ft. 17. 108 sq. ft. 54 sq. in.
18. 1 cu. yd. 10 cu. ft. 64 cu. in. 19. 72 cu. yd.
20. 68 cu. yds. 14 cu. ft. 21. 17280.
22. 20412. 23. 10530. 24. Rs. 2, 12*a*. 5*p*.
25. Re. 1, 13*a*. 6*p*. 26. Rs. 8, 1*a*.
27. 52 tons 14 cwt. 2 qr. 21 lb. 28. 4 tons 16 cwt. 1 qr. 20 lbs
29. 234 gallons. 30. Rs. 46. 31. 10 cu. ft.
32. 1 cwt. 1 qr. $11\frac{2}{3}$ lb. 33. 2970,
34. 4 hrs. 3 min. 35. $4\frac{1}{2}$ miles. 36. 630. 37. 60.

EXERCISE 85.

1. $2\frac{1}{2}$ ft. 2. 2 ft. 3. 2 ft. 4. 1 ft. 8 in.
5. 4 ft. 6. 1 ft. 8 in. 7. 4 ft. 6 in, 8. 5 ft. 6 in.
9. 2 ft. 6 in. 10. 6 ft. 11. $1\frac{1}{2}$ ft. 12. $1\frac{1}{4}$ ft.
13. 18 ft. 14. $1\frac{1}{2}$ in. 15. $2\frac{1}{2}$ ft. 16. 11 ft.
17. 6 in. 18. 2 in. 19. 3 ft. 9 in. 20. $1\frac{1}{4}$ in.

EXERCISE 86.

1. 105 cu. ft. 2. 960 cu. ft. 3. 150 cu. in.
4. 1540 cu. in. 5. 44 cu. in. 6. 25 cu. in.
7. 8 cu. ft. 9 cu. in. 8. 756 cu. ft. 9. 240 cu. in.
10. 30 cu. ft. 11. 594 cu. in. 12. 792 cu. in.
13. 4620 cu. ft. 14. 1925 cu. ft. 15. 35 ft.
16. $3\frac{1}{2}$ ft. 17. 704 cu. ft. 18. 14080.
19. 11 cu. ft. 1188 cu. in. 20. 18 cu. ft. 576 cu. in.
21. 7 in. 22. 8 in. 23. 2420 gallons.
24. $1\frac{2}{5}$ ft. 25. $3\frac{1}{3}$ ft. 26. 100 ; 153.

EXERCISE 87.

1. Re. 1, 15*a*. 2. 11*a*. 3. 1 min. 48 sec.
4. Rs. 26, 4*a*. 5. Rs. 196, 8*a*.
6. Rs. 2, 1*a*. ; Rs. 30, 15*a*. 7. Rs. 3, 10*a*. 6*p*,
8. Rs. 6. 9. Rs. 45. 10. 6035 meters nearly.
11. Rs. 97, 2*a*. 12. $127\frac{1}{2}$. 13. Rs. 3, 1*a*.
14. 26. 15. 304. 16. £ 1, 15*s*.
17. Rs. 21, 14*a*. 18. 2*s*. $2\frac{1}{4}d$. 19. Rs. 17, 8*a*.
20. 1 min. $22\frac{1}{2}$ sec. 21. Rs. 5, 14*a*. $7\frac{1}{2}p$.
22. 24 miles 4 fur. 28 poles. 23. 60.
24. Rs. 403, 9*a*. 25. $\frac{15}{34}$. 26. Rs. 101, 4*a*.
27. Rs. 1009, 5*a*. 28. Rs. 407, 11*a*. 4*p*.
29. Rs. 984, 15*a*. 4*p*. 30. 3 hrs. 10 min.
31. 9. 32. Rs. 517, 8*a*. 33. $183\frac{1}{8}$ miles.
34. $7\frac{1}{2}$ ac. 35. 3 ac. 36. Rs. 30. 37. Rs. 60.

EXERCISE 83.

1. 20, 2. $28\frac{1}{3}$. 3. $38\frac{2}{5}$. 4. 42 days.
5. 10 days. 6. $8\frac{1}{3}$ days. 7. $25\frac{2}{3}$ days. 8. $11\frac{1}{4}$ days.
9. 28. 10. 700. 11. $38\frac{1}{4}$ days. 12. 28 days.
13. 48 men. 14. 6 months. 15. 5 hrs. 12 min.
16. 30. 17. 24. 18. 64.

19. £3,7*s*. 10*d*. 20. $2\frac{2}{3}$ chk. 21. Rs. 87, 7*a*. $8\frac{92}{253}p$.
22. 101. 23 4 miles. 24. 14 days.
25. $202\frac{1}{2}$ miles. 26. $11\frac{1}{3}$. 27. 1600 men.
28. $13\frac{1}{3}$ days, 29. $26\frac{2}{3}$ days.

EXERCISE 89.

1. $8\frac{1}{3}$ days. 2. 21 men. 3. 9 men.
4. $8\frac{2}{3}$ hrs. 5. Rs. 480. 6. £545, 6*s*. 3*d*.
7. Rs. 33, 5*a*. 4*p*. 8. 6 days. 9 $12\frac{1}{2}$ days.
10. 60 days. 11. Rs 37, 8*a*. 12. 100 months.
13. $\frac{1}{2}$ hr. a day. 14. 6 days. 15. 9 men.
16. 36 days. 17. 2 men. 18. 7 men.
19. Rs. 34, 6*a*. 20. 63 days. 21. 48 oz.
22. 15 pumps. 23. 15 pumps. 24. 18 days.
25 Re. 1, 8*a*. 26. 9 months. 27. 2 days.
28. 540 guns. 29. 48 days. 30. $25\frac{5}{7}$ days.
31. 32 days. 32. 14 women. 33. 45 men.
34. $7\frac{1}{2}$ hrs 35. 27 days. 36. 125 men.
37. 1250 men. 38. Rs. 5096. 39. 84 men. 40. 32 men.

EXERCISE 90.

1. Rs. 72. 2. 48 marks. 3. 36 dollars. 4. Rs. 16.
5. 10 lbs. 6. 24 mds. 7. Rs. 68, 12*a*. 8. 9 days.
9. $7\frac{1}{2}$ days. 10. 82 fr. 50 c.

EXERCISE 91.

1. 3 : 1. 2. 3 : 4. 3. 4 : 5. 4. 5 : 9.
5. 12 : 13. 6. 3 : 1. 7. 11 : 4. 8. 4 : 1.
9. 1 : 15. 10. 8 : 9. 11. 5 : 6. 12. 15 : 20.
13. 16 : 27. 14. 81 : 64. 15. 216 : 343. 16. 9 : 12.
17. (*a*) $4\frac{4}{5}$ (*b*) $3\frac{3}{8}$. 18. 25 : 39. 19. 91 : 81. 20. 9 : 16.
21. (*i*) 21 : 32 (*ii*) 105 : 126 : 144 : 160.
22. 21 : 18 : 20 : 25. 23. 20, 25. 24. 6, 10.
25. 21, 42, 35. 26. 5 : 4. 27. 3 : 28. 28. 15, 25.
29. 25 : 24. 30. 15 : 30 : 40 : 48 and 5 : 16.

EXERCISE 92.

1. 12. 2. 9. 3. 30. 4. 18. 5. 16.
6. 84. 7. 42. 8. 54. 9. 120. 10. $3\frac{1}{2}$.
11. 16. 12. 12. 13. No. 14. Yes.
15. 217 : 216. 16. 50 ; 3rd. term $=\frac{3}{500}$.
17. 1 : 3. 18. 217 : 865. 19. £3. 20. $\frac{3}{5}$ lb.

EXERCISE 93.

1. 6 days. 2. $3\frac{7}{11}$ days. 3. 60 days.
4. $9\frac{3}{8}$ days. 5. $10\frac{2}{7}$ days. 6. $13\frac{1}{3}$ days.
7. 8 10, 12 days. 8. 42 days. 9. 4 days.
10. $7\frac{1}{2}$ days. 11. 60 days. 12. 15 days.
13. $16\frac{2}{3}$ days 14. 16 days. 15 3 days.
16. $33\frac{3}{5}$ days. 17. 6 days. 18. 5 days.
19. 8 days. 20. $11\frac{8}{13}$ days. 21. 72, 36 days.
22. $93\frac{1}{3}$, 40 days. 23. 24, 72, 36 days. 24. 20, 25, 30 days.
25. 20, 30 days. 26. $43\frac{3}{4}$, $58\frac{1}{3}$ days. 27. 40, $18\frac{2}{11}$, 50 days
28. $28\frac{4}{5}$, 36, 48 days. 29. 5 days. 30. $10\frac{10}{11}$ days.
31. 16 days. 32. 2 : 1. 33. $21\frac{3}{17}$ days.
34. 8 hours. 35. 10 days. 36. 16 days.
37. $7\frac{7}{30}$ days. 38. 60 days. 39. 480, 600 days.
40. 8 days. 41. 51 %.

EXERCISE 94.

1. Rs. 9 ; Rs. 7, 8*a.* ; Rs. 6.
2. *A* Rs. 15 ; *B* Rs. 10 ; *C* Rs. $12\frac{1}{2}$. 3. Rs. 12.
4. $9\frac{3}{4}$ weeks ; £ 341, 5*s.* 5. Rs. 14586.
6. 30 days. 7. 15 days.

EXERCISE 95.

1. $1\frac{1}{8}$ min. 2. 8 minutes. 3. $\frac{19}{24}$.
4. 11 A.M. 5. 24 hours. 6. 21 hours.
7. 20 minutes after noon. 8. 45 minutes past 12.
9. 6 hours. 10. 12 minutes past 5.
11. Afte 4 min. 12. $12\frac{6}{7}$ minutes. 13. $10\frac{5}{6}$ minutes.
14. 161 minutes. 15. 53 minutes.

EXERCISE 96.

1. $7\frac{1}{2}$ minutes. 2. $51\frac{1}{3}$ ft.
3. 5 P. M. ; 14 and 18 miles. 4. $26\frac{1}{4}$ miles ; $1\frac{1}{4}$ miles.
5. 21 hours ; $2\frac{5}{8}$ hours. 6. 11 P. M. ; 60 miles.
7. 29 miles. 8. $37\frac{1}{8}$ miles. 9. $39\frac{3}{8}$ miles.
10. 150 yards. 11. 3 O'clock night 12. After 14 hours.
13. 9 hrs. $37\frac{3}{11}$ min. 14. $1\frac{1}{2}$ miles. 15. 15 miles.
16. 12 miles. 17. $2\frac{1}{2}$ miles. 18. 400 leaps.
19. 540 leaps. 20. $3\frac{7}{16}$ minutes. 21. $20\frac{4}{7}$ miles.
22. 20 miles. 23. 15 min past 5. 24. 7 miles.
25. $4\frac{1}{2}$ miles. 26. $\frac{11}{60}$ mile. 27. $463\frac{13}{17}$ hours.
28. 10 min. 29. $13\frac{1}{3}$ miles. 30. 90 miles.

EXERCISE 97.

1. 440 yards. 2. 40 miles. 3. 176 yards.
4. $5\frac{5}{8}$ seconds. 5. 9 seconds. 6. (*i*) 45 sec. (*ii*) 5 sec.
7. $44\frac{4}{9}$ seconds. 8. $41\frac{1}{3}$ sec. ; $40\frac{1}{2}$ sec. 9. 1 minute.
10. $16\frac{1}{4}$ miles. 11. $21\frac{1}{4}$ miles. 12. 110 yards.
13. $80\frac{5}{8}$ yards. 14. $58\frac{1}{3}$ yards. 15. 138 yards.
16. 385 miles. 17. 444 miles. 18. 32 miles.
19. $15\frac{3}{22}$, $29\frac{19}{22}$ miles. 20. 121 yards ; $6\frac{7}{8}$ sec.
21. 3 hours 40 min. 22. $11\frac{1}{3}$ seconds.
23. 88, 55 yards, 24. 16 : 13. 25. 15, 60 miles.

EXERCISE 98.

1. $4\frac{8}{25}$ hours. 2. $37\frac{1}{2}$ minutes. 3. $2\frac{4}{5}$ hours.
4. 6 hours. 5. $1\frac{7}{8}$ minutes. 6. $\frac{9}{16}$ mile per hr.
7. $\frac{3}{10}$ mile per hr. 8. $26\frac{2}{3}$ minutes. 9. $3\frac{1}{2}$, $1\frac{1}{2}$ miles per hr.
10. 3 times.

EXERCISE 99.

1. $20\frac{2}{45}$. 2. 10 minutes. 3. 29 yards.
4. $222\frac{2}{9}$. 5. $53\frac{1}{3}$ yards. 6. $5\frac{1}{3}$ minutes.
7. $583\frac{1}{3}$ yards. 8. $79\frac{1}{11}$ yards. 9. $5\frac{1}{6}$ minutes.
10. 28 yards. 11. *B*. 12. *C*.
13. 27 points. 14. 100. 15. *C* ; 40. 16. 28.

EXERCISE 100.

1. (*i*) 20 hrs. (*ii*) $3\frac{1}{3}$ hrs. 2. 12 hrs. ; $2\frac{2}{5}$ hrs.
3. (*i*) 20 hours. (*ii*) 20 hrs. 4. 2 hrs. in each case.
5. 300 days.

EXERCISE 101.

1. (*i*) $21\frac{9}{11}$ (*ii*) $5\frac{5}{11}$, $38\frac{2}{11}$ (*iii*) $54\frac{6}{11}$ minutes after 4.
2. (*i*) $27\frac{3}{11}$ (*ii*) $10\frac{10}{11}$, $43\frac{7}{11}$ minutes after 5. (*iii*) at 6.
3. (*i*) $10\frac{10}{11}$ (*ii*) $27\frac{3}{11}$ (*iii*) $43\frac{7}{11}$ minutes after 2.
4. (*i*) $43\frac{7}{11}$ (*ii*) $27\frac{3}{11}$ (*iii*) $10\frac{10}{11}$ minutes after 8.
5. (*i*) $49\frac{4}{11}$ (*ii*) $32\frac{8}{11}$ (*iii*) $16\frac{4}{11}$ minutes after 9.
6. (*i*) $32\frac{8}{11}$ (*ii*) $16\frac{4}{11}$, $49\frac{1}{11}$ minutes after 6.
7. $32\frac{8}{11}$ minutes after 4. 8. $43\frac{7}{11}$ minutes after 7.
9. $23\frac{1}{13}$ minutes after 5. 10. $23\frac{1}{13}$ minutes after 3.
11. 2 min. 36 sec. to 4. 12. 6 P.M.
13. 3 hrs. 58 min. 8 sec 14. 11 hrs. 47 min. 55 sec.
15. 160 hrs. ; 3 hrs. 58 min. $53\frac{1}{3}$ sec. ; 4 hrs. $53\frac{1}{3}$ sec.
16. 80 hrs. ; $13\frac{1}{3}$ min. past 4 ; $16\frac{2}{3}$ min. to 4 on Tuesday.

17. March 16, 1941, 9 A. M. 18. August 18 at noon.
19. Diff. 33 min. 35 sec.; $110\frac{10}{13}$ days.
20. $8\frac{62}{361}$ minutes to 9. 21. 10 hrs. 14 min. 24 sec.
22. At the same time on the after noon of Aug. 23, when the first clock will show 1-46′ and the second 2-16′.
23. Real time midnight 6 and 7 July; time indicated 5 hrs. 58 min. $7\frac{1}{2}$ sec. 24 1·13 p m. on July 2.
25. They last agreed at 10 hrs 30 min. p. m. when they both indicated 10 hrs. 30 min. 50 sec.
26. $37\frac{49}{143}$ minutes past 5 27 $32\frac{4}{13}$ minutes past 5.
28. $21\frac{57}{143}$ minutes past 3. 29. 2 hours $7\frac{7}{11}$ minutes..
30. 12 days before; after 528 days.

EXERCISE 102.

1. $\frac{3}{20}$. 2. $\frac{1}{4}$ 3 $\frac{7}{25}$. 4. $\frac{3}{8}$.
5. $\frac{3}{16}$ 6. ·16. 7. ·45. 8. ·075.
9. ·175. 10. 28 11. $37\frac{1}{2}$. 12. $12\frac{5}{7}$.
13. $6\frac{2}{5}$. 14. 9. 15. 33. 16. $1\frac{1}{4}$.
17. $7\frac{1}{2}$ 18. Rs. 96. 19. Rs. 84. 20. Rs. 273.
21. £ 40, 10*s*. 22. £ 21, 4*s*. 3*d*. 23. Rs. 16, 1*a*.8*p*.
24. (*i*) 1600 (*ii*) 1500. 25. (*i*) 20 (*ii*) 20 (*iii*) 50.
26. 216. 27. Ch. 114, Moh. 190, Hindus 456.
28. Rs. 72 29. $4\frac{1}{6}$. 30. 20. 31. £ 562, 10*s*.
32. $13\frac{1}{3}$. 33. 20. 34 798. 35. Rs. 500.
36. Rs. 1036. 37. 2100. 38 192. 39. 1500.
40. Rs. 6000. 41. Rs. 1906, 4*a*. 42. 50%.

EXERCISE 103.

1. 27783. 2. 331776. 3. 9000. 4. 25194.
5. 4. 6. 10. 7. 5. 8. 9112·5.
9. Rs. 675. 10. $18\frac{2}{11}$. 11. $9\frac{1}{11}$. 12. 10*a*.
13. 15*a*. 14. 1000. 15. 500. 16. $16\frac{2}{3}$%.
17. $23\frac{1}{13}$%. 18. 3 mds. $37\frac{1}{2}$ sr. 19. 20000.
20. 630. 21. 200. 22. 23%. 23. £ 1, 6*s*. 8*d*.
24. 81%, 19%. 25. 297391, 520139.

EXERCISE 104.

1. $16\frac{1}{2}$. 2. 15. 3. 8. 4. $7\frac{2}{3}$.
5. $16\frac{1}{4}$ years. 6. 7376. 7. Rs. 41, 8*a*. 6*p*.
8. 11·9 years. 9. 15 years, 10. 115. 11. $55\frac{1}{2}$.
12. 15 years 11 months. 13. 14 years. 14. $32\frac{1}{2}$ years.
15. Rs. $80\frac{1}{2}$. 16. 15·16 years. 17. 25 years. 18. 9 runs.
19. 28. 20. Rs. 468. 21. 95°. 22. Rs. 4.
23. Rs. 21. 24. 120. 25. Rs. 105.

EXERCISE 105.

1. Rs. 10. 2. Rs. 225 gain. 3. Rs. 84.
4. Rs. 2, 9*a*. 8*p*. gain. 5. gain 2*a*. 6. £ 65.
7. 9*s*. 4*d*. loss. 8. Rs. 217. 9. 2*a*. per sr.
10. 1*s*. 8*d*. per lb.

EXERCISE 106.

1. 20% 2. 25% 3. $33\frac{1}{3}$% 4. $12\frac{1}{2}$%
5. $33\frac{1}{3}$% 6. 4%. 7. 25% loss. 8. $25\frac{5}{7}$% gain.
9. 50% gain. 10. $26\frac{4}{11}$% loss. 11. $12\frac{1}{2}$%. 12. 20% loss.
13. 25%. 14. $16\frac{2}{3}$% gain. 15. 4% loss. 16. $33\frac{1}{3}$%.
17. 20% 18. 8%. 19. 25% 20. $2\frac{2}{11}$% loss.

EXERCISE 107.

1. Rs. 72. 2. Rs. 483, 12*a*. 3. Rs. 61, 5*a*, 4*p*.
4. 2680. 5. 10*s*. 10*d*.
6. Rs. 1152 ; Rs. 7, 3*a*. $2\frac{2}{5}p$. 7. 9*a*. $7\frac{1}{5}p$.
8. Rs. 6, 7*a*. $2\frac{2}{5}p$. 9. 10. 10. 20.

EXERCISE 108.

1. Rs. 25. 2. Rs. 33, 5*a* 4*p*. 3. Rs. 90.
4. Rs 133, 5*a*. 4*p*. 5. Rs. 200. 6. 4*a*. 7*p*.
7. Rs. 145, 15*a*. 8. Rs. 70, 10*a*. 9. Rs. 86, 10*a*. 8*p*.
10. Rs. 210, 15*a*. 11. £ 5. 12. Rs. 106.
13. 18 for a rupee. 14. 98 for a rupee. 15. Rs. 50.

EXERCISE 109.

1. Rs. 200. 2. Rs. 50. 3. £ 3373, 6*s*. 8*d*.
4. Rs. 55, 8*a*. $10\frac{2}{3}p$. 5. Rs. 2, 1*a*. 4*p*. 6. Rs. 4000.
7. Rs. 250.

EXERCISE 110.

1. Rs. 8. 2. Rs. 100. 3. Rs. 112, 8*a*.
4. Rs. 191, 4*a*. 5. £ 5, 7*s*. 6*d*. 6. £ 96.
7. Rs. 10, 8*a*. 8. £ 11, 6*s*. 9. 2560.

EXERCISE 111.

1. $10\frac{5}{7}$ p. c. loss. 2. $8\frac{1}{3}$ p. c. gain. 3. $2\frac{1}{2}$ p. c. loss.
4. 16 p. c. gain. 5. $14\frac{1}{3}$% gain. 6. 13 p. c. gain.
7. $28\frac{4}{7}$ p. c. gain. 8. $3\frac{1}{3}$ p. c. gain.

EXERCISE 112.

1. £ 70. 2. Rs. 48. 3. Rs. 9. 4. $2\frac{1}{2}$ p. c. loss
5. Rs. 8 per md. 6. Rs. 9, 6*a*. $2\frac{1}{4}p$.
7. 35. 8. $11\frac{1}{4}$.

EXERCISE 113

1. Rs. 4. 2. Rs. 15. 3. Rs. 26. 4. Rs. 280.
5. Rs. $3\frac{1}{4}$. 6. Rs. 25.

EXERCISE 114

1. 4*s*. 2. $21\frac{7}{8}$. 3. Re. 1. 1*a*. 6*p*. 4. Rs. 10, 4*a*.
5. 4 cwt. 6. 5 srs. 7. $\frac{1}{3}$ quart. 8. $7\frac{1}{2}$ srs.

EXERCISE 115.

1. 960. 2. 1920. 3. 240. 4. 10 for 1*a*. 9*p*. ; 40 doz.

EXERCISE 116.

1. Rs. 16. 2. Rs. 25. 3. Rs. 16, 10*a*. 8*p*. 4. Rs. 15.
5. 15 for 2*a*. 6. Rs. 200 7. Rs. 230. 8. 200*s*.
9. £8. 10. Rs. 50.

EXERCISE 117.

1. Tea 2*s*. 6*d*. ; sugar 6*d*.
2. Tea Re. 1, 14*a*. ; coffee 13*a*. 4*p*.
3. Tea Rs. 2, 8*a*. ; sugar 2*a*. 8*p*.
4. Tea Re. 1, 2*a*. 4*p*. ; sugar 1*a*. 8*p*.
5. Rice Rs. 5 ; wheat Rs. $3\frac{1}{2}$.

EXERCISE 118.

1. $40\frac{5}{8}$. 2. Rs. 24, 6*a*. 3. Rs. 6776.
4. Rs. 1000. 5. $21\frac{2}{13}$.

EXERCISE 119.

1. Rs. 12, 8*a*. 2. Rs, 7, 8*a*. 3. Rs. 45, 8*a*.
4. Rs. 7. 5. Rs. 5, 7*a*. 6*p*.

EXERCISE 120.

1. Rs. 60. 2. Rs. 55. 3. Rs. 30.
4. Rs. 50. 5. Rs. 15.

EXERCISE 121.

1. 22 mds. 2. 117 mds. 2. 60. 4. 680.

EXERCISE 122.

1. Rs. 310, 250. 2. Rs. 2100, 1500. 3. Rs. 60, 75.
4. Rs. 150, 75. 5. Rs. 175, 150. 6. Rs. 320, 200.

EXERCISE 123.

1. $12\frac{1}{2}$. 2. $43\frac{3}{4}$. 3. 20. 4. $12\frac{1}{2}$.

EXERCISE 124.

5*a*. 2. 9*a*. 3. Rs. $2\frac{1}{2}$. 4. £ 2, 4*s*. $5\frac{1}{3}$*d*.

EXERCISE 125.

1. Rs. 6400. 2. Rs. 5333, 5*a*. 4*p*. 3. Rs. 200, 250.
4. Rs. 2100, 1400. 5. Rs. 5. 6. Rs. 50, 40.

EXERCISE 126

1. Rs. 31, 14*a*. 2. $6\frac{2}{3}$. 3. $10\frac{1}{4}$ p. c. gain.
4. Rs. 8, 5*a*. 4*p*. 5. 8. 6. 7 p. c. loss.
7. Rs. 5. 8. Rs. 33, 5*a*. 4*p*. 9. Rs. 46; 60.
10. 960. 11. Rs. 80. 12. $62\frac{1}{2}$ p. c.
13. $28\frac{6}{13}$. 14. 48 lbs. 15. £ $4\frac{1}{3}$.
16. 72 eggs for 7*s*. 17. $15\frac{15}{16}\%$ 18. Rs. $26\frac{1}{10}$.
19. Rs. 54, 4*a*. 20. Rs. 3, 15*a*. 4*p*. 21. 6*a*.
22. $4\frac{1}{6}$ p. c. loss. 23. 80 lb. 24. Rs 31, 4*a*.
25. 24 yds. 26. 25 p. c. loss. 27. 21% profit.
28. Rs. 600, 750. 29. Rs. 250. 30. Rs. 90 loss.
31. $\frac{9}{10}$ sr. 32. Rs. 25. 33. 5*a*. 4*p*.
34. 1 p. c. loss. 35. 16 dozen. 36 Rs. 185, 200.
37. $17\frac{13}{40}$*a*. 38. Rs. 150, 125. 39. 6 scores.
40. Rs. 120, 80.

EXERCISE 127.

1. Rs. 87, 12*a*. ; Rs. 131, 10*a*. ; Rs. 307, 2*a*.
2. Rs. 225 ; Rs. 150 ; Rs. 112, 8*a*.
3. Rs. 50, 2*a*. ; Rs. 300, 12*a* ; Rs. 150, 6*a*. ; Rs. 125, 5*a*.
4. 60°, 100°, 20°. 5. 120. 6. Rs. 39.
7. Rs. 207, 245, 286.
8. Rs. 152, 8*a*. ; Rs. 189 6*a*. ; Rs. 244. 2*a*.
9. Rs. 132 ; Rs. 165, ; Rs. 220.
10. Rs. 63, 3*a*. ; Rs. 42, 2*a*. ; Rs. 21, 1*a*.
11. Rs. 120 ; Rs. 150 ; Rs 180.
12. Rs. 151, 8*a*. ; Rs. 75, 12*a*. ; Rs 25, 4*a*.
13. £ 60 ; £ 90 ; £ 100.
14. Rs. 540 ; Rs. 270 ; Rs. 180 ; Rs. 135.
15. Rs. 80 ; Rs. 120 ; Rs. 150.
16. *A* Rs. 113, 5*a*. 4*p*. ; *B* Rs. 268, 5*a*. 4*p*. ; *C* Rs. 243, 5*a*. 4*p*.
17. Rs. 120; Rs. 150, Rs. 125. 18. Rs. 625, Rs. 500, Rs. 400.
19. 42, 36 and 30 years. 20. $3\frac{6}{25}$*s*. ; $4\frac{8}{25}$*s*. ; $5\frac{10}{25}$*s*. ; $7\frac{17}{25}$*s*.
21. Rs. 145, Rs. 116, Rs. 290.

22. 36 rupees, 54 eight anna pieces, 72 four-anna pieces.
23. Sov. 250, cr.300, fl. 400 24. 80, 1200, 9600.
25. Rs. 1080, Rs. 600, Rs. 420. Rs. 340.
26. Rs. 1881, Rs. 1140, Rs. 3249.
27. Rs. 560, Rs. 660, Rs. 1860.
28. Rs. 3762, Rs. 2280, Rs. 6498. 29. £4, 4*s*.; £3 ; £1, 16*s*.
30. Rs. 167. 1*a*.; Rs. 97, 7*a*. 3*p*.; Rs. 69, 9*a*. 9*p*.; Rs. 41, 12*a*.3*p*.
31. Rs. 300, Rs .200, Rs. 400, Rs. 225, Rs. 125.
32. Rs. 140, 160, 144. 33. Rs. 320, 480, 640.
34. Rs. 250, 300, 400. 35. Rs. 1200, 1800, 800.
36. 40, 36 yrs. 37. 52, 40 yrs.
38. 4, 35 yrs. 39. 56, 41 yrs. 40. 120, 108, 84.

EXERCISE 128.

1. Rs. 120, Rs. 160.
2. Rs. 242. 2*a*,; Rs. 605, 5*a*,; Rs. 1331, 11*a*.
3. Rs. 3432, Rs. 2860, Rs 10458.
4. Rs. 155, Rs. 145. 5. Rs. 375.
6. Rs. 1750; Rs. 2450; Rs. 3500 and Rs. 4200.
7. *A* Rs. 800, *B* Rs. 900, *C* Rs. 750.
8. Rs. 288 ; Rs. 402 ; Rs. 440. 9. Rs. 450 ; Rs. 540.
10. Rs. 3150. 11. Rs. 736.
12. Rs. 1600. 13. Rs. 329 ; Rs. 121.
14. Rs. 1600 ; Rs. 1800. 15. Rs. 3400 ; Rs. 8500.
16. £350 ; £450. 17. *A* £540 ; *B* £900.
18. Rs. 360 ; Rs. 225. 19. Rs. 330 ; Rs. 2100.
20. *A* Rs. 1228 ; *B* Rs. 1542 21. 10 months.

EXERCISE 129.

Nate—In many questions e.g. 10, 11, 12, 13, 18, 19 etc., other answers are also possible.

1. 3 : 1, 2. 1 : 5, 3. 7 : 5. 4. 2 : 1.
5. 4 : 1. 6. 7 : 9. 7. 7 : 5. 8. 3 : 5.
9. 2 : 5. 10. 1 : 1 : 6. 11. 1 : 1 :2. 12. 1 : 1 : 4.
13. 1 : 1 : 4. 14. 9 sr. 15. 8 sr. 16. 10 sr.
17. $10\frac{2}{3}$ sr. 18. 36, 12, 12, 24 gallons. 19. 1 : 1 : 2.
20. 52 : 78 : 51 : 68. 21. 45 gallons. 22. 30 gallons.
23. 79 : 49.

EXERCISE 130.

1. 23 : 4 ; Rs. 2, 9*a*. 2. 40 apples, 20 pears.
3. 31 : 1. 4. Ratio of equality.
5. 16·2 gallons 6. 13 : 9 7. 72% and 56%.

EXERCISE 131.

1. Boys 60, girls 90. 2. 100, 80. 3. 35 florins, 40*s*.
4. 80 lbs. 5. 25 yds. 6. $86\frac{2}{3}$, $56\frac{2}{3}$, $56\frac{2}{3}$ lbs.

EXERCISE 132.

1. $3\frac{3}{8}$ months. 2. $7\frac{3}{5}$ months. 3. $92\frac{4}{23}$ days.
4. 7 months 5. 80 months. 6. 8·7 months.

EXERCISE 133.

1. 70 sheep. 2. 70 oxen. 3. 14 days.
4. $51\frac{1}{3}$ acres. 5. 190 oxen. 6. 9 minutes.
7. 3 acres. 8. 65 gallon, 13 hours.

MISCELLANEOUS EXERCISES II.

I

1. 5. 2. 69 eggs. 3. (*a*) Rs. 44, (*b*) 1·00001. 4. 30 days
5. Rs. 1435, 7*a*. $9\frac{1}{4}p$. 6. 30 days. 7. 195.
8. £6, 10*s*. 8*d*. 9. 88 yds. 10. 20·4 yrs.

II

1. 99679. 2. $\frac{5}{7}$. 3. Rs. 17, 2*a*. 4. Rs. 40
5. 165 mangoes 6. $2\frac{4}{7}$ days. 7. 36·284. 8. 42 days.
9. 4 : 5. 10. 5*a*. 11*p*.

III

1. $\frac{21}{665}$. 2. $1\frac{1}{24}$. 3. 8 days. 4. 3*d*.
5. 24000 males ; 26000 females.
6. Rs. 63, 10*a*. $2\frac{2}{3}p$. 7. 76·2, 38·1 yds.
8. 4 sov., 6 h. cr., 22*s*. 9. 1367 Km.
10. At 8 p. m. on Monday ; 5 hrs. 59 min. $48\frac{3}{4}$ sec.

IV

1. £8, 1*s*. 2. 9990, 10125. 3. 1849854486324
4. £27, 0*s*. $5\frac{1}{2}d$. 5. 150 marks. 6. 5 days,
7. 42 miles 726 yds. 8. £1, 0*s*. 10*d*.
9. $99 \times 807 = 806193$. 10. $1\frac{1}{3}$ cwt., £1, 1*s*. 6*d*.

V

1. (1 and 5) or (6 and 0). 2. *A* 48 ; *B* 84.
3. 1296 cu. in.; 5*a*. 9*p*. 4. Rs. 31, 4*a*.
5. 8320 men. 6. ·9998.
7. Rs. 5, 8*a*. 6*p*.; Rs. 5. 13*a*. 6*p*.; Rs, 6. 2*a*. 6*p*.; Rs. 6, 7*a*. 6*p*.
8. Rs. 11865, 3*a*. $2\frac{1}{3}p$. 9. 156 boys.
10. Length 27 ft., breadth 18 ft. height 12 ft.

VI.

1. 1. 2. ·0203125. 3. 5. 4. 15 ft.
5. 20% gain 6. 32 days. 7. 1·0001. 8. 10 yds., 22 yds.
9. $4\frac{1}{6}$ days. 10. 13 stones 4 lbs.

VII.

1. $4\frac{1}{4}$, ·63. 2. $1\frac{3}{140}$ ft. 3. Rs. 1440. 4. 1232 yds.
5. 30·557 ft. 6. 186. 7. 17*s.* 3*d.* 8 2·068501.
9. $2\frac{1}{2}$ ft. 10. 20, 21.

VIII.

1. Rs. 3, 1*a.* 6*p.* 2. G. C. M.=·279 ; L. C. M.=25234713.
3. 178 hrs. 52 min. 30 sec. 4. 16 days.
5. $\frac{1}{791}$, $\frac{355}{113}$. 6. 1 : 8. 7. C passes.
8. Rs. 4, 6*a.* 9. 39 yds. 10. 4 hrs. $36\frac{12}{13}$ min.

IX.

1. $3\frac{5}{124}$. 2. 7 hrs. 3. $1\dot{3}84615\dot{}$.
4. 12 min. $40\frac{10}{11}$ sec. 5. 6 yds. 6. 2 : 3.
7. 3000 leaps of greyhound. 8. Nothing. 9. $4\frac{1}{2}$ lb.
10. (*a*) $2\frac{1}{4}$, 8·729. (*b*) Rs. 180.

X.

1. 6 times 2. $11\frac{9}{41}$ in.
3. $39\frac{3}{8}$ miles from the starting place.
4. Rs. 2, 7*a.* 9*p.* 5. Rs. 72. 6. 6 men.
7. (*a*) 33, 44 ft. (*b*) 125%, 149%. 8. After 9 days.
9. 125 subscribers. 10. 40 men.

EXERCISE 134.

1. Rs. 16. 2. Rs. 18. 3. Rs. 80. 4. Rs. $8\frac{1}{3}$.
5. Rs. 16. 6. Rs. $6\frac{1}{4}$. 7. Rs. 23, 7*a.*
8. Rs. 16, 6*a.* 6*p.* 9. Rs. 72. 10. Rs. 150.
11. Rs. 26, 10*a.* 8*p.* 12. £117, 6*s.*
13. £197, 1*s.* 3*d.* 14. Rs. 228, 3*a.* 8*p.*
15. Rs. 351, 4*a.* $8\frac{1}{4}$*p.*

EXERCISE 135.

1. Rs. 126. 2. Rs. 63. 3. Rs. 146, 4*a.*
4. Rs. 56, 4*a.* 5. Rs. 144. 6. £97, 4*s.*
7. Rs. 160. 8. Rs. 42, $6\frac{3}{4}$*p.* 9. Rs. 168, $5\frac{5}{8}$*p.*
10. Rs. 335, 1*a.* 5*p.*

EXERCISE 136.

1. Rs. 45. 2. Rs. 80. 3. Rs. 42.
4. Rs. 103, 2*a*. 5. Rs. 159, 4*a*. 6. Rs. 42.
7. Rs. 120, 11*a*. $1\frac{1}{3}p$. 8. Rs. 241, 8*a*. 9. Rs. 37, 8*a*.
10. Rs. 106, 5*a*. 8*p*. 11. Rs. 182, 3*a*. 4*p*.
12. Rs. 189, 4*a*. 6*p*. 13. Rs. 356, 3*a*. 1*p*.
14. Rs. 126, 3*a*. 11*p*. 15. Rs. 44, 15*a*. 6*p*.

EXERCISE 137.

1. Rs. 185, 15*a*. 2. Rs. 86, 11*a*. $7\frac{1}{136}p$.
3. Rs. 208, 3*a*. $10\frac{32}{55}p$. 4. Rs. 116, 4*a*. 10*p*.
5. Rs. 148, 6*a*. 9*p*. 6. Rs. 226, 14*a*. 6*p*.
7. Rs. 1694, 5*a*. 4*p*. 8. Rs. 1464, 10*a*. 7*p*.
9. Rs. 2111, 1*a*. 3*p*. 10. Rs. 1587, 12*a*. 2*p*.

EXERCISE 138.

1. Rs. 5. 2. Rs. 13. 3. Rs. 5, 10*a*.
4. Rs. 20, 14*a*. 5. Rs. 38, 6*a*. $4\frac{4}{5}p$. 6. Rs. 7, 3*a*. $2\frac{2}{5}$.
7. Rs. 20. 8. £ 4, 6*s*. $4\frac{4}{5}d$. 9. £ 5, 4*s*.
10. £ 4, 3*s*. 4*d*. 11. £ 25, 4*s*. 12. £ 8, 8*s*. 9*d*.
13. £ 14, 6*s*. 8*d*. 14. £ 27, 6*s*. 3*d*. 15. Rs. 146, 10*a*. 8*p*.
16. Rs. 7487, 3*a*. $2\frac{32}{55}p$. 17. *B* paid Rs. 10, 8*a*. more.
18. Rs. 2032. 19. £ 3, 1*d*. 6*d*. 20. Rs. 500.

EXERCISE 139

1. Rs. 520. 2. Rs. 833, 5*a*. 4*p*. 3. Rs. 1050.
4. Rs. 640. 5. Rs. 533, 5*a*. 4*p*. 6. Rs. 266, 4*a*.
7. Rs. 600. 8. Rs. 600. 9. Rs. 1000.
10. Rs. 2400. 11. Rs. 7800. 12. £ 7500.
13. £ 168. 14. Rs. 1600. 15. Rs. 785.

EXERCISE 140.

1. 4. 2. 4. 3. 5. 4. 4. 5. $3\frac{1}{2}$.
6. $2\frac{1}{4}$. 7. $7\frac{1}{2}$. 8. 4. 9. 7. 10. 5.
11. 3. 12. 5. 13. 5. 14. $4\frac{11}{16}$. 15. $2\frac{1}{2}$.

EXERCISE 141.

1. 4 years. 2. 3 years. 3. 4 years.
4. $6\frac{2}{3}$ years. 5. 7 years. 6. 4 years.
7. 5 years. 8. 10 years. 9. 25 years.
10. $6\frac{2}{3}$ years. 11. $2\frac{1}{2}$ years. 12. 32 months.
13. 16 years. 14. Sept. 16. 15. June 27. 16. 8.

EXERCISE 142.

1. Rs. 6, 13*a*. $7\frac{4}{5}p$.
2. Rs. 11, 5*a*. 6*p*.
3. Rs. 2594, 8*a*. $11\frac{13}{25}p$.
4. Sundar Rs. 77, 15*a*. 6*p*. more.
5. Rs. 500.
6. 3%
7. $12\frac{1}{2}$ years.
8. Rs. 625.
9. $3\frac{1}{3}\%$
10. 5 years.
11. Rs. 500 ; 3%
12. Rs. 500; 5%
13. Rs. 500 ; 5 p. c.
14. Rs. 400 ; 4 p. c.
15. Rs. 900.
16. Rs. 1075.
17. 24 Nov.
18. $3\frac{1}{3}\%$
19. 5%.
20. Rs. 825.
21. Rs. 3881.
22. Rs. 745, 8*a*.
23. Rs. 897.
24. 5%
25. 4%
26. 5%
27. £ 855, 11*s*. $10\frac{1}{2}d$.
28. Rs. 1200.
29. Rs. 900, Rs. $1200\frac{2}{3}$.
30. Rs. 6500.
31. Rs. 1300.
32. Rs. 1125, 875.
33. 4%, 5%
34. $7\frac{1}{2}\%$, 5%
35. Rs. 900.
36. Rs. 300.
37. £ 63 ; £ 60, 13*s*. 4*d*. ; £ 58, 10*d*.
38. *A* Rs. 3575 ; *B* Rs. 3146 ; *C* Rs. 3025.
39. 3 years ; 4%
40. 4 years ; 5%.
41. £ 650 ; 4 years.
42. $44\frac{4}{9}\%$.
43. Rs. 700, Rs. 900.
44. Rs, 1239.

EXERCISE 143.

1. Rs. 10, 4*a*.
2. Rs. 16, 5*a*. $1\frac{11}{25}p$.
3. Rs. 39, 6*a*. 6*p*.
4. Rs. 61, 12*a*. $9\frac{3}{6}p$.
5. Rs. 118, 3*a*. 6*p*.
6. Rs. 187, 4*a*. $8\frac{104}{125}p$.
7. Rs. 331, 0*a*. $2\frac{2}{5}p$.
8. £ 60, 15*s*.
9. £ 175, 3*s*. $1\frac{47}{64}d$.
10. £ 265, 10*s*. 9·93627*d*.
11. Rs. 64, 1*a*. 5*p*.
12. Rs. 156, 13*a*. 11*p*.
13. £ 37, 2*s*. 6*d*.
14. £ 88, 6*s*.
15. Rs. 85, 6*a*. 8*p*.
16. Rs. 43, 9*a*. 2*p*.
17. Rs. 328, 15*a*. 6*p*.
18. £ 152, 8*s*. 10*d*.
19. £ 136.
20. £ 447, 19*s*. 1*d*.
21. Rs. 579, 2*a*. 6*p*.
22. Rs. 692, 1*a*. 8*p*.
23. Rs. 1766, 14*a*. 4*p*.
24. Rs. 1732, 12*a*. 2*p*.
25. £ 1930, 2*s*. 11*d*.
26. £ 1919, 13*s*. 3*d*.
27. Rs. 77, 14*a*. 3*p*.
28. Rs. 1408, 1*a*. 11*p*. nearly.
29. Rs. 3, 15*a*. 3·2256*p*.
30. £ 1415, 4*s*. 7·68*d*.

EXERCISE 144.

1. Rs. 500.
2. Rs. 1250.
3. Rs. 1500.
4. Rs. 3125.
5. Rs. 12000.

EXERCISE 145.

1. Rs. 800. 2. Rs. 3750. 3. Rs. 860. 4. £ 700.
5. Rs. 6875. 6. Rs. 14285. 11*a* $5\frac{1}{7}p$. 7. £ 1500.
8. Rs. 625.

EXERCISE 146.

1. 2 years. 2. 3 years. 3. 3 years.
4. 2 years. 5. $3\frac{1}{3}$ years 6. $3\frac{1}{3}$ years.

EXERCISE 147.

1. 10% 2. 5% 3. 4% 4. 5% 5. 4% 6. $3\frac{1}{3}\%$.

EXERCISE 148.

1. £ 2000. 2. £ 800. 3. Rs. 2000 ; 10 p. c.
4. £ 250 ; 4 p. c. 5. £ 4600. 6. Rs. 32000.
7. £ 315. 8. £ 13, 12*s*. 9. £ 1000.
10. £ 1156. 11. 4 p. c. 12. Rs. 4410. 13. £ 4394.
14. £ 420 £ 400. 15. Rs. 3200; Rs. 3889, 9*a*. 11·04*p*.
16. Rs. 16400. 17. £ 3125, £ 3250. 18. £ 500.
19. £ 2205, £ 2100, £ 2000. 20. £ 3497, 5*s*. $4\frac{6}{61}d$.
21. *A* Rs. 23409; *B* Rs. 22500

EXERCISE 149.

1. Rs. 70. 2. £ 51, 5*s*. 3. £ 70, 17*s*. 6*d*.
4. Rs. 19, 4*a*. 5. £ 296, 13*s*. $4\frac{1}{2}d$. 6. £ 200.
7. £ 800. 8. Rs. 216, 10*a*. 8*p*. 9. £ 576, 18*s*. 9*d*.
10. £ 575. 11. £ 8000. 12. 504, 8*a*. 11*p*.
13. £ 188, 13*s*. $5\frac{1}{2}d$. 14. Rs. 260, 15*a*. 15. Rs. 8000.
16. £ 237, 10*s*. 17. Nothing. 18. £ 963, 19*s*. $10\frac{54}{287}d$.
19. Rs. 6250.

EXERCISE 150.

1. Rs. 41204, 4*a*. 8*p*. 2. £ 520, 17*s*. 6*d*. 3. £, 621, 3*s*. 4*d*.
4. £ 505, 5*s*. 5. Rs. 872, 5*a*. 6. £ 652, 11*s*.
7. £ 450, 13*s*. 4*d*. 8. £ 10780, 14*s*. 8*d*.

EXERCISE 151.

1. 2 years. 2. 9 months. 3. 146 days.
4. $3\frac{4}{5}$ years. 5. 4 months. 6. $3\frac{1}{2}$ yrs. 7. 20 months.

EXERCISE 152.

1. 4 p. c. 2. 4 p. c. 3. $5\frac{1}{4}$ p. c.
4. 4 p. c. 5. $4\frac{1}{2}$ p. c. 6. $3\frac{1}{4}$ p. c.

EXERCISE 153.

1. 4 years. 2. 4 months. 3. Rs. 3218, 12*a*.
4. 2 years. 5. $4\frac{1}{2}\%$. 6. 15 years.
7. £ 3750 ; $4\frac{1}{2}\%$. 8. $6\frac{1}{4}\%$. 9. Rs. 180.
10. 12 months. 11. (*i*) Rs. 45, 13*a*. 4*p*. (*ii*) Rs. 13. 1*a*. $6\frac{2}{7}p$.
12. 80 : 83 , Rs. 320. 13. $9\frac{1}{11}$.
14. 10 months. 15. £ 130. 16. Rs. 9, 9*a*. $10\frac{2}{13}p$.
17. £ 2500. 18. $1\frac{1}{2}$ p. c. gain. 19. $8\frac{2}{5}\%$.
20. 25 p. c. 21. £ $17\frac{23}{41}$. 22. £ 720 ; $4\frac{1}{6}$ p. c.
23. £ 375, 10*s*. 24. 17*s*. 6*d*. 25. Rs. 4556, 4*a*.

EXERCISE, 154.

1. 5 per cent. 2. $6\frac{1}{4}$ p. c. 3. Rs. 108000.
4. £ 1, 2*s*. 10*p*. 5. £ 515. 6. £ 65, 13*s*. $6\frac{1}{2}d$.
7. $4\frac{3}{4}\%$ 8. £ 508, 15*s*. ; 3% 9. Rs. 9.
10. Rs. 255. 11. £ 6020. 12. £ 59, 7*s*. 6*d*.
13. 31st May. 14. $11\frac{1}{9}$ per cent. 15. $4\frac{3}{8}\%$.
16. 73 days. 17. $2\frac{1}{2}$ per cent.
18. Rs. 20150 ; 3 p. c. 19. £ 17, 8*s*. $5\frac{7}{8}d$. ; £ 1742, 5*s*.
20. $8\frac{1}{3}\%$; £ 300.

EXERCISE 155.

1. 8*a*. 2. 12*a*. 3. 5*a*. 4*p*.
4. 10*a*. 8*p*. 5. 12*a*. ; Rs. 5346.
6. 5*a*. 4*p*. ; Rs. 8840 ; Rs. 15446 ; Rs. 5038, 2*a*. 8*p*.
7. 10*a*. 8*p*. ; Rs. 5931, 5*a*. 4*p*. 8. Rs. 25200.
9. Rs. 13900. 10. £ 900. 11. Rs. 26891.
12. Rs. 245, 1*a*. 13 Rs. 14400, Rs. 12000.
14. Rs. 3000, Rs. 2500. 15. £ 2760.
16. £ 7500. 17. 10*a*. 8*p*. 18. 15*s*.
19. £ 320. 20. £ 267.

EXERCISE 156.

1. Rs. 1473, 8*a*. 2. Rs. 1144, 13*a*. 8*p*. 3. Rs. 39, 1*a*.
4. Rs. 2697, 14*a*. 8*p*. 5. Rs. 4700. 6. Rs. 1750, $8\frac{1}{2}a$.
7. Rs. 1460, 15*a*. 8. Rs. $1600\frac{17}{31}$. 9. £ 2500.
10. £ $2252\frac{55}{472}$. 11. Rs. 1259, 6*a*. 12. £ 1800.
13. Rs. 1844. 14. 4*d*. 15. Rs. 4.
16. £ 20. 17. £ 5. 18. Rs. 855.

EXERCISE 157.

1. Rs. 4, $9\frac{5}{7}p$. 2. Rs. 528, 13*a*. $\frac{12}{25}p$. 3. 2%
4. Rs. 3700. 5. Rs. 600. 6. Rs. 15000.
7. $3\frac{1}{2}$% 8. Rs. 90. 9. Rs. 73, 7*a*.
10. Rs. 7525. 11. Rs. 25000. 12. Rs. 14550.
13. £ 1900. 14. Rs. 700. 15. Rs. 766, 10*a*. 8*p*.

EXERCISE 158.

1. Rs. 23400. 2. £ 1867, 10*s*. 3. Rs. 52800.
4. $10\frac{3}{4}d$. 5. 7*s*, $6\frac{3}{4}d$. 6. 1·5 frank per rupee.
7. £ 85, 17*s*, 4*d*. 8. Rs. 3972. 9. $\frac{1}{5}$.
10. Rs. 200. 12. 1*s*. 2*d*. 13. Rs. 120000.
14. Rs. 10020, 5*a*. 15. $\frac{25}{1000}$, Rs. 64 = £ 5.
16. Rs. $13\frac{5}{7}$, Rs. 960, Rs. $28\frac{4}{17}$ 17. $15\frac{10}{21}$ guinea.
18. 2·5 frank. 19. 1*s*. $10\frac{1}{4}d$. 20. Rs. 62, 9*a*. $\frac{12}{23}p$.

EXERCISE 159.

1. Rs. 1840. 2. Rs. 2472. 3. Rs. 3960.
4. £ 5418, 3*s*. 9*d*. 5. £ 588, 5*s*. 6. Rs. 3078, 13*a*. 6*p*.
7. £ 9175, 12*s*. 6*d*. 8. Rs. 1226, 1*a*. 6*p*.
9. Rs. 1881, 4*a*. 10. £ 2412, 5*s*. $7\frac{1}{2}d$.

EXERCISE 160.

1. Rs. 14100. 2. £ 15573, 8*s*. $10\frac{1}{2}d$.
3. £ 9104, 6*s*. 8*d*. 4. Rs. 10793, 12*a*. 5. Rs. 68

EXERCISE 161.

1. Rs. 9000. 2. £ 1000. 3. Rs. 4500. 4. £ 74
5. Rs. 110. 6. 3419 fr. 58 c. 7. £ 5300.

EXERCISE 162.

1. £ $238\frac{7}{8}$. 2. Rs. 50, 13*a*. 4*p*. 3. $112\frac{7}{32}$.
4. Rs. 70. 5. Rs. 6250. 6. Rs. 7500.
7. Loss £ 5. 8. Loss Rs. 75.

EXERCISE 163.

1. Rs. 1368, 8*a*. 2. £ 2, 1*s*. 8*d*. 3. Rs. 43, 8
4. £ 83, 6*s*. 8*d*. 5. £ 5750.

EXERCISE 164.

1. Rs. 44072, 1*a*. 4*p* 2. Rs. 9000. 3. Rs. 16800.
4. Rs. 11250. 5. £ 1942, 10*s*. 6. Rs. 122169.

EXERCISE 165.

1. 84. 2. $94\frac{3}{2}$. 3. Rs. 104, 4*a*.
4. $77\frac{1}{2}$. 5. 90. 6. 105.

EXERCISE 166.

1. Increase Rs. 20. 2. £ 26, 13*s*. 4*d*.
3. Increase Rs. 34, 8*a*. 4. No change.
5. Rs. 1250; Rs. $2\frac{1}{2}$ less. 6. £ 60 increase.
7. Rs. 47 increase. 8. Rs. 100 increase.
9. Ro. 31, 4*a*. 10. Increase Rs. 14, 8*a*.
11. Increase £ 6, 13*s*. 4*d*. 12. Rs. 355.
13. Increase Rs. 300. 14. Decrease 7*s*.
15. Increase Rs. 310. 16. £ 53, 6*s*. 8*d*. increase.
17. Rs. 3800; Rs. 11 decrease. 18. £ $3\frac{3}{29}$.
19. Rs. 150 increase. 20. £ 1, 12*s*.
21. Rs. 19, 13*a*. 4*p*. decrease.

EXERCISE 167.

1. 122. 3*s*. 4*d*. 2. 80. 3. The latter.
4. 1st.; £ 1342, 10*s*.; $3\frac{63}{179}$ p. c. 5. £ 100.
6. Equal. 7. £ 78. 8. Rs. 19992.
9. $110\frac{13}{16}$. 10. $4\frac{1}{2}\%$ 11. The latter; Rs. 60.
12. $4\frac{10}{0}$. 13. £ 4725. 14. Rs. 23400.
15. £ 3429, ; £ 135 ; £ 139, 14*s*. 16. $86\frac{31}{54}$.
17. Rs. 34 decrease. 18. no change. 19. £ 2500.
20. Rs. 1638. 21. Rs. 21735. 22. $106\frac{11}{19}$.
23. Rs. 49700. 24. 4% 25. £ 25000.
26. £818, 8*s*. 27. Rs 25200. 28. Rs. 151710.
29. Rs. 27000 ; Rs. 185 more. 30. £ 24 increase.
31. Rs. 16000 ; Rs. 24000. 32. £ 397 increase.
33. Rs. 13200 34. (*i*) £ 4, 16*s*. (*ii*) 35 : 34.
35. Rs. 39333, 5*a*. 4*p*. 36. Rs. 38200.
37. $88\frac{7}{8}$. 38. £ 215. 39. Increase £ 47.
40. Rs. 30000. 41. $85\frac{1}{3}$. 42. $94\frac{7}{8}$.
43. £6000. 44. £ 257, 5*s*. 5*d*.
45. Rs. 4800 ; Rs. 5200. 46. $82\frac{1}{4}$.
47. Rs. 30780. 48. 156. 49. 10 p. c.
50. Rs. 945 ; Rs. 35190. 51. £ 2852 ; £ 25935.
52. Rs. 12960 ; Rs. 11220. 53. Rs. 480000.
54. £ 25935. 55. Rs. $136\frac{1}{8}$.

EXERCISE 168.

1. 11344000. 2. 1·526. 3. 9. 4. 810000.
5. 1. 6. ·0625. 7. 100. 8. $17\frac{13}{16}$.
9. ·5. 10. 1. 11. ·2. 12. $13\frac{5}{6}$.
13. 100. 14. 1. 15. ·1. 16. $\frac{1}{6}$.
17. $\frac{11}{36}$. 18. 1. 19. ·05. 20. 1.

EXERCISE 169.

1. 102. 2. 100. 3. 17*a*. 8*p*.
4. Rs. 15. 5. Rs. 40. 6. Rs. 7; Rs. 10.
7. 60 days. 8. 18 days. 9. 25 days.
10. 8 days. 11. $2\frac{2}{3}$ days. 12. $266\frac{2}{3}$ days.
13. Man $7\frac{1}{2}$ hours, boy 18 hours, together $5\frac{1}{7}$ hours.

EXERCISE 170.

1. ·309. 2. 3·146. 3. 3·968. 4. 17·796.
5. 1·366. 6. 3·827. 7. ·434. 8. 4·051.
9. 1. 10. ·972. 11. 0. 12. $\sqrt[3]{3}$.
13. $\sqrt[4]{15}$. 14. 294·151. 15. $9\sqrt{77}$.

EXERCISE 171.

1. 153 miles. 2. 35 ft. 3. 5 ft. $7\frac{1}{2}$ in. 4. $32\frac{1}{2}$ in.

EXERCISE 172.

1. 60°C. 2. 9·4 miles. 5. 30 m.; 12 m.
6. $9\frac{3}{4}$ hrs. from *A*'s start; $7\frac{1}{2}$ hrs. and $12\frac{1}{4}$ hrs. from *A*'s start.
8. 9 sec. 9. 6·7 hrs.

EXERCISE 173.

1. Wednesday. 2. Tuesday. 3. Friday.
4. Sunday. 5. Monday. 6. Thursday.
7. Friday. 8. Tuesday. 9. Sunday.
10. Monday. 11. Tuesday. 12. Thursday.
13. Wednesday. 14. Sunday. 15. Sunday.
16. 2, 9, 16, 23, 30. 17. Yes.

MISCELLANEOUS EXERCISES III,

I.

1. £ 10, 8*d*. 2. Quotient 17430, Rem. 13.
3. ·03212. 4. £ 1682. 5. 72 men; 288 women.
6. Rs. 1500. 7. 8*s*. 4*d*. 8. 10-15 A. M.
9. $11\frac{8}{7}$*s*. 10. £ 3990.

II.

1. 14 days 7 hrs. 11 min. 17 sec. ; 2674 days 9 min. 59 sec.
2. Tuesday. 3. $\frac{1}{2}$. 4. 5*s*. 3*d*. ; ·0037115625.
5. 46·947177. 6. 25640000. 7. Rs. 9672.
8. £ 1, 13*s*. 4*d*. 9. £ 120. 10. £ 1350.

III.

1. 30 ; ·75. 2. $\frac{1}{184}$ and $\frac{1}{3}$. 3. Rs. 86, 10*a*. 8*p*.; 266·6̇.
4. Thursday, 7th February 1833.
5. 999984. 100149, 753. 6. 6 pies in a Re.
7. $33\frac{1}{3}$. 8. 320. 9. 4 years. 10. £ 15400.

IV.

1. 2*s*. 7*d*. 2. ·08125, ·000̇3, ·0̇39896̇1̇. 3. 8.
4. 160 yards. 5. 80. 6. ·491824. 7. 104.
8. 49 yds. 9. £ 3, 7*s*. 9*d*. ; 364 of each.
10. Decrease Rs. 1000.

V.

1. $1\frac{1}{4}$. 2. £ 50. 12*s*. $6\frac{7}{13}d$. 3. ·03.
4. $611\frac{1}{9}$. 5. $\frac{3}{8}$ gallon. 6. 5 p. c.
7. 96000 yds. 8. $26\frac{6}{19}$. 9. Loss 80 p. c. 10. £ 31, 5*s*.

VI.

1. 101249·7. 2. 105570 ; 950370. 3. £ 1.
4. 1*a*. 4*p*. ; ·114583̇. 5. Loss $16\frac{2}{3}$ p. c. 6. £ 1, 14*s*.
7. 180 ; 160 ; 200. 8. £·5565. 9. $2\frac{3}{4}$ miles.
10. Rs. 39440 ; Rs. 1560.

VII.

1. Men Rs. 72, 14*a*. $2\frac{2}{3}p$. ; Women Rs. 85, 5*a*. 4*p*.
Boys Rs. 81, 12*a*. $5\frac{1}{3}d$.
2. ·0725. 3. 20150. 4. 10*a*.
5. Rs. 510. 6. $16\frac{1}{2}$ hours. 7. Rs. 80.
8. 19*s*. 3*d*. 9. 8*s*. 4*d*. 10. $3\frac{1}{8}$ p. c.

VIII.

1. 720. 2. Rs. 12500. 3. $\frac{1}{16}$. 4. $\frac{5}{19}$.
5. 0. 6. Rs. 15000. 7. £ 690. 8. 6 months.
9. 3. 10. £ 13800.

IX.

1. 5. 2. Rs. 60. 3. 2 seconds. 4. $\frac{3}{4}d$., 20.
5. 112428. 6. 65 p. c. 7. Rs. 640. 8. £ 4800.
9. 5*a*. 4*p*. 10. 54.

X.

1. 0 ; 565. 2. $13\frac{11}{13}$ min. and $16\frac{4}{11}$ min. past 3.
3. $1\frac{1}{2}$ ft. 4. 12. 5. *B* ; *A* will be 80 yds. off.
6. *A* 120 sec., *B* 132 sec. 7. Three. 8. 4·075.
9. 4 of inferior with 5 of superior. 10. £ 2387.

PUNJAB UNIVERSITY.

1942.

1. (*a*) ·007 (*b*) 99 2. Rs. 625, 11*a*. 6*p*. 3. Rs. 1, 13*a*. 3*p*.
4. £ 8 increase. 5. £ 4. 6. Wednesday. 7. Rs. 981.
8. *A* Rs. 1875, *B* Rs. 1125, *C* Rs. 2625.

1943.

1. (*a*) ·0025 (*b*) ·19 2. Rs. 432, 4*a*.
3. $2\frac{1}{2}$ in. 4. $56\frac{1}{4}\%$ 5. 6 months.
6. 5 days. 7. Rs. 171, 14*a*. 8 Rs. 518, 7*a*.

1944.

1. (*a*) 17 (*b*) $1\frac{1}{24}$. 2. Rs. 709, 12*a*. 3*p*. 3. 225.
4. Rs. 1050 ; Rs. 36, 12*a*. 5. 6 yds.
6. Rs. 208, 10*a*. 2*p*. 7. 1020. 8. $1\frac{1}{2}$ ft. 9. Sunday.

1945.

1. (*a*) 4983300 (*b*) 8825. 2. $\frac{1}{12}$. 8. Rs. 1159, 2*a*. $10\frac{1}{2}$*p*.
4. $37\frac{1}{2}\%$ 5. Rs. 9. 6. 7 months.
7. 15 days. 8. Rs. 320 or Rs. 9 increased income.
9. Rs. 82, 8*a*. 10. Friday.

1946.

1. (*a*) $1\frac{1}{3}$. (*b*) ·09375. 2. (*a*) Rs. 17687, 9*a*. 4*p*.
(*b*) 16 for a rupee. 3. (*a*) Rs. 18. (*b*) Rs. 175, 140, 210.
4. (*a*) Rs. 715. (*b*) Rs. 450. 5, (*a*) Rs. 24000, Rs. 190 decrease. (*b*) 1 md. 5 sr. 8 ch. 6. (*a*) 11 men. (*b*) $117\frac{1}{3}$ ft.

1947.

1. (*a*) $1\frac{5}{12}$*s*. (*b*) Rs. 78, 2*a*.
2. (*a*) $8\frac{1}{2}\%$ (*b*) Rs. 1020, 1040, 780.
3. (*a*) £ 2330. (*b*) Rs. 2812, 8*a*.
4. (*a* 200 sq. ft. (*b*) Rs, 10088, 13*a*. 9*p*.
5. (*a*) $\frac{2}{3}$. (*b*) 11664.
6. (*a*) Sunday. (*b*) Rs. 36600.

1948.

1. (*a*) 2 (*b*) ·007
2. (*a*) Rs. 495, 6*a*. $11\frac{1}{2}$*p*.
(*b*) G. C. M. $= \frac{5}{12}$ L. C. M. $= \frac{1320}{7}$.
3. (*a*) Rs. 1550, 1550, 1240 (*b*) £ 3125 ; 4%.
4. (*a*) $12\frac{1}{2}\%$ (*b*) 30 days. 5. (*a*) first is poorer by Rs. 4, 4*a*. (*b*) Rs. 14000.
6. (*a*) Rs. 11, 12*a*. 5*p*
(*b*) 10800.

RAJPUTANA BOARD.

1940.

1. (*a*) 850 votes. (*b*) 35. 2. (*a*) Rs. 8, 2*a*. $10\frac{10}{11}p$.
   (*b*) 4 : 5. 3. (*a*) 18 miles per hour (*b*) 2 miles.
4. (*a*) 240 sq. ft. (*b*) ·706 or ·71 in.
5. (*a*) Rs. 518, 10*a*. $4\frac{2}{25}p$. (*b*) $346\frac{2478}{3087}$.
6. (*a*) 5 years. (*b*) £ 87, 10*s*.

1941.

1. (*a*) 785 miles. (*b*) $\frac{321}{361}$.
2. (*a*) ·400665. (*b*) 297 boys
3. (*a*) 10 days. (*b*) 32%.
4. (*a*) 1 year (*b*) £138.
5. (*a*) Ox Rs. 30. (*b*) Rs. 300, Rs. 200.
6. (*a*) 12 mid night on Monday (*b*) $\frac{AP}{P-q}$ miles

1942.

1. (*a*) £ 7, 10*s*. (*b*) 65°.
2. (*a*) Rs. 3, 12*a*. (*b*) 576 students ; Rs. 3.
3. (*a*) Rs. 26, 8*p*. (*b*) 1500.
4. (*a*) Rs. $9\frac{1}{4}$%. (*b*) A is gainer by Rs. 10
5. (*a*) $3\frac{1}{8}$. (*b*) Increase $7\frac{1}{10}$.
6. (*a*) $13\frac{1}{3}$. (*b*) 220 : 171.
7. (*a*) £ 56, 5*s*. (*b*) 300 at 25%, 700 at 1

1943.

1. (*a*) 29700. (*b*) Re. 1. (*c*) ·143.
2. (*a*) 391. (*b*) 37.
3. (*a*) 26 ft. (*b*) $6\frac{2}{3}$ oz. per day.
4. (*a*) Rs. 24. (*b*) £ 40400 & 2%.
5. (*a*) £ 499, 1*s*. (*b*) $2\frac{2}{3}$ seers.

1944.

1. Rs. $68\frac{2}{11}$. 2. 37044.
3. (*a*) Rs. 1120. (*b*) $3\frac{1}{5}$ ft.
4. (*a*) 25 men. (*b*) $9\frac{1}{11}$ yards.
5. (*a*) $13\frac{3}{5}$%. (*b*) Rs. 1500, Rs. 1250.